

A beacon of new physics: the Pioneer anomaly modelled as a path based speed loss driven by the externalisation of aggregate non-inertial QM energy

Paul G. ten Boom

School of Physics, University of New South Wales, Sydney NSW 2052, Australia

Email: ptb@phys.unsw.edu.au, paul.tenboom@unswalumni.com

Abstract: This treatise outlines how a non-systematic based Pioneer anomaly, with its implied violation (re: ‘low’ mass bodies only) of both general relativity’s weak equivalence principle and the Newtonian inverse-square law, can be successfully modelled. These theoretical hurdles and various awkward observational constraints, such as the low value of Pioneer 11’s anomaly pre-Saturn encounter, have (to date) not been convincingly modelled. Notwithstanding the recent trend to embrace a non-constant Sun/Earth-directed heat based explanation of this anomalous deceleration, the actual: nature, direction, and temporal and spatial variation of the Pioneer anomaly remain an open arena of research. Working backwards from the observational evidence, and rethinking: time, mass, quantum entanglement and non-locality, we hypothesise a mechanism involving a quantum mechanical energy source and a new type of ‘gravitational’ field; neither of which lie within general relativity’s domain of formulation/application. By way of a systemic conservation of energy principle, an internally inexpressible (aggregate) non-inertial energy discrepancy/uncertainty — involving a myriad of quantum (lunar/third-body residing) atomic and molecular systems moving in analog curved spacetime — is (non-locally) re-expressed externally as a (rotating) non-Euclidean spatial geometry perturbation. At a moving body each “rotating space-warp” induces sinusoidal proper acceleration and speed perturbations, as well as a path-based constant (per cycle) rate of speed shortfall relative to predictions that omit the additional effect. ‘Solutions’ of the new model may extend to: the Earth flyby anomaly, solar system related large-scale anomalies in the CMB radiation data, the nature of dark energy, and how a theory of everything unification agenda is inadvertently impeding a deeper understanding of physical reality and quantum entanglement.

Keywords: Pioneer anomaly — gravitation — time — Earth flyby anomaly — dark energy

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1 Introduction

The Pioneer 10 and 11 spacecraft, launched in 1972 and 1973 respectively, represent an ideal system to perform precision celestial mechanics experiments (Anderson, Laing, Lau, Liu, Nieto, & Turyshev 2002). In the quiescent outer solar system (and beyond), where the effects of solar radiation pressure are minimal, it was strikingly counter to expectations that the radio-metric Doppler tracking data of these spacecraft unambiguously indicated the presence of a small, anomalous, blue-shifted frequency drift — ‘uniformly’ changing with a rate of $\approx 6 \times 10^{-9}$ Hz/s. This drift (applicable to both spacecraft) has generally been interpreted as a ‘constant’ (mean) ‘inward’ deceleration of magnitude $a_P = (8.74 \pm 1.33) \times 10^{-10}$ m/s². This deviation away from expected/predicted spacecraft navigational behaviour has become known as the Pioneer anomaly.

Although the anomaly had been apparent in the data from as early as 1980, the first publication of this well deliberated “Pioneer anomaly” was Anderson, Laing, Lau, Liu, Nieto, & Turyshev (1998). Subsequently, there has been ongoing debate as to whether this anomaly is a harbinger of ‘new’ physics or merely an overlooked or unappreciated systematic effect. Throughout this debate, the latter and more conservative position has understandably (and rightfully) been favoured; with recent publications from: Francisco, Bertolami, Gil, & Páramos (2012), Rievers & Lämmerzahl (2011), Turyshev, Toth, Ellis, & Markwardt (2011), and Turyshev, Toth, Kinsella, Lee, Lok, & Ellis (2012) increasingly confident that the ‘solution’ lies in rectifying the modelling of onboard (anisotropic) heat emissions, particularly radiative momentum transfer. Over the years, this confidence has been bolstered by the failure of all conceivable alternative hypotheses to convincingly explain the anomaly.

In Section 2, as well as presenting a fuller overview of the Pioneer anomaly and its primary observational features, concerns with this conventional anisotropic heat based explanation are presented, not the least of which is the minor qualitative inconsistencies of the various models with each other. John D. Anderson, first author of the initial comprehensive study of the anomaly, remains sceptical of this approach (Kerr 2011). Supporting his stance is the disparity of these thermal-radiation/heat based hypotheses with both the general pre-2010 consensus of a long-term *constant* anomaly, and the minor role ascribed to anisotropic/non-isotropic heat effects in the initial comprehensive investigation (Anderson et al. 2002).

The main purpose of this paper is to construct a model that is able to fully describe a non-systematic based Pioneer anomaly. This is achieved by way of utilising: a process of elimination, a process of abductive reasoning (i.e. guessing/hypothesising), and inference to the best conceivable/possible explanation — that is free of preconceptions and primarily based upon *all* facets of the (awkward) observational evidence. As such, and in light of the fact that motion in the solar system has historically provided fertile ‘ground’ for scientific advancement, the (outer solar system) Pioneer anomaly is conceivably a bellwether of new physics —

as the inner solar system anomalous precession of Mercury’s perihelion once was. Herein, it is *not* the case that general relativity (GR) is (in any way) quantitatively wrong or in need of modification; rather, GR’s domain of application is seen as incomplete in that a unique supplementary source-type of non-Euclidean geometry or gravitation (in its widest sense) — that is not describable by way of GR’s formalism (and conceptual basis) — is hypothesised. Importantly, the unusual observational constraint of long-term constancy in the Pioneer anomaly contributes to this new supplementary acceleration/gravitational field being compatible with the invariance requirements of relativity. This additional/secondary ‘gravitational’ field type is seen to (perturb and) “piggyback” upon the standard gravitational field. The Pioneer anomaly arises from a ‘low’ mass moving body encountering several of these supplementary (effectively oscillatory) acceleration/gravitational fields — that coexist in superposition. Above a maximum “cut-off mass” value — noting that there are several different, distance dependent, and coexistent “cut-off mass” values — ‘high’ mass bodies such as: large asteroids and comets, moons, and planets are not additionally/anomally affected.

In broad terms, a Heisenberg uncertainty principle (“wobble room”) based extremely low energy situation is hypothesised as a root cause. This hypothesis pertains to the role of the intrinsic angular momentum of every elementary fermion particle within (the prodigious number of) atoms/molecules constituting/comprising a lunar (*third*) celestial body — that itself is part of a celestial global system. Furthermore, we take issue (at a ‘fractional’ quantum mechanical level) with general relativity’s stance that: celestial *geodesic* motion is (without exception) *always* inertial motion. The paper builds, Section by Section, a conceptually intricate physical model that mathematically is fairly straightforward. Five distinct ‘sizes’/levels of physical ‘particles’ and/or systems are important to the model; these are: (1) elementary fermion particles; (2) atoms and molecules; (3) (bulk matter) moons; (4) Sun-planet-moon systems; and (5) the universe. A hybrid mechanism, *arising* from both (a many-particle and spin based) quantum mechanical energy and curved spacetime *together*, is implied by the observational data/evidence. Thus, the conceptualisation and mathematical formalisms of general relativity and/or quantum mechanics in isolation are insufficient to apprehend and appreciate the mechanism proposed herein. Mathematically, the use of a single system Lagrangian or Hamiltonian is not viable in this ‘lunar’ *geodesic motion* based situation, that encompasses and ‘spans’ both the microscopic *and* macroscopic realms; nor is a metric (tensor) based approach appropriate/viable. A ($\Delta t \gg 0$) *process* based externalisation of (fractional and ‘internally’ inexpressible) *non-inertial* quantum mechanical spin/intrinsic angular momentum rate (i.e. energy) is involved, with this (externalisation process) dependent upon a new type of (non-local) ‘quantum’ entanglement between: the ‘overall’ non-inertial (and virtual) *quantum mechanical* spin angular momentum (per process cycle), and the (acceleration/gravitational) amplitude of the

new/secondary *macroscopic* non-Euclidean geometry (i.e. space-time curvature) field perturbation effect.

The model's development is best described as a three-stage process. Primary model development specifics and the violation of GR's equivalence principles are addressed in the second and third stages respectively. In Section 3 (the first/inceptive stage), the awkward observational constraints of: a constant long-term mean value for the (outer solar system and beyond) anomalous acceleration, as well as the temporal variation about this constant value, drive the model's origination. By way of a process of elimination, first order *constant amplitude* sinusoidal acceleration/gravitational field perturbations (in one dimension) upon the pre-existing gravitational field, that each induce a type of (perturbation based) "celestial 'simple' harmonic motion" in the spacecraft's 'translational' motion, are implied. Interestingly, the phases associated with the (four) *sinusoidal perturbations* (around their non-stationary 'equilibrium' values) — i.e. the gravitational field and a body's: (proper) distance, speed, and acceleration (responses) — are all different and offset from each other by either 90 or 180 degrees (see subsections 3.2.4 and 3.2.6). Furthermore, additional observational evidence implies the acceleration/gravitational fields are actually (in two and three dimensions) rotating curvature/deformations of spacetime, i.e. rotating "space-warps" (RSWs) of cosmological extent, and that their rotation and energy source is related to Sun-planet-moon (three-body) celestial motion. The axis of rotation of the ('thick' planar or cylindrical-like) space-warp is coexistent with the spin axis of the moon with/to which it is associated/affiliated. Both this axis, and each plane of space-warping orthogonal to it, extend to 'infinity'. Note that one side of the rotating space-warp is above, and the opposite side is below, the equilibrium curvature of spacetime produced by general relativistic gravitation, and as such the (overall) "net spacetime curvature" remains unchanged. Close examination of the Pioneer spacecrafts' diurnal and annual (post-fit) residuals facilitates and enlightens this modelling process. The units/dimensions (of the physical 'quantities') involved necessitates that these RSWs have a conjoint mass aspect (discussed in Sections 6 and 7, the third and 'formulated' stage) that accounts for: the *dispersion of energy* at increasing distances from a RSW's source 'region', and the restriction of the oscillatory perturbation motion effect to "low mass" bodies — i.e. bodies whose mass is below at least one of the individual "cut-off mass" values associated with the numerous rotating space-warps. The unsteady/oscillatory speed (and acceleration) components induced by the coexistence of several of these (superpositioned) oscillatory/perturbation fields account for: (1) the (Pioneer) anomalous deceleration as a constant rate of speed shortfall, relative to predicted/expected spacecraft motion, i.e. relative to 'steady' motion in the absence of these superpositioned supplementary (or secondary) fields; (2) the quasi-stochastic temporal variation of (both) the model's (supplementary) 'gravitational' field strength, and (Pioneer 10) spacecraft speed (and deceleration) about/around their respec-

tive mean values, as well as how these temporal variations relate to measurement noise and overall observational variation in the data; and (3) a lower value of the Pioneer 11 based anomaly's magnitude pre-Saturn encounter (cf. Pioneer 10's value at >20 AU), and its 'rapid' increase post-Saturn encounter.

Sections 4 and 5 (the model's second, formative and innovative stage) addresses: (1) the apparent incompatibility and coexistence of these rotating field deformations (or RSWs) with (special and general) relativity theory; as well as (2) how this secondary (type of) gravitational field (or non-Euclidean geometry) is 'generated'; and (3) how each rotating space-warp necessarily coexists with an appropriate initial "non-local mass" value $[m^*(r_1)]$, and a non-local mass distribution (NMD) of cosmological extent that determines the (distance/spherical-radius dependent) "cut-off mass" (particular to its associated RSW). "Non-local mass", a new concept and new physical quantity demanded by the model's use of quantum (spin) entanglement and quantum non-locality, is enacted in order to provide a "best/only fit" to the observational evidence. The Pioneer anomaly is mainly accounted for by way of five main RSWs, emanating 'from' (or rather, by way of) Jupiter's four Galilean moons and Saturn's large moon Titan. Note that the amplitude/strength of a rotating space-warp is dependent upon (both) a moon's mass, and the orbital progression angle (around the Sun) of the moon's (host) planet per lunar orbital cycle.

In the spirit of the historian and philosopher of science Thomas Kuhn (Kuhn 1970), the requirement of some kind of "new physics" (in response to a genuine anomaly) necessitates Section 4's 'non-destructive' *conceptual* (both physical and philosophical based) re-think of certain foundational features of physics, particularly: mass, space and time, energy, and a theory of everything unification agenda. Concerns of philosophers of science (and physics) regarding the physical status of general covariance in general relativity, and the non-relativistic aspects of rotation and acceleration, are explored. Combining these concerns with the issue of time's treatment in quantum mechanics as compared to time's treatment in special (and general) relativity (Albert & Galchen 2009), we promote a Vladimir Fock-like stance/attitude towards relativity (Fock 1959); in the sense that special relativity is more appropriately viewed as a theory of invariance, and general relativity as a relativistic theory of gravitation. Responding to the absence of a (classical) global/systemic perspective in relativistic theorisation, and the existence (and global/systemic implications) of non-local entanglement in quantum mechanics, leads to the proposal of a ("noumenal" cf. 'phenomenal'/observational) *supplementary* and background/hidden ontological¹ stance (or perspective) regarding space and time. By virtue of the "noumenal-

¹"Ontology" (a philosophical term and branch of philosophy) is concerned with the nature of: being, existence, and reality. Herein its usage is largely synonymous with "physical reality", but its usage additionally implies a depth of enquiry — into physical existence and reality — that reaches beyond (current and standard) 'scientific' means/methods.

phenomenal² ‘bifurcation’ arising from this ontological/physical supplementation it is shown that: this paper’s proposed (unique) second/new ‘type’ of gravitation (i.e. a second means to non-Euclidean geometry) can coexist with general relativity’s (standard) ‘coverage’ of gravitation. Further, in order to appease quantum non-locality and entanglement, this ‘complementary’ (and background/hidden) ontological stance features a (universe-encompassing) background/*hidden* systemic and digital (cf. analog) process-based approach to the “passing of time” — in the sense of a sequential arrangement of events. This allows the conservation of energy principle to be applicable on a cosmological scale, but only in the context of the model’s entanglement and non-local based circumstances.

In Section 5 (and early in Section 6) the specifics of this (‘boutique’) model/mechanism are further developed and the emphasis is decidedly quantum mechanical, specifically involving (lunar-based) *discrete* quantum *systems* (i.e. individual spin-orbit coupled atoms and molecules dominated by electromagnetic forces) — moving (along a geodesic) in *continuous/analog* curved spacetime — acquiring a non-inertial frame status *relative* to a systemic/global background inertial frame (and other unaffected solar system bodies). It is proposed that under appropriate circumstances (composite/whole) atoms and molecules in the (spin-orbit resonant) *lunar/third body* of a macroscopic three-body Sun-planet-moon celestial system — by way of their constituent fermion elementary particles and a (closed loop and path based) relative (spin-based) *geometric phase* offset — attain (or acquire) an extraordinarily small ($\sim 10^{-34} \times 10^{-6}$ via $\leq \frac{1}{2}\hbar\Delta t^{-1}$) *virtual* rate of intrinsic (spin) angular momentum offset relative to inertial (*spin*) frame conditions; i.e. $\sim 10^{-40}$ J of energy per atom or molecule, which is an *extraordinarily small amount* of atomic/molecular based energy.

Importantly, for conceptual explanatory/heuristic reasons, this virtual/‘fictitious’ (quantum mechanical) geometric spin phase offset — that entails a virtual/fractional intrinsic angular momentum offset — is hypothetically conceptualised (i.e. conceived of) as a relative spin phase *precession* ‘induced’ by the closed-*orbital* path of a moon’s *geodesic* motion; with this motion being ‘governed’ by its host planet’s curved spacetime, that (in turn) is *nested* ‘within’ the solar system’s dominant central Sun based spacetime curvature. Furthermore, (lunar based) intra-atomic/molecular *orbital* (phase and) angular momentum are *not* affected, and electromagnetic spin-orbit coupling thwarts/denies any possible actual/real spin-based response; thus ensuring a non-inertial spin configuration is acquired/attained. This virtual spin offset’s magnitude and (*externally* ‘imposed’) projection plane is common to all the elementary fermion particles within (effectively) all $\sim 10^{49}$ atoms/molecules in a large non-rigid ‘solid’ moon — with this being an *extraordinarily large number* of atoms and molecules.

²Use of the adjective ‘phenomenal’ as a noun is intentional, so as to indicate its juxtaposition alongside ‘noumenal’ (as defined/modified herein, see subsection 4.1.3), which has traditionally been used (in philosophy) as either an adjective or a noun.

In order to ensure and maintain global/universal and systemic *conservation of energy*, the (additive) total amount of this (non-decohered) internally inexpressible ‘fractional’ and ‘fictitious’ quantum *energy* — which (along with its underlying geometric phase offset) can *not* be ‘carried over’ into the following closed loop cycle — is necessarily expressed *externally* as both: a rotating acceleration/gravitational field space-perturbation/deformation or ‘rotating space-warp’, and a (conjointly existing) non-local mass distribution. Facilitating this (cyclic) externalisation process there is — for each individual Sun-planet-moon (or barycentre-planet-moon) system — a (newly proposed) non-local entanglement relationship between (the *en masse* total value of shared/common) unresolved and non-inertial spin/intrinsic angular momentum and the perturbation amplitude of the (‘gravito-quantum’) rotating space-warp (RSW).

Scientific knowledge and concepts related to this aforementioned physical mechanism/effect are: (1) a non-locality associated with fermion quantum waves (as is similarly the case with the Aharonov-Bohm effect); (2) “a [closed loop/orbit based relative] geometric phase [also known as *Berry’s phase*] associated with the motion [i.e. kinematics] of the state of the quantum system and *not* with the motion of the Hamiltonian [as was the case with the approach of Sir Michael Berry] (Anandan & Aharonov 1988, p.1864)”; (3) the inertial circumstances associated with the ‘intrinsic spin’ of elementary fermion particles (and atomic/molecular systems) within a lunar/third-body, specifically a geometric phase based exception to: “...the tendency of intrinsic spin to keep its aspect with respect to a global background inertial frame (Mashhoon & Kaiser 2006)”; (4) a new (‘reversed’) instantiation of the energy-time version of Heisenberg’s uncertainty principle (HUP), noting that Δt in this version of HUP is not/never an operator belonging to a particle — and in the model’s case it is the closed (lunar spin-orbital) loop/cycle duration ($\sim 10^6$ s); and (5) because (intrinsic angular momentum) entanglements are generated between the many-atomed/moleculed (lunar) ‘quantum system’ and its environment (i.e. the surrounding universe), quantum mechanical *energy* (cf. information) is effectively transferred to (or re-expressed in) the surroundings/environment. Finally, (6) the proposed mechanism — which also involves fermion wave self-interference — requires *quantum decoherence* to be averted; this requires/involves (model based) spin geometric phase offsets that are below a tiny (2π radian) half fermion wavelength decoherence onset (and spinor sign change) threshold. This final concept/feature is (indirectly) associated with a denial of the existence of the graviton particle — as a (quantum) decoherence ‘agent’ at least. To ensure (whole/entire moon) decoherence is not ‘triggered’, a number of rather stringent pre-conditions must be met, including: that moons be predominantly (non-rigid) solid bodies, and celestial moon-planet 1:1 (or synchronous) *spin-orbit resonance* — with the latter also referred to as: (lunar) synchronous rotation, tidal locking, and ‘phase’ lock.

Section 6 returns to a largely (conventional) physics based approach; it begins with an overview

of the model's major features³ and a brief discussion of errors. A variety of implications of the hypothesised (mathematical and conceptual) model are then outlined, primarily within the solar system but also further afield, e.g. an ecliptic plane based signature in the cosmic microwave background radiation. A surprisingly simple (qualitative) resolution of the *Earth flyby anomaly* is proposed, involving (geocentric inbound and outbound) trajectory based geometric angles in cooperation with observational ramifications arising from the influence of the model's RSWs upon spacecraft motion — with each RSW's plane of rotation ('near to' Earth) having been 'refracted' by the Earth's (~ 10 billion times stronger) gravitational field so as to be parallel with the Earth's equatorial plane. This proposal is consistent with the (quantitative) findings/model proposed by Anderson, Campbell, Ekelund, Ellis, & Jordan (2008). Subsequent to this, the (Pioneer anomaly based) model is fully quantified. Particularly relevant is an energy equality — specific to each participating Sun-planet-moon (three-body) system — involving an exact value of total (virtual) non-inertial quantum mechanical (spin) energy and the 're-expression' of this energy magnitude in the external environment as two scalar fields: (1) a *constant* (across all space) 'gravitational' *amplitude* (gravito-quantum) rotating (non-Euclidean geometry) space-warp, and a *conjointly* existing (2) 'non-local mass' distribution. The geometrical/orbital configuration and kinematics of each Sun-planet-moon system determines (in a quasi-empirical manner) the relative geometric phase offset (per closed loop/cycle), which then quantifies the efficiency with which the minimum real (i.e. maximum virtual) quantum mechanical intrinsic spin ($\frac{1}{2}\hbar$) — of all elementary (fermion) particles within a composite (lunar-based) atom or molecule — is affected per loop/cycle. Note that (for geometric reasons) the Earth's moon is not a RSW 'generator'⁴. Also determined are: the various (individual) constant space-warp/space-deformation 'gravitational-acceleration' amplitudes Δa [which are collectively/non-singularly represented as $(\Delta a)_i$] and their (attendant/conjoint) variable non-local mass distributions; the latter by way of a constant/invariance relationship involving the product of non-local mass and enclosed volume. We show/argue that each RSW's specific energy is proportional to $\frac{1}{2}(\Delta a)^2$, and that a root sum of squares (RSS) approach determines the *model's* overall (Pioneer) anomalous rate of speed shortfall (a_p). This value provides a fit well within the error range of the experimental/*observational* based value (a_P). Finally (in section 6.7), the model's ap-

peasement of the apparent violation of general relativity's equivalence principles (especially the weak equivalence principle) and the general principle of relativity are examined and discussed in considerable detail.

Section 7 utilises and applies aspects of the preceding model/mechanism to argue that the currently favoured interpretation of observations pertaining to (distant) type Ia supernovae and baryon acoustic oscillations, which implies accelerated cosmological expansion and (hence) *dark energy*, is quite conceivably misguided. It is proposed that the model's (cosmological scale) non-local mass scalar field distributions, that have been active for approximately four and a half billion years — i.e. from when the large moons (of Jupiter and Saturn) attained spin-orbit resonance 'with' their (respective) host planet — subjects (incoming) propagating electromagnetic (EM) radiation to an increasing energy field over the duration of its journey. Upon recognising/correcting for this additional (distance/radius dependent and *spherical* volume based) foreground 'effect' — that slightly increases (i.e. blueshifts) the frequency of all ('incoming' and) received EM radiation — the resultant relationship between the 'observed' redshifts and (cosmological) comoving spatial coordinates now includes a signature that matches/mimics the (additional) effect/effects attributable to dark energy's ('distance' dependent) presence. Subsequently, an *accelerating* expansion of the universe (interpretation) is no longer implied by the data/evidence. This conclusion is supported by the (presumed) onset of accelerated expansion (at $z_t \approx 0.44$) being remarkably consistent with the initial attainment of (gas giant) planet-moon spin-orbit resonance approximately 4.56 billion ('lookback') years ago. The new model's implications for the (cosmological) "flatness problem" and other features of the 'concordance model' are (necessarily) also discussed.

Section 8 summarises and discusses the paper's major findings, both qualitative and quantitative. Major quantitative results, predictions, applications, and equations of the proposed model and mechanism are presented; as well as its pertinent (primary) conceptual features, and circumstantial simplifications that greatly benefited the model's formulation. Broader implications of the model/mechanism are also reviewed and discussed. As is the case with this introductory Section, Section 8 is quite extensive. This has been deemed necessary because an investigation into an anomalous physical phenomenon (and physical reality in general) that seeks to be progressive — thereby questioning the existing and generally accepted state of affairs (within physics) — is easily misconstrued.

2 Background and discussion of the Pioneer anomaly

In this section a brief background of the Pioneer anomaly, and how it is to be addressed in this paper, is outlined. Additionally, the primary observational evidence and what constitutes a "new physics" is discussed. Other solar system motion concerns are mentioned, followed by comments on the anomaly's status.

³As such, and due to the length of this paper, the first time reader may wish to jump from the end of Section 3 to the beginning of Section 6.

⁴The 'excessive' (i.e. $> 2\pi$ rad) geometric spin phase offset associated with the Sun-Earth-Moon system's (collision-originated) orbital configuration triggers a (lunar) decoherence effect/event. This event, that encompasses every one of its atoms/molecules, renders the (whole) Moon a non-quantum/classical body. We note that the decoherence of large-scale macroscopic (celestial) bodies is qualitative, and not (purely) size/quantitatively based — as is mistakenly assumed/envisioned by some physicists (subsection 5.1.5).

2.1 Stance taken in this paper: real anomalous spacecraft motion

The Pioneer anomaly is the *difference* between the predicted behaviour of the Pioneer spacecraft and their observed or measured behaviour. Radiometric tracking and navigation of the Pioneer 10 and 11 spacecraft was performed using (electromagnetic wave based) Doppler observations. By way of a raw measurement involving a phase count⁵ divided by the count time duration, the anomaly is indicated by a steady anomalous Doppler frequency⁶ (blue-shift) drift (of magnitude $\approx 6.0 \times 10^{-9} \text{ Hz s}^{-1}$), which implies that the spacecraft speed is *less* than the speed theoretically predicted (i.e. expected). This speed offset/anomaly occurs at a constant/steady rate, which may be interpreted as a constant anomalous acceleration. Thus, as the Pioneer 10 and 11 spacecraft travel away from the inner solar system and into deep space, their speed retardation is greater than that theoretically predicted; or in other words, the speed of the spacecraft (and distance covered) is less than the predicted values.

In the Introduction the direction of the Pioneer anomaly was (somewhat) ambiguously referred to as ‘inward’. The word “inward” was used to highlight that analyses of the observational data do *not* conclusively favour a Sun-directed anomalous deceleration, to the extent of precluding either (or both) an Earth-directed or/and a (spacecraft) *path*-directed anomaly (i.e. directed along the spacecraft’s “velocity vector”).

The stance taken in this paper is that this anomalous measurement is not an observational or systematic artifact; rather, the anomaly is considered to be an indication of real anomalous spacecraft (S/C) motion and, on the whole, the observational measurements are considered to be reliable⁷. This stance is based upon an appraisal of the observational evidence that yields an “open verdict”, and the underdevelopment of models acknowledging and addressing a non-systematic based (‘real’) Pioneer anomaly.

The anomaly is well described in the literature, with the comprehensive Turyshev & Toth (2010) review paper and the brief Nieto (2007) review paper recommended. The Pioneer spacecraft yield a unique test/measurement, of a very small effect, spanning a long period of time (i.e. years) cf. tests of (general relativistic) time delays involving electromagnetic radiation propagation.

Herein it is accepted that the Pioneer anomaly represents a strange and significant non-conformity with the usual Newtonian/Einsteinian gravitational

behaviour of solar system bodies. Note that the Pioneers’ *predicted* motion (and measurement thereof) incorporates general relativity and very sophisticated modelling of the spacecrafts’ behaviour, with the anomaly itself being in excess of five orders of magnitude greater than the corrections to Newtonian gravitation associated with General Relativity at 50AU (Rathke & Izzo 2006, p.808).

At 20AU the Pioneer anomaly is approximately 0.006% of the Sun’s Newtonian gravitational value; at 40 AU it is 0.024%, and at 80AU it is 0.094%. By way of comparison, Mercury’s (inner solar system) observed precession of the perihelion is ≈ 5600 seconds of arc per century, with 43 seconds of arc per century attributed to general relativity, representing $\approx 0.774\%$ of the (5557 seconds of arc) non-relativistic value. Considering Mercury’s anomalous precession was first reported by Urbain Le Verrier in 1859, and that observational accuracy and precision has come a long way since then, we begin to appreciate the ‘significance’ of the Pioneer anomalous measurement⁸.

2.2 On heat emission as the cause of the Pioneer anomaly

The remaining alternative hypothesis⁹ to a real Pioneer anomaly is a thermal (radiation) recoil force hypothesis, arising from an anisotropic emission of waste heat generated on board the spacecraft. This waste heat is dominated by the radioisotopic thermoelectric generators (RTGs), but other internal heat sources are also relevant — particularly the electrical equipment located inside the spacecraft compartments.

The RTGs of non-Pioneer spacecraft are mounted much closer to their spacecraft’s mainframe, e.g. Cassini, and also the New Horizons space probe (*en route* to Pluto). For all spacecraft with radioisotopic thermoelectric generators the heat generated by the RTGs overwhelms the tiny (Pioneer) anomalous acceleration. Although the magnitude of the thermal radiation (or heat) from the Pioneer S/C RTGs is much larger than the anomaly itself [approximately 2200W vs. 63W (List & Mullin 2008)¹⁰], the symmetric and perpendicular to the spin axis nature of the released heat (Toth & Turyshev 2009, p.7), is herein considered to negate this factor as the primary cause — although this continues to be a subject of ongoing debate.

A web post by *Symmetry Magazine* — a joint Fermilab/SLAC publication, concerning a presentation at the American Physical Society’s April 2008 conference — reported that no more than one third of the anomaly (21W) could conceivably be heat related. In

⁵The Doppler phase difference between transmitted and received (S-band frequency) phases is counted (Turyshev & Toth 2010, p.58).

⁶The Doppler data (in and of itself) is *not* a frequency measurement, but this data does share the same unit as frequency which is cycles per second or Hertz [Hz].

⁷Certainly, the anomaly’s: error, variance, and temporal evolution remain somewhat ambiguous. This is not surprising considering the navigational accuracy of the *in situ* observations of the Pioneers is unsurpassed. The Voyager spacecraft with three-axis stabilization are much less precise navigators than the spin stabilised Pioneer 10 and 11. For a discussion of a heat basis to the anomaly see section 2.2.

⁸As the story goes: in 1994 physicist Michael Martin Nieto, who had a good knowledge of the observational accuracy of gravitational theory, almost fell off his chair when JPL’s John Anderson quantified the discrepancy in the Pioneer spacecrafts’ position and speed.

⁹Over time all other conceivable hypotheses have been gradually eliminated.

¹⁰In so much as a 63 Watt collimated beam of photons will produce an acceleration upon the S/C equivalent to the Pioneer anomaly. Turyshev & Toth (2010, p.75) determine the value to be 65W.

contrast, Bertolami, Francisco, Gil, & Páramos (2008) argued that up to two thirds (67%) of the anomaly’s magnitude may be heat related; even though the general consensus (in 2008) of a long-term constant anomaly was in conflict with their stance, because over time there is RTG radioactive decay and a (pronounced) reduction in available electrical power. Prior to 2011, the standard response to this (constancy) concern was to cite Markwardt (2002), who found that a temporal variation of the Pioneer anomaly could not (at that stage) be ruled out. As of 2011, one may now also cite Toth (2009) and the long-awaited paper of Turyshev et al. (2011): “Support for temporally varying behaviour of the Pioneer anomaly from the extended Pioneer 10 and 11 Doppler data sets”.

By way of their numerical analysis and discussion, Levy, Christophe, B  rio, M  tris, Courty, & Reynaud (2009, Section 3) provide a strong counter argument to accepting a non-constant (decaying/diminishing) anomaly; thus supporting the stance of Anderson et al. (2002, Section VIII D) — reaffirmed in a recent interview (Kerr 2011) — and our stance herein, that anisotropic heat effects upon the Pioneer spacecraft are not considered to be of great significance.

Counter to these sentiments, (most recently) the three independent and mutually supportive models of: Francisco et al. (2012), Rievers & L  mmerzahl (2011), and Turyshev et al. (2012) have claimed or strongly suggested that (thermal) radiative momentum transfer effects can *fully* account for the Pioneer anomaly. Regarding the first of these three models/papers, the comments of Rob Cook¹¹ in a Physics arXiv blog¹² highlight some (potentially grave) concerns regarding the authors’ modelling of diffusive and specular reflection effects. The other two models/papers — simply by assuming the veracity of a “known physics” approach — may well have fallen victim to the “liberties of parametrisation” afforded to the ‘maker’ of such a many-faceted model, with this complexity evident in the abstract of Rievers & L  mmerzahl (2011). Subsequently, and regardless of the fact that one or more of these models may be ‘correct’, we shall pursue our hypothesis that the Pioneer anomaly indicates a ‘real’ (and non-systematic based) phenomenon associated with some kind of ‘new’ (or overlooked) physics.

2.3 Why a temporally diminishing Pioneer anomaly is not assured

One would assume that the major shift in stance from a long accepted/established (time ‘variable’ but) temporally long-term-mean *constant* anomaly to a temporally *diminishing* (or decaying) anomaly — implied by Turyshev et al. (2011) — would be based upon a clear and decisive scientific case for embracing the new stance over the former stance. This aim of this section is to argue — by citing specific remarks in Turyshev et al. (2011) — that this latter stance has not

been decisively attained; nor has the previous stance been found wanting. Indeed, “[the] batched stochastic [method or] model [as used by Anderson et al. (2002)], ... as the model with the most estimated parameters, it is [most] likely to produce the best possible fit (p.2).”

Bearing in mind that: “the [post-fit] residuals show significant [temporal] structure (p.2)” — which is a well credentialed feature of the Pioneer anomaly — and that: the “gradually decreasing linear and exponential decay models [and the stochastic model] yield [only] *marginally* [italics added] improved fits when compared to the [steadily] constant acceleration model (pp.3-4)”; this finding is only significant in that the extended data set challenges the acceptance level of the anomaly’s long-term mean constancy. Noise and errors inherent in the data ensure that neither case prevails.

Subsequent to Anderson et al. (2002), it was common sense to expect that an extended Doppler data set would improve our understanding of the Pioneer anomaly. However, “the addition of earlier data arcs [see footnote 1, p.2], with greater occurrences of [spin axis orientation] maneuvers [and greater solar radiation pressure], did not help as much as desired (p.4).” Additionally, the standardisation of the heroically saved/retrieved older Doppler data into a common format was laden with challenges and complexity (e.g. see <http://www.newscientist.com/article/dn11304>).

Significantly, by way of the extended Pioneer 11 deep space (P11 DS) data, Turyshev et al. (2011) declare that: “We can [now] exclude an anomaly directed along the spacecraft [path or] velocity vector (p.4)”, even though the quality of the additional data is (somewhat) inferior to the latter/original data. Further, “for Pioneer 11, the rms residuals [in 3D cf. 1D] improve when considering an *unknown* [italics added] constant force, perpendicular to the spacecraft-Earth direction (p.3)”; and with regard to the possible onset (or ramping up) of the anomaly post-Saturn encounter, results for Pioneer 11’s “Saturn approach” were disappointing: $a_{P11} = (4.58 \pm 11.80) \times 10^{-10} \text{ m/s}^2$. Consequently, the possibility of a path-directed anomaly is *retained*.

2.4 Primary observational evidence

Any viable model, that assumes the Pioneer anomaly is real, must satisfy the following three primary observational constraints.

1. Only low mass bodies are affected. Planets, moons, larger comets¹³ and larger asteroids (whose mass is known) appear to not display the anomaly¹⁴ — see Iorio (2007); Whitmire & Matese (2003); Anderson et al. (2002, p.41); and Wallin, Dixon, & Page (2007, p.11).

¹³Assuming a real anomaly, Whitmire & Matese (2003) assert that cometary bodies of mass $\geq 10^{14} \text{ kg}$ ($\approx 7 \text{ km}$ diameter, assuming a comet density of $0.5 \times 10^3 \text{ kg m}^{-3}$) are *not* affected — at least between 20 and 70 AU.

¹⁴The 21st century NASA ephemerides, overseen by E. Myles Standish, includes effects from over 300 ‘larger’ asteroids. Successful modelling of the motion of larger celestial bodies does *not* require the inclusion of the Pioneer anomaly correction. At present, the masses of ‘smaller’ asteroids and comets have yet to be determined with good accuracy.

¹¹http://www.iop.org/careers/workinglife/profiles/page_50796.html

¹²<http://www.technologyreview.com/blog/arxiv/26589/>

2. General constancy (of the *mean* anomalous ‘deceleration’) and isotropy of the anomaly, at larger heliocentric distances, i.e. $r > 15\text{AU}$. The magnitude of the anomaly¹⁵ is usually expressed as: $a_P = (8.74 \pm 1.33) \times 10^{-8} \text{ cm s}^{-2}$ which equals $a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ m s}^{-2}$.
3. The pre-Saturn flyby values of the anomaly (a_P), associated with the Pioneer 11 spacecraft, are much less than the “headline” constant value — see Figure 7 in Anderson, Laing, Lau, Liu, Nieto, & Turyshev (2002).

While a variety of solutions have been proposed, no model has to date successfully addressed all three of these constraints together. Modified general relativity, e.g. Brownstein & Moffat (2006) omits constraint 1, whereas an electromagnetic wave/photon propagation effect (Mbelek, Mosquera Cuesta, Novello, & Salim 2007) omits constraint 3. Other approaches are similarly restricted; e.g. cosmological stretching of space-time, clock acceleration, dark matter, and MOND¹⁶, to name a few.

A surprising number of proposals restrict themselves simply to constraint 2. Such disregard for the subtleties of the observational evidence, by way of only addressing the headline result, is quite alarming — in that the observational evidence is paramount to any explanation¹⁷.

Assuming the observations were reliably obtained, and also not a result of systematics¹⁸, the appeasement of these three *inconvenient* constraints (together) necessitates the introduction of some form of “new physics”.

2.5 Veracity and impediments to new physics.

What is new physics? In this paper it is considered to be the application of known physical principles and concepts to a particular system and/or set of circumstances in a previously unforeseen manner.

Why new physics? The Pioneer 10 and 11 spacecraft are relatively simple¹⁹, and the quiescent²⁰ outer solar system is an ideal place to test the ‘truth’ of gravitational theory. With the permutations of the more likely explanations conceivably exhausted, by process of elimination new physics becomes a distinct option.

¹⁵It has been common practice to express the Pioneer anomaly (motion *shortfall* rate) in terms of a positive acceleration magnitude, and to use centimetres in the units.

¹⁶Modified Newtonian dynamics (Milgrom 2009).

¹⁷When the observational evidence is unprecedented, as in constraint 1, and awkward as in constraint 3, disbelief is somewhat understandable, but not strictly an objective/scientific approach — notwithstanding the success of general relativity.

¹⁸Whose causal basis is either onboard or external to the spacecraft.

¹⁹They were based somewhat upon the amazingly successful and reliable Pioneer 6, 7, 8 and 9 space probes, although Pioneer 10 and 11 are more complicated.

²⁰As far as radiative forces acting upon the spacecraft are concerned, e.g. direct solar radiation pressure. Other non-gravitational influences such as the: Poynting-Robertson, Yarkovsky, and YORP effects do not affect the Pioneer S/C.

This option may indeed be misguided, but it is the essence of science to attempt such things, especially when “the jury is out” on an anomalous phenomenon.

The alternative (approach) of seeking a missing systematic merely seeks to maintain the “status quo”. It is a conservative approach, albeit reasonable, but not at present supported by a convincing and progressive²¹ explanation.

Was it too soon (circa 2005-10) to attempt to model the anomaly using new physics? Certainly the detailed re-analysis of the full Pioneer 10 and 11 data sets (then underway) was to be insightful, especially regarding the Saturn flyby; but the formulation and presentation of a hypothesis/model need not wait for the final results of this analysis to be published. The analysis covering 1987-1998 was rigorous, and it utilised the highest quality Doppler data set.

Does existing gravitational experimental evidence deny a “real” Pioneer anomaly? Authorities such as Clifford M. Will and NASA’s E. Myles Standish²² are inclined to be sceptical. This is understandable because General Relativity (GR) has passed all tests to date²³. Further, the failure of modified GR, with its flexibility of parametrization, to appease the anomaly has led many to take refuge in a sceptical or non-committal stance. Nevertheless, the onus lies upon the sceptic to find a (quantifiable) chink in the observational evidence, once a purportedly reliable scientific analysis is completed/presented.

Additionally, very accurate assessment of low mass bodies such as spacecraft travelling at speed²⁴ in deep space, over substantial periods of time, is a situation *not* previously (or subsequently) encompassed by other gravitational experiments.

Two impediments to new physics. (1) Inadequate appreciation of the observational evidence, and (2) conceptual rigidity regarding the theoretical *interpretation* of physical observations. Regarding the latter, Rathke & Izzo (2006, p.808) correctly cite that an additional gravitational *force* is in contradiction with the planetary ephemerides²⁵ and the weak equivalence principle; but to then exclude an unforeseen contributor to spacetime curvature assumes a complete and final understanding of gravitation, *and* its interaction with quantum mechanical energy. Although apparently unlikely, something may still be missing from our understanding that intervenes in an unexpected manner. This paper seeks the reader’s open-mindedness to illustrate how such an oversight may (and can) exist.

²¹Progressive in the sense of fruitful, in that other issues/anomalies are positively affected; i.e. benefits and advancement arise — e.g. insight into the Earth flyby anomaly — as compared to just “putting the fire out”.

²²Private communication.

²³Including impressively accurate tests within the solar system. For example: Shapiro electromagnetic wave propagation delay (Cassini spacecraft), and lunar laser ranging experiments that test the strong equivalence principle.

²⁴The Pioneers exceed 10 km s^{-1} .

²⁵Further, and more recently, supported clearly at Jupiter and most likely beyond in an article by E. M. Standish (2008) titled: “Planetary and Lunar Ephemerides: testing alternate gravitational theories”.

2.6 Further concerns regarding the solar system

A brief list of concerns is presented. These may or may not be relevant to a non-systematic (Pioneer) anomaly. Nevertheless, they should be of at least peripheral interest to anyone investigating the Pioneer anomaly.

1. The Earth flyby anomaly (Anderson, Campbell, & Nieto 2007; Anderson, Campbell, Ekelund, Ellis, & Jordan 2008), involving an anomalous increase (and decrease) in kinetic energy (from a geocentric perspective.).
2. The timescale problem, involving the too rapid formation of the ice giants Uranus and Neptune — assuming the core accretion hypothesis. See Boss (2002) amongst numerous sources. Planetary migration is considered to appease this concern.
3. A less than expected number of small comets — i.e. less than 1 km in diameter. See Kuzmitcheva & Ivanov (2002, Figures 1 & 2), Francis (2005) and Zahnle, Schenk, Dones, & Levison (2004).
4. The fading problem, involving a major shortfall in the number of returning comets predicted by dynamical models of the solar system. See Levison, Morbidelli, Dones, Jedicke, Wiegert, & Bottke (2002) and Rickman (2005).
5. The (counter to expectations) drastic drop-off in the number of large objects beyond 50 AU (Malhotra, Allen, & Bernstein 2001; Luu & Jewitt 2002, p.76) — known as the “Kuiper cliff”.

Finally, note that the extrapolation of Newtonian gravitation to galaxies assumes that the solar system’s mechanics is well understood. A real and non-systematic based Pioneer anomaly, i.e. anomalous motion for low mass bodies, denies this assumption and complicates the appealing ‘invocation’ of dark matter — which has not yet been directly detected.

2.7 Author’s comments on the anomaly’s status

A model that is inconsistent with any aspect of the observational evidence is a model that is falsified by the observations themselves. Assuming a real Pioneer anomaly and the invalidity of a thermal-radiation/heat based explanation, the anomaly (conceivably) remains unexplained and in need of a suitable model. As such, the solar system effectively becomes (in a sense) a new “terra incognita” (Turyshv, Nieto, & Anderson 2007).

Sceptics of a non-systematic based and unresolved Pioneer anomaly are asked to consider the (new) model presented herein (simply) as a *contingency plan*.

The extraordinary observational evidence necessitates an extraordinary model²⁶, that will need to be:

²⁶Certainly, as the saying goes: “extraordinary claims require extraordinary evidence”. Unfortunately, there is a short to medium-term inability to improve S/C navigational accuracy and precision in the outer solar system. Nevertheless, the current observational evidence is not insufficient for attempts at an original model to be pursued. Acceptance of such a model is another thing entirely.

consistent with *all* the observational evidence, predictive, progressive, and not in conflict with physical principles, theory, and other astrophysical observations. An alternative to the anomaly’s ‘preferred’ explanation as a conjectured hidden (or “dark”) systematic is sought. Explaining the apparent violation of the (weak) Equivalence Principle (in the case of low mass bodies) is crucial.

3 First stage modelling of the Pioneer anomaly

This Section begins the piecemeal task of modelling a real Pioneer anomaly (a_p). A lowercase ‘p’ subscript is used when dealing with the model’s acceleration determination(s), as distinct from the observation based Pioneer anomaly’s standard nomenclature (a_P).

The aim of this Section is restricted to establishing a basic (S/C path-based, i.e. velocity vector based) model that has the ability to match the full suite of observational evidence, with a number of lesser observational aspects of the Pioneer analysis being introduced. A new domain of application of *known* mathematical physics (Rayleigh’s energy theorem²⁷) is introduced.

Partial theoretical appeasement of the observationally implied model is the basis of the Section following this one (i.e. Section 4), which then paves the way for further development of the model in Section 5. Full quantification, and some further conceptualisation, of the model is given in Section 6. This Section, and the three Sections to follow, alter in stages what is in need of explanation. The complexity of the model (to be presented) demands such a piecemeal approach. This Section begins with an (assumed) real a_P devoid of *any* suitable explanation, and incompatible with present theoretical constraints²⁸.

3.1 Loosening the subtle shackles of General Relativity

In this section (i.e. 3.1) the initial conceptual landscape of the model is outlined in a rough preliminary/outline form. The detail of the model/picture shall then be added slowly, step by step. Subsequently, the reader should not expect full clarity of the final product at this early/formative stage; this is because the model covers vast ground, extending into a number of different academic territories.

At this stage we simply need to appreciate that the relationship between General Relativity (GR) and Quantum Mechanics (QMs) has not been finalised. Herein, General Relativity is accepted as the correct theory of gravitation — at least for planets, moons, and larger asteroids and comets. The existence of a ‘real’ Pioneer anomaly implies that GR may be *incomplete* as a theory describing celestial motion; i.e. a

²⁷Also known as Parseval’s theorem.

²⁸Thermal recoil force may play a minor role, but (herein) its influence is considered to comprise no more than 20% of the anomalous acceleration (a_P).

minor supplementation of spacetime curvature²⁹, beyond the scope of GR (alone), may exist — for example involving (celestial) three-body systems and (celestial) spin-orbit resonance. To proceed, such a limitation needs to be provisionally accepted. Section 4 shall more fully investigate this assertion of limitation, and how the GR ‘fortress’ is structured to deny any such limitation and/or supplementation. By process of elimination, only a *system* involving some subtle aspect of quantum mechanical [internal (spin-orbit coupled) motion/momentum-based] energy, in curved spacetime, can conceivably overcome this restriction.

The invariance and general covariance³⁰ associated with Special and General Relativity (respectively) must not be violated — at least not beyond a minimum amount associated with Heisenberg’s uncertainty principle, nor above a physically relevant quantum mechanical *first* energy level. If such a (‘fractional’ quantum mechanical) effect is shared by a vast number of atoms/molecules³¹ the sub-quantum or *virtual* energy involved is not insignificant. Later, we shall see that a significant amount of quantum mechanical (QM) excess (virtual) internal energy is necessarily expressed externally (and singularly) as a large-scale rotating warp-like curvature of space. One side of this “space-warp³²” (or space-fold³³) is above, and the opposite side is below, the equilibrium curvature of spacetime, so that on average (and ‘overall’) there is no deviation from the background gravitational field’s strength.

It is well known that in GR gravitational energy cannot be localised. We shall exploit the logical loophole that when QMs is involved, there is no impediment to a (supplementary) energy associated with spacetime curvature being non-localised in some manner³⁴. In what follows, we need to clearly distinguish (and ‘divide out’) mass from gravitational acceleration and (then) mass from *specific* energy. By way of quantum mechanics (QMs), the notion of non-local (or distributed³⁵) ‘mass’ demands consideration³⁶.

In these unusual and highly restrictive supplementary circumstances, we may: (1) (fortunately) neglect high-speed *special* relativistic effects; and shall (2) give credence to the notion of “the total energy of space curvature” — but only as regards the supplementary space curvature being pursued.

²⁹In GR “spacetime curvature” is loosely used to describe non-Euclidean geometry.

³⁰Also known as diffeomorphism covariance or general invariance.

³¹Of the order of 10^{50} , in the case of large solar system moons — indeed a “large number”.

³²In the sense of aeroplane wing warping, which involves twisting in opposite directions, but involving space ‘deformation’.

³³In the geological sense of the word ‘fold’, which implicitly assumes an initial surface that is essentially level.

³⁴As Paul Davies says: “... QMs is ‘non-local’ because the state of a quantum system is spread throughout a region of space (Davies 2004, p.20).”

³⁵As is the case with force fields, energy fields, and wave phenomena.

³⁶As such, this non-local mass (occupying space) is not considered ‘matter’ *per se*.

3.2 Pioneer Anomaly as a shortfall relative to *predicted* motion

3.2.1 Introductory remarks

Observational evidence rules out any additional direct *force*, but the conceptual notion of (intrinsic) curved spacetime remains viable. Awkwardly, the three observational constraints presented in section 2.4 effectively shrink any ‘conceptual modelling space’ to zero, i.e. no conceivable model. To proceed with a (non-systematic based) “real anomaly” hypothesis, at least one unconventional extra or missing ingredient is required.

One, and possibly the only, viable way forward is to conceive of the anomaly as a (constant rate of) shortfall in predicted motion, associated with an additional and unforeseen perturbation of spacetime curvature — whose influence is restricted to lesser masses. How can this be achieved in our solar system?

Initially, we shall employ the idealisation of spacecraft (S/C) motion as a radially (i.e. outwardly) directed point mass, in conjunction with a systemic reference frame centered at the solar system barycentre³⁷. Subsequently, and in conjunction with the fact that supplementary curvature is to be determined by way of an energy magnitude³⁸, we restrict the visual conceptualisation of the analysis to curved space cf. curved spacetime.

We examine the difference between a ‘steadily’ moving mass and one additionally subjected to a supplementary (and global) undulatory “gravitational” field³⁹. It shall be seen that a difference in translation motion arises between the two cases — as “observed” from/at (or near) the solar system barycentre. Actually, the Pioneer anomaly is modelled as a simple linear superposition of a (multiple) number of these effects (subsection 3.2.13), but the bulk of this section is concerned with examining a *single* instantiation of (such) an undulatory acceleration/gravitational field.

It shall become evident that the covariant formalism of GR is unsuited to describing this situation, and cannot attain the ensuing (systemic and) barycentric result. The alternative, and simpler, point-based formalism of Eulerian (or Continuum) Mechanics, as used in fluid mechanics for example, is required. We shall now examine the kinetic energy of, and the *specific* gravitational energy affecting, a moving point mass (*at* the point mass) over a *single* oscillatory cycle.

³⁷Note that the *rate* of time (for the system as a whole) can be either as per the Earth’s surface (Terrestrial time), or as per the barycentre (Barycentric Coordinate Time).

³⁸Note that atomic/molecular mass and (a minimum) angular momentum (over a cyclic process time) are involved in this energy magnitude. The reduced Planck constant (\hbar), i.e. Dirac’s constant, shall also play a key role.

³⁹The term “gravitational” is being used in its broadest sense; thus it includes hypothesised and unforeseen additional space curvature — which may not necessarily be representable by GR’s ‘manifestation’ of non-Euclidean geometry. Later, to avoid confusion, the expression “acceleration/gravitational field” is usually preferred. The expressions “supplementary gravitational field” and “acceleration field” are occasionally used.

3.2.2 Conceptual aspects of a shortfall in motion relative to predicted motion

For a moving body, an unsteady (i.e. undulating around a mean value) acceleration/gravitational field, on top of the standard gravitational field, necessarily induces a component of unsteady motion. These newly postulated field undulations are seen to lie or “piggy-back” upon the pre-existing gravitational field, much in the manner of ocean waves upon an otherwise calm sea. Clearly, these undulations are *completely unrelated to GR’s gravitational waves*, and their influence relates especially to the long-term behaviour of (low mass) moving bodies. In what (immediately) follows, it is (a *single* idealised instantiation of) this new/supplementary field perturbation that shall only concern us — not the underlying pre-existing gravitational field.

The following shall show that from a barycentric/systemic perspective a supplementary (oscillatory) acceleration/gravitational field leads to a *redistribution* of some steady translational energy, into a *longitudinal* oscillatory motion (about a mean speed); which then detracts from the (maximum possible) translational speed of the moving body. Such a scenario results in a shortfall (per oscillation cycle), when *compared to* predictions that omit or overlook the unsteady ‘acceleration’ field aspect. Consideration is initially restricted to a moving point mass subject to a (single and pure) sinusoidal field influence. The shortfall directly *opposes the* (linear outward) *path* and/or velocity vector of the motion.

We shall see that consistency with GR’s invariance requirements necessitates the field (acceleration) undulations have constant amplitude throughout space. Subsequently, energy dispersion, with increasing radius from the undulation source, necessarily arises from a ‘new’ variable and distributed/‘non-condensed’ (non-local) mass aspect that has a precise value at all places/points in the field⁴⁰. If multiple sources of such undulations exist, then their (linear) superposition may explain the Pioneer spacecraft observations — although their sum will need to mimic a roughly constant (i.e. low variance) effect, and account for Pioneer 11’s low a_P value around 6AU (pre-Saturn flyby). These characteristics markedly constrain any prospective model.

To validate this scenario, firstly the spacecraft’s (specific) kinetic energy ‘shortfall’ needs to be quantified; and secondly, the (until recently) generally accepted “pure-constancy” of the anomaly shall need to be found wanting. The former follows and the latter is clarified in sections 3.3, 3.4 and 3.5.

⁴⁰We shall see later (in section 6.6) that the value of the supplementary field’s “non-local mass”, whose existence is related to quantum mechanical *non-locality*, is inversely proportional to the spherical volume associated with a radius from the energy source. Also of interest shall be the conditional response of “inertial spacecraft mass” — which incidentally always equals (passive and active) gravitational mass — to the field’s acceleration undulation magnitude *and* the non-local mass value ‘at’ the spacecraft. Subsequently, only bodies whose mass is below the field’s (non-local) “cut-off mass” will experience the retardation effect.

The global or systemic distribution of the additional space curvature/undulation, and the nature of the driving mechanism that generates the undulation, are established later in the paper. The reader is asked to consider that both of these are physically viable at this stage (see subsection 3.2.15 for further comment).

3.2.3 General preliminaries, idealisations, and undulation features of the model

The following *idealisation* suits the Pioneer 10 and 11 spacecraft’s motion quite well. Initially, a simple linear (one-dimensional) situation is considered. The approach is decidedly classical, even though it utilises the notion of a (deformable) space continuum. Note that the (deeper) nature of the field strength’s sinusoidal variation is elucidated in subsection 3.5.2.

1. The Pioneer 10 and 11 spacecraft are considered to be in purely linear-radial (outward, one dimensional) motion from the Sun/Barycentre⁴¹.
2. Barycentric: radius (x), speed (v), and Sun-dominated gravitational acceleration (g) are very much greater (\gg) than undulation-based perturbation magnitudes (Δx , Δv , Δg) around the mean x , v , and g — over cycle time (Δt).
3. Undulation time (Δt) is considered to be (celestial) small/short. It is of the order of a week (7 days).
4. Change in g over Δt is considered negligible — indicative of a very low gradient gravitational field/well (in the S/C’s immediate vicinity).
5. Thus, barycentric speed (v) is considered effectively constant over Δt — in the absence of a (Δg) undulation. It is of the order of 10 km s^{-1} , and as such $v \ll c$ (the speed of light).
6. The (supplementary) acceleration/gravitational field undulations are assumed to exist on a large scale, i.e. the order of or greater than solar system size.
7. Subsequently, an idealised *equivalence* can be envisaged. Over Δt the field’s undulation at the *moving* spacecraft approaches the acceleration/gravitational field’s undulation at a position *fixed* with respect to the barycentre.
8. Thus, our idealisation permits an equivalence between (supplementary) gravitational (field) undulations and an object’s physical (or proper) acceleration⁴², such that: $\Delta g = \Delta a$. From here on we use Δa to signify both.
9. Let us also distinguish between two types of speed and acceleration perturbation: a *local* variation relative to the *moving* body’s mean value, and a perturbation as measured from the (global/systemic) *inertial* and *fixed* barycentre.

⁴¹Later we shall see that S/C (and “low mass” bodies) in non-radial motions are equally affected, but line-of-sight *observations* of these non-radial motions are less than (or equal to) the actual magnitude of the (path-based) effect.

⁴²I.e. (an object’s) measurable acceleration — achievable by way of an onboard accelerometer.

Of primary interest is whether the spacecraft's barycentric mean speed (v) remains unchanged over Δt , in the presence of a (Δa) field undulation/sinusoid — that induces a ‘measurable’ speed sinusoid (magnitude Δv) at the spacecraft. In other words, by way of a cyclic field perturbation, has a barycentric speed shortfall (δv) occurred (after and) during Δt ?

3.2.4 Briefly introducing celestial simple harmonic (proper) motion

We now briefly consider the qualitative effect of the introduction of a *single* (sinusoidal) field undulation upon the motion of a mass moving in otherwise uniform motion. In general, the sign of the *field* acceleration undulation/perturbation (relative to a mean value) opposes the sign of the *spacecraft's* speed perturbation⁴³. The acceleration/gravitational field perturbation/sinusoid $a_{\text{field}} = a_{\text{field}}(t) = \Delta a \sin(\omega t - \varphi)$ induces a (proper) speed perturbation $v_{\text{proper}} = -\Delta v \sin(\omega t - \varphi)$; which in turn can be associated with a (proper) position perturbation $x_{\text{proper}} = \Delta x \cos(\omega t - \varphi)$ and an acceleration undulation $a_{\text{proper}} = -\Delta a \cos(\omega t - \varphi)$. We note that the phase of a_{proper} lags a_{field} by 90° or $\pi/2$ radians, and that the phase of v_{proper} lags a_{field} by 180° or π radians.

The phase of each of the *four* sinusoids (at a given time) is offset by 90° relative to their ‘neighbouring’ sinusoids — which (with 360° in a full cycle) is arguably an elegant ‘set’ of circumstances. Upon correcting for non-sinusoidal motions also influencing the spacecraft⁴⁴, the spacecraft moves ‘to and fro’ about a *mean position* (and mean speed and acceleration). From the perspective of a barycentric coordinate system, this mean position is in (translational) motion i.e. non-stationary. Similarly, Earth-based (inertial frame-based) simple harmonic motion neglects extraneous motions associated with its location, e.g. motion associated with the spin of the Earth on its axis.

Note that there is no restoring *force* (directed towards the mean position) associated with the (spacecraft's speed or position) undulation itself as is the case with the position undulation of a spring, pendulum

⁴³The variation of space curvature, i.e. (supplementary) acceleration/gravitational field strength, may also be thought of as a sinusoidal variation in a ‘tilt’ angle (i.e. small inclination) — around its mean or flat equilibrium condition — *encountered* by the moving spacecraft. Maximum tilt angle (representing maximum field strength) is associated with minimum spacecraft speed and vice versa. Alternatively, imagine a fast moving ice skater on a frictionless (ice) surface comprising a series of sinusoidal (small height) undulations. As they reach the top of each hill (representing maximum field strength and tilt angle) their speed is minimised, whereas at the bottom (*representing* minimum field strength and extreme negative tilt angle) their speed is maximised. Stripping out the overall mean speed leaves a sinusoidal speed perturbation/undulation, in the same manner that we have removed the mean speed (v) of the moving spacecraft to only consider the speed undulations/sinusoids (amplitude Δv) of the spacecraft.

⁴⁴Primarily the spacecraft's motion along its trajectory, and (later) also the ‘real’ monotonic (Pioneer) anomalous acceleration (i.e. anomalous rate of speed change).

or ocean wave for example. Nevertheless, the spacecraft's (position, speed, and acceleration) undulations are considered to be driven, by way of the supplementary gravitational field undulation — which also possesses an associated total energy. The (immediate) basis, rather than the (ultimate) source or nature, of this acceleration/gravitational field undulation is the “rotating space-warps” (superficially) introduced in section 3.1.

3.2.5 Agenda and (relative) undulation maximum amplitude relationships

The mathematical relationships between: the *hypothesised* acceleration/gravitational field undulation amplitude (Δa), speed undulation amplitude (Δv), translational speed shortfall per cycle (δv), and the *rate* of translational speed shortfall (δa) are to be established in a systemic reference frame. It will be eventually shown (noting that $\delta a_{\text{spacecraft}} = \delta a_{s/c}$) that:

$$\Delta a_{\text{field}}^2 = \delta a_{\text{proper}}^2 \text{ (and in idealised 1D } \Delta a_{\text{field}}^2 = \delta a_{s/c}^2 \text{)}$$

and thus — assuming we are dealing with a *linear* system — superposition of the newly proposed field's wave-like nature⁴⁵ and effect upon spacecraft motion is conceptually non-problematic⁴⁶.

To proceed we shall need to make use of the following two (*magnitude only*) amplitude relationships, that are based upon $v_{\text{proper}} = v_{\text{proper}}(t) = -\Delta v \sin(\omega t - \varphi)$ and (celestial) simple harmonic (proper) motion:

$$\Delta a = \omega \Delta v \tag{1}$$

$$\Delta v = \omega \Delta x \tag{2}$$

A constant Δa amplitude and constant ω ensure that the Δv and Δx amplitudes are also constant, and (as such) they are not functions of time.

Equation 1 has been discussed in Anderson et al. (2002, p.37 Section IX, Part C). Its form relates to Equations 1, 13 and 50 in Anderson et al. (2002). Note that Anderson et al. (2002, Equation 50) uses A_0 (cf. Δa) to denote (apparent angular) acceleration amplitude, and Δv represents the (amplitude of) change/variation in the *line-of-sight* Doppler velocity data. *Constant* angular frequency (ω) was indicative of either an Earth day based ($\omega_{d.t.}$) or an Earth year based ($\omega_{a.t.}$) angular velocity. For Anderson et al. (2002, p.38) Equation 1 was representative of “errors in any of the parameters of the spacecraft orientation with respect to the chosen reference frame”, e.g. Earth equator orientation; and as such, these *Earth-based* errors ‘generate’ undulatory (idealised to sinusoidal) diurnal and annual residuals in the Doppler data. In section 3.4 we re-access this account of the annual residual, and discuss a major concern with the *magnitude* ascribed to it.

⁴⁵Note that wave ‘height’ is somewhat analogous to the field's (spatially-constant) ‘acceleration’ amplitude/magnitude.

⁴⁶The physical validity of this superposition is further discussed in subsection 3.2.12 and elsewhere.

3.2.6 Relative undulation amplitude relationships: for two different cases

A crucial aspect of our hypothesis of a real Pioneer anomaly is that these relationships (Equations 1 and 2) are seen to also apply to: real Pioneer spacecraft (i.e. physical or proper) motion fluctuations (Δv) — *at the spacecraft*. The (constant) angular frequency (ω) is the magnitude of a (vector) angular velocity associated with an (as yet to be clearly discussed) rotational (space-warp) mechanism (see subsection 3.5.2). This mechanism produces a sinusoidal variation in the acceleration/gravitational field strength (amplitude Δa); and subsequently a sinusoidal undulation in proper motion speed (Δv), position/‘range’ (Δx) and proper acceleration (also Δa). Recall that these are undulatory/oscillatory *amplitude-only* expressions.

Indeed, the kinematics of the moving body’s sinusoidal variation/perturbation in position, speed, and acceleration — relative to a ‘comoving’ (equilibrium) reference frame/point⁴⁷ — is essentially that of (one dimensional) simple harmonic motion⁴⁸; but the kinetics and dynamics involved in the situation presented are quite different. At this stage, not even the Δv value can be unambiguously related to barycentre-based, or (with appropriate corrections) Earth-based, observations. We shall examine two different cases.

Firstly, for the no speed shortfall case ($\delta v = 0$) — designated “case 1” — we accept that in addition to:

$$\Delta g = \Delta a_{\text{field}} = \Delta a = \Delta a_{\text{proper}} \quad (3)$$

we also have — with the provision/caveat that mean speed v is removed and Δt is relatively short — that:

$$\Delta a_{\text{proper}} = \Delta a_{\text{spacecraft}} (= \Delta a_{s/c}) \quad (4)$$

Thus, we have declared that *observed* spacecraft acceleration (perturbation) amplitude $\Delta a_{s/c}$, *after* correcting for actual spacecraft (S/C) motion, is also equal in magnitude to proper acceleration amplitude Δa_{proper} . The sinusoids associated with Δa_{field} and $\Delta v_{s/c}$ are 180 degrees out of phase, with this being the primary *physical* relationship; whereas the lesser physical relationships involving sinusoidal variations/amplitudes of $\Delta x_{s/c}$ and $\Delta a_{s/c}$, relative to $\Delta v_{s/c}$, are both 90 degrees out of phase (with respect to the latter) — as is the case with standard simple harmonic motion.

In terms of ‘causation’ of events/effects we have: $\Delta a_{\text{field}} \Rightarrow \Delta v_{s/c} \Rightarrow \Delta a_{s/c}$ and $\Delta x_{s/c}$; and in terms of 90° ($\pi/2$) phase offset jumps, the sequence (reference to lagging) is ‘signified’ by: a_{field} , $a_{s/c}$, $v_{s/c}$, then $x_{s/c}$ (noting/recalling the four interrelated time dependent sinusoidal expressions given in subsection 3.2.4).

Secondly, in the case of (our hypothesised) monotonic spacecraft speed shortfall ($\delta v \neq 0$), these sinusoidal amplitude/magnitude equalities still apply, but only if the δv monotonic effect has *also* been removed; i.e. compensated for in observed $\Delta v_{s/c}$ and $\Delta a_{s/c}$ — in

addition to the (previous) removal of the mean speed of the spacecraft (over suitably short time scales). Later, we shall refer to this (speed shortfall) case as “case 2”. We note that $\delta v \ll v$.

By way of the adjustments associated with case 1 and case 2 (alternatively situation-1 and situation-2), we’ve ensured that we are simply dealing with Δa_{field} and three (celestial) simple harmonic motion amplitudes. With this restriction in place, we have: $\Delta v_{\text{proper}} = \Delta v_{s/c}$ and $\Delta x_{\text{proper}} = \Delta x_{s/c}$, as well as: $\Delta a_{\text{field}} = \Delta a = \Delta a_{\text{proper}} = \Delta a_{s/c}$. Subsequently, the various subscripts may be dropped and we may simply use the nomenclature of: Δa , Δv , Δx , and δv from here on — unless we seek to highlight a particular feature of the model.

3.2.7 Celestial (simple) harmonic motion and its influence upon average speed

We (once again) enquire: does the *average* translational speed (v) of a body change when subjected to this undulatory acceleration field effect? In (*symmetric*) GR the default answer is ‘no’. The Eulerian-like analysis of subsection 3.2.8, which examines conditions at a moving point mass, implies the purely relative (barycentre-to-S/C) relationship of Equation 1, as employed in Anderson et al. (2002) and case 1, has neglected a (system-based) shortfall in motion (δv) — as mentioned in case 2. Note that this shortfall is quantified by way of a comparison to circumstances in the *absence* of an acceleration/gravitational field undulation; i.e. it is quantified relative to predicted/expected (non-anomalous) spacecraft motion.

We shall assume that it is the spacecraft — by way of a new supplementary field — and *not* an Earth-based error [as assumed by Anderson et al. (2002)], that is responsible for both the Δv oscillation signal *and* the (rate of) speed shortfall ($\delta a = \delta v / \Delta t$). Thus, we (need to) appreciate that Equation 1 only describes the oscillation/undulatory aspect of what is actually a two pronged (overall) speed variation — as measured by an Earth-based observer or (in our idealised case) a barycentric-based observer.

In the next subsection (3.2.8) a second (and new) relationship, between field undulation amplitude (Δa) and spacecraft speed variation is proposed; with the latter involving the monotonic δv shortfall as compared to the sinusoidal/undulatory Δv variation. This (monotonic aspect) requires the S/C be considered as moving *relative to the barycentre*, which is the fixed central point of a solar-systemic reference frame. Importantly, recalling subsection 3.2.3, our point of analysis ‘attached’ to the moving body, effectively experiences the same undulation of acceleration/gravitational field as a fixed point in the vicinity — over the (short duration) cyclic undulation time Δt .

3.2.8 Rayleigh’s Power Theorem in curved space (at a point mass)

Rayleigh’s Energy Theorem is sometimes known as Parseval’s theorem, and *vice versa*. This theorem is

⁴⁷Alternatively, a frame not influenced by the undulatory gravitational/accelerational field effect.

⁴⁸Particularly the fact that the undulatory proper acceleration of the spacecraft is proportional to, and oppositely directed to, its displacement from a (non-stationary) “mean position”.

an energy conservation theorem and (for a continuous function) may be written as:

$$\int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

It says that a signal⁴⁹ contains the same amount of energy regardless of whether that energy is computed in the time domain or in the frequency domain⁵⁰.

Bracewell (2000, p.120) points out that: "...the theorem is true if [only] one of the integrals exists." and he reassures us that Fourier transforms of physically real, yet finite sinusoidal waves, do mathematically exist.

Rayleigh's theorem, by way of "the power theorem", is also applicable to the rate of *energy transfer*, or power, of a 'signal' (Bracewell 2000, p.121).

Applying this mathematical equality to a space curved into sinusoidal undulations⁵¹, and the resultant motion of a moving point mass⁵² (with a systemic/global reference frame implicit), implies:

$$\int_{-\infty}^{\infty} a^2(t) dt = \int_{-\infty}^{\infty} |v(f)|^2 df$$

One can distinguish two *physical* aspects: the moving point mass and the (sinusoidal) field acting upon it. Ignoring the steady (average/mean) power of the "signal" and restricting ourselves to the 'power' in the undulations, this mathematical coexistence relationship appears to also describe a physically relevant (coexistence) relationship — which we can alternatively think of as a (one-way) '*transfer*'/*causal* effect. Note that the (disregarded) mean acceleration field and mean speed terms are indicative of conditions arising from standard gravitational theory, i.e. general relativity.

It is hypothesised that: when a body, moving with a constant/steady translational speed, is subjected to (i.e. affected by) a *single* (constant amplitude) sinusoidal acceleration/gravitational field undulation ($\Delta a = \Delta a_{\text{field}}$), over a *solitary* cycle (time Δt), the power theorem implies:

$$\frac{1}{2} \Delta a^2 \Delta t = \frac{1}{2} \delta v^2 f \quad (5)$$

where $f = \Delta t^{-1} = \omega (2\pi)^{-1}$ (a single exact value), and the unit in (the per cycle relationship that is) Equation 5 is $[\text{m}^2/\text{s}^3]$ i.e. *specific* power. Recall that: the point of analysis is in motion within a much larger system. Note that: $\frac{1}{2} \delta v^2$ is a fixed magnitude quantity, and because negative frequency ($-f$) is non-physical it has been neglected — effectively halving the (mathematical) value of the frequency-based 'integral'.

⁴⁹The terminology describing Fourier transforms is biased towards electrical engineering interests.

⁵⁰An alternative interpretation is that: the total energy contained in a waveform $x(t)$ summed across all of time t is equal to the total energy of the waveform's Fourier transform $|X(f)|$ summed across all its frequency components f .

⁵¹By way of (for example) the (constant angular velocity) "rotating space-warps" briefly introduced in section 3.1.

⁵²We shall need to examine the role of mass more closely at a latter stage; but for now, only specific energy, acceleration and speed are considered.

Physically, we consider $\frac{1}{2} \delta v^2$ (i.e. $\frac{1}{2} \delta v_{s/c}^2$) to be indicative of the specific (kinetic) energy associated with the body/spacecraft's *unsteady* motion energy component; and $\frac{1}{2} \delta v^2$ is also seen to be indicative of a (barycentric referenced) kinetic energy 'short-fall' (per Δt cycle) relative to a fully-steady motion case. Thus, δv is seen to represent a speed short-fall (per Δt cycle), relative to (predicted) fully-steady motion — which assumes the absence of an undulatory/sinusoidal gravitational/accelerational field effect. Note that the body/spacecraft's total kinetic energy is predominantly in "steady" (i.e. non-oscillatory) motion. Rearranging Equation 5 gives:

$$\Delta e = \frac{1}{2} \Delta a^2 \Delta t^2 = \frac{1}{2} \delta v^2 \quad (6)$$

where this common specific energy aspect (Δe) of the point mass motion (is seen to) also physically quantify the *specific* ('potential') energy of the supplementary oscillatory acceleration/gravitational field (*per cycle*), i.e. the 'rotating space-warp' mentioned previously⁵³.

The compound motion⁵⁴ involved and the ensuing asymmetric resultant motion effect (δv) is beyond both: GR's conceptual and representational scope, and Newtonian gravitation's perceived content.

3.2.9 Physical interpretation of the Power Theorem (over a single cycle)

Physically, introducing the field undulation redirects some "given" steady kinetic energy⁵⁵ to unsteady energy, and thus the *initial* steady motion is incrementally decreased *each cycle* (by δv). The field has effectively transformed, by way of a redistribution, a small proportion of the body's (pre-cycle) energy of motion.

Another way of saying this is: since the equilibrium 'point' of the longitudinal motion oscillation is stationary (relative to the steady motion), the unsteady energy cannot contribute to directed translational energy. Note that *within* each (undulatory/oscillatory) cycle a partial analogy to pendulum motion is apparent, in that increasing gravitational potential energy detracts from kinetic energy and *vice versa* — with the (constant) speed oscillation amplitude (Δv) being quite distinct from the asymmetric (and monotonic) retardation effect (δv) attributed to the existence of the *full* cycle (see Figure 1). Note that, the same loss of (steady) translation speed occurs for each and *every* cycle. In brief, constant amplitude Δa gives a constant δv shortfall per (Δt) cycle (i.e. a constant δa).

It is also worth noting that for a frictionless pendulum or spring the system's *total* energy re-

⁵³Note the similar expressions for specific (field energy) $\Delta e = \frac{1}{2} \Delta t^2 \Delta a^2$ and the (specific) *total mechanical* energy of (spring-based) simple harmonic motion: $e = \frac{E}{m} = \frac{1}{2} \frac{k}{m} A^2 = \frac{1}{2} \omega^2 A^2$ — where A is the amplitude (i.e. maximum *displacement* from the equilibrium position).

⁵⁴Steady and unsteady motion, arising from standard GR based celestial motion and the new supplementary field undulations respectively.

⁵⁵Arising from rocket propulsion and a gravitational assist manoeuvre at Jupiter — in the case of Pioneer 10 cf. Jupiter and Saturn in the case of Pioneer 11.

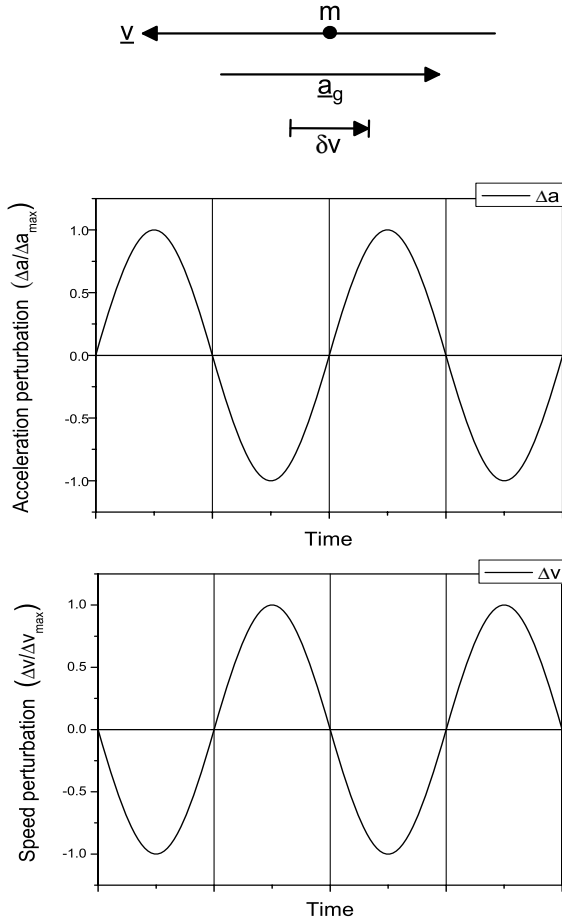


Figure 1: The vector schematic diagram (very top) shows the relative magnitude and direction of a body's motion and the speed loss (δv) (per cycle), with the (approximate) direction of the Sun's gravitational acceleration also shown — assuming the body is moving outwards from the barycentre. The (lower) two diagrams illustrate the fact that oscillatory acceleration/gravitational *field* perturbation amplitude (Δa) and the coexisting speed perturbation amplitude (Δv) of a moving body are 180 degrees out of phase. Thus, a stronger acceleration/gravitational field (relative to its average value) results in a reduced speed for the moving body — and *vice versa*.

mains constant, whereas for a spacecraft in undulatory curved space, a (simple) constant total energy notion (K.E.+P.E.) is no longer physically meaningful. From a barycentric perspective, *the system* (i.e. frictionless S/C motion) is (effectively) *dissipative*, with 'given' (predominantly steady) K.E. being incrementally eroded (per cycle) by the unsteady (energy) component of the motion. Further, Equation 6 indicates that (the perturbation based) 'specific gravitational

field energy' ($\frac{1}{2}\Delta a^2\Delta t^2$) and specific kinetic energy loss per cycle ($\frac{1}{2}\delta v^2$) have *equal* magnitudes.

3.2.10 Two different ways in which celestial simple harmonic motion occurs

Before we establish a link between systemic (translational) speed reduction per cycle (i.e. δv) and (constant) speed undulation amplitude (Δv) in subsection 3.2.11, we shall need to distinguish two quite distinct physical situations associated with this unsteady motion; which is essentially a (perturbation-based) "celestial simple harmonic motion"⁵⁶ (CSHM).

Firstly, we can have a *pseudo effect* arising from the (one-dimensional) projection of (errors associated with) uniform (Earth-based) 'circular' motion⁵⁷ onto the observed/apparent position (and motion) of a spacecraft. This results in a (pseudo) sinusoidal position variation, that upon differentiation gives the associated speed and acceleration 'sinusoidal' oscillations. The Pioneer's diurnal and annual variations are generally seen to be based upon such a scenario.

Secondly, we have a *real effect* arising from the influence of the (constant amplitude) sinusoidally varying acceleration/gravitational field associated with one of a number of (constant angular frequency) rotating space-warps (RSWs). Both situations, but more so the first situation, are largely analogous to the relationships that describe spring-based simple harmonic motion; notwithstanding the fact that Newton's second law and Hooke's law play no direct role. In Einstein's gravity there is no *force* of gravity (*per se*), only spacetime curvature. In both *situations* we retain the 'truth' that: the (perturbation) acceleration of an object is proportional to its (perturbation) displacement and is oppositely directed — which is a defining characteristic of simple harmonic motion (SHM).

In the first (pseudo) situation the trigonometric differentiation process (to speed and then acceleration 'sinusoids') has its basis in a position/displacement-based sinusoid, whereas in the (second) situation/scenario a differentiation from a (proper) speed sinusoid to (proper) acceleration sinusoid is paramount — with this (further) relying on the equivalence of Δa_{field} and Δa_{proper} magnitudes. In the second situation, which is pivotal to our explanation/modelling of the Pioneer anomaly, the integration from a speed sinusoid to a displacement 'sinusoid' (around a 'moving' mean position) is of lesser importance in the model.

With particular emphasis upon the second (RSW-based) situation, a subtlety involving (specific) energy arises — regarding its analogy to (spring-based) SHM;

⁵⁶Note that the orbital motion of planets around the Sun, and moons around a planet can be considered as a means of inducing a sinusoidal motion perturbation upon a freely moving body in deep space. This *distance dependent* amplitude scenario is negligible cf. the (similar) *constant amplitude* celestial *simple* harmonic motion being proposed.

⁵⁷For example: "The transformation from a Earth fixed coordinate system to the International Earth Rotation Service (IERS) Celestial System is a complex series of rotations that includes precession, nutation, variations in the Earth's rotation (UT1-UTC) and polar motion (Turyshv & Toth 2010, p.57)."

particularly concerning the role of terms/quantities involving rotation. Firstly, for springs, we note that: ‘A’ designates the amplitude (i.e. the maximum *displacement* from the equilibrium position), $E_{\text{tot}} = \text{constant} = \text{P.E.} + \text{K.E.} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$, $\omega^2 = \frac{k}{m}$, $x(t) = A \cos(\omega t - \varphi)$, and $v(t) = -A\omega \sin(\omega t - \varphi)$; with the *total mechanical* energy of (spring-based) SHM (per unit mass) represented by: $e = E_{\text{tot}}/m = \frac{1}{2}\frac{k}{m}A^2 = \frac{1}{2}\omega^2 A^2$. Secondly, from Equation 6 we have the expression for the specific (field energy) of a RSW: $\Delta e = \frac{1}{2}\Delta t^2 \Delta a^2$. These two energy expressions, representing different types of SHM, are somewhat similar. In the first specific energy expression, ω actually has the unit of $[\frac{\text{rad}}{\text{s}}]$ and thus the energy per unit mass of SHM has a non-standard (energy) unit of $[\frac{\text{m}^2 \text{rad}^2}{\text{s}^2}]$ — where m is metres, s is seconds, and rad is radians.

By way of comparison, (from Equation 5) specific power *per cycle* has ‘units’ (alternatively meaning “the unit of”) $[\frac{\text{m}^2}{\text{s}^3 \text{cycle}}]$; and thus the (specific) energy of a rotating space-warp (itself) has (model/theoretical) units of $[\frac{\text{m}^2}{\text{s}^2 \text{cycle}^2}]$, with duration Δt having the (non-standard) unit of $[\text{s/cycle}]$ which is the inverse of ‘unit’ frequency $[\text{cycle/s}]$ ⁵⁸. Further, we need to appreciate a distinction between: the (constant/fixed) total specific energy of the physical phenomenon that is an entire rotating space-warp, and the temporal sinusoidal influence of this RSW upon a moving mass/body. The former retains the same specific energy throughout time, whereas in the latter case the cyclic nature of the influence results in a (δv) speed shortfall *per* full (2π radian) cycle, i.e. per revolution of the rotating space-warp — with δv having units of $[\frac{\text{m}}{\text{s cycle}}]$.

This subtlety, regarding the not specifically stated radian unit in angular frequency — occurring in SHM, and our (projection of Earth-based measurement error) situation-1, by way of the trigonometric differentiation process — necessitates that when Equations 1 and 2 are applied in situation-2 — which concerns the (real/non-pseudo) celestial simple harmonic motion (of a body/spacecraft) — a unit-based modification of Equations 1 and 2 is required. In a manner of speaking, we do this so as to reconcile the (radian-based) mathematics with the (cyclic-based) physics that both describe aspects of the phenomenon.

In effect, when dealing with spacecraft (or proper) speed amplitude (i.e. $\Delta v = \Delta a/\omega$), the angular frequency needs to have a cyclic basis rather than a radian basis. Thus, we need to convert the units of speed amplitude, i.e. $[\frac{\text{m}}{\text{s}^2 \text{rad}}] = [\frac{\text{m}}{\text{s.rad}}]$ to $[\frac{\text{m}}{\text{s.cycle}}]$. This is achieved by multiplying the magnitude of Δv (per radian) by 2π . Think of converting from a number in metres per second to a number in metres per minute;

⁵⁸The National Institute of Standards and Technology (NIST) recognises the radian [rad] as a derived unit, but *not* the (unit of) cycle [cycle]. Herein, the model’s introduction and discussion of celestial simple harmonic motion, in conjunction with a cycle-based (rotating space-warp) specific energy, necessitates the introduction of this (new) derived unit; which is also (loosely) indicative of, and similar to: a (full) ‘turn’, full circle, (single) revolution, (completed) rotation, or (a single and completed) cycle.

the longer time involves a numerically larger quantity, and similarly a larger angular measure (cycles cf. radians) requires a larger numerical value for the quantity.

Thus, $\Delta v = (2\pi)\Delta a/\omega_{\text{rad}} = \Delta a/\omega_{\text{cyc}} = \Delta v_{\text{cyc}}$. Situation-2 (or case 2) also requires a similar correction to the simple harmonic motion and situation-1 relationship: $\Delta x = \Delta v/\omega$. This change in numerical *magnitude* by way of a conversion of units, yields ‘per cycle’, rather than ‘per radian’, values of Δv and Δx (as regards a moving spacecraft for example). Subsequently, these ‘adjusted’ units make the situation-2 values of Δv and Δx consistent with: the Rayleigh power theorem based determination of the physically real RSW specific energy (Δe), and particularly the associated velocity shortfall *per cycle* (δv) induced by the rotating space-warp field strength variation at a ‘point’.

Note that with Δv being necessarily a sinusoid-based ‘wave’ amplitude, and *actual* spacecraft motion (cf. a theoretical description) being described by speed $[\text{m/s}]$, the use of units $[\text{m/s}]$ for Δv , when dealing with the relationship $\Delta x = \Delta v/\omega$, is considered appropriate (and indeed necessary); whereas in the case of (non-sinusoidal/monotonic) δv , with units of $[\frac{\text{m}}{\text{s cycle}}]$, such an alteration would be inappropriate (and inaccurate at best). Similarly, sinusoidal-based $\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2$, when describing a rotating space-warp (itself), can be thought of (simply) as a specific energy $[\frac{\text{m}^2}{\text{s}^2}]$.

With Δa effectively equal to $f\Delta v$ (and Δv effectively equal to $f\Delta x$) it is (somewhat) tempting to re-write the sinusoids as $a_{s/c}(t) = -\Delta a \cos(ft - \varphi)$ and $v_{s/c}(t) = -\Delta v \sin(ft - \varphi)$ (for example); but radians play a unique role in trigonometric differentiation and in the form taken by a solution of the spring-based simple harmonic motion (second-order linear) differential equation: $m \frac{d^2 x}{dt^2} = ma = -kx$. In short, simple trigonometric derivatives and integrals only ‘work’ if the argument of sine and cosine functions are expressed in *radians*. Consequently, with regard to quantities that vary sinusoidally, this frequency-based notation, and possible consequences thereof, is neither promoted nor pursued.

Finally, we recall and note that the analogy to SHM involving the (*pseudo* cf. *real*) first situation — which involves Earth-based diurnal-spinning⁵⁹ and annual-orbiting ‘projected’ motion (errors) — does *not* require this (radians to cycles) unit correction.

To proceed we summarise the above by noting that: in (pseudo) situation-1: $\omega = \omega_{\text{rad}}$, whereas in (real) situation-2: $\omega = \omega_{\text{cyc}} = f = \Delta t^{-1}$. Also note that: $\omega_{\text{rad}} = 2\pi\omega_{\text{cyc}} = 2\pi f$, and that ω (with no subscript) is ambiguous. The relationship of acceleration amplitude to velocity amplitude is best encapsulated by:

$$\Delta a_{\text{field}} = \Delta a_{\text{proper}} = \omega \Delta v = \omega_{\text{rad}} \Delta v_{\text{rad}} = \omega_{\text{cyc}} \Delta v_{\text{cyc}}$$

3.2.11 Amplitude to shortfall relationships and an equality of specific energies

We are now in a position to establish a link between systemic translational speed reduction per cycle (δv)

⁵⁹With the observer on the Earth’s surface at a fixed radius from its centre.

and speed undulation amplitude (Δv), when it is a rotating space-warp that is causing the celestial simple harmonic motion (i.e. situation-2 as discussed in subsection 3.2.10).

As a preliminary we need to appreciate that by way of $\Delta a_{\text{field}} = \Delta a_{\text{proper}} = \Delta a_{s/c}$ (recall Equations 3 and 4 in subsection 3.2.6) Equation 1, i.e. $\Delta a = \omega \Delta v$, can also be used to describe a relationship between (relative) speed amplitude (per cycle) and acceleration perturbation/wave amplitude. Furthermore, Equations 5 and 6 describe a relationship *at* the moving point mass. This appreciation, allows us to substitute (Equation 1) into either Equation 5 or 6, and vice versa.

Using the situation-2 result obtained in subsection 3.2.10, i.e. $\Delta a = \omega_{\text{cyc}} \Delta v = f \Delta v = \Delta v / \Delta t$, which is an altered form of Equation 1, and upon substituting $\Delta a = \Delta v / \Delta t$ into Equation 6, i.e. $\Delta e = \frac{1}{2} \Delta a^2 \Delta t^2 = \frac{1}{2} \delta v^2$, we obtain:

$$\Delta v = |\delta v| \quad (\text{per cycle}) \quad (7)$$

Note that if we had simply substituted Equation 1 (situation-1) into Equation 5 or 6 the result would have been: $2\pi \Delta v = |\delta v|$; with the units of Δv (or rather Δv_{rad}) being $[\frac{\text{m}}{\text{s}} \frac{1}{\text{rad}}]$ rather than $[\frac{\text{m}}{\text{s}} \frac{1}{\text{cyc}}]$ as required by the model — which necessarily employs a (per cycle) situation-2 based definition of Δv . An alternative way of expressing this is: $2\pi \Delta v_{\text{rad}} = \Delta v_{\text{cyc}}$ with $\Delta v_{\text{cyc}} = |\delta v|$ (per cycle).

Also note that we have not assumed δv is positive, and from subsection 3.2.6 we recall that (with some caveats — particularly one dimensional radial motion): $\Delta v = \Delta v_{\text{proper}} = \Delta v_{s/c}$. Further, by way of defining:

$$\delta a = \frac{\delta v}{\Delta t} \quad (8)$$

it follows that:

$$\Delta a = |\delta a| \quad (9)$$

Note that the (model's) unit for Δv and δv is $[\frac{\text{m}}{\text{s}} \frac{1}{\text{cycle}}]$, whereas the unit for Δa and δa is $[\frac{\text{m}}{\text{s}^2}]$. Next, by way of Equations 7, 8 and 9 we confirm (re: situation-2):

$$\Delta a = \frac{\Delta v}{\Delta t} = f \Delta v = \omega_{\text{cyc}} \Delta v = \frac{\omega_{\text{rad}} \Delta v}{2\pi} \quad (10)$$

A discussion of this acceleration result is presented in subsection 3.2.12. The terminology of Equations 7 and 9 is a bit confronting, i.e. two 'deltas' (upper and lower case) in each of the two equalities, but they are indicative/representative of two different types of *change*: a sinusoidal perturbation (maximum) amplitude and a scalar (monotonic) difference.

Equation 7 indicates that: the total loss of (solar-systemic) steady speed in a *single cycle* (δv), relative to a case where only non-undulatory steady speed (v) persisted throughout the cycle time, equals the amplitude of the sinusoidal speed variation (Δv). Importantly, *the speed shortfall always opposes the direction of motion*. The idealised actual situation is: $v_{\text{final}} - v_{\text{initial}} = \delta v$ with $\delta v < 0$; and additionally $|\delta v| \ll v$ such that: $v_{\text{final}} \approx v_{\text{initial}}$.

By way of Equation 7 it follows that per cycle:

$$\frac{1}{2} \Delta v^2 = \frac{1}{2} \delta v^2$$

This result is significant in that: the *unsteady* specific energy associated with the celestial harmonic motion of an otherwise steadily moving body is *equal in magnitude* to the (monotonic) *shortfall* (cf. predictions) in the specific energy of its motion. In other words, the energy of the body's oscillatory variation in motion is equal to the (per cycle) loss in kinetic energy of the body; with this loss being relative to the kinetic energy the body would otherwise have had — in the absence of the sinusoidal field's influence. In short, and in agreement with conservation of energy, a moving body's loss of steady (translational) kinetic energy equals its gain in unsteady (oscillatory) kinetic energy. Furthermore, the two (kinetic) energy magnitudes also equal the (specific) 'potential' energy of the rotating space-warp phenomenon (itself); that is:

$$\Delta e = \frac{1}{2} \Delta a^2 \Delta t^2 = \frac{1}{2} \Delta v^2 = \frac{1}{2} \delta v^2$$

Thus, the model requires that there be *three* different (and equal) specific energy effects 'in play' at the same time (and over the same cyclic duration), as compared to spring-based simple harmonic motion with its solitary total energy equation. Finally, it is worth noting: firstly, the *constancy* of the specific energies, and the spring-based total mechanical energy per unit mass, involved in the preceding discussion; and secondly, that two very different *transfer* of energy *mechanisms* are 'active' in (this) celestial simple harmonic motion (scenario).

Beyond this section's largely accurate idealisation, in reality, a *non-constant* equilibrium speed (v) value exists because both: the Sun-based gravitational field (g), and kinetic energy redistribution (δv per cycle) act to alter the S/C's steady translational speed (v).

3.2.12 Acceleration field strength to spacecraft acceleration relationship

In subsection 3.2.11 it was established (Equation 9) that: $\Delta a = |\delta a|$.

This result can *also* be established from *situation-1 conditions*: i.e. the (angular frequency) undulatory relationship $\Delta a = \omega \Delta v$ (i.e. Equation 1), and $2\pi \Delta v = |\delta v|$ — as discussed in subsection 3.2.11. Subsequently:

$$\Delta a = \omega \frac{|\delta v|}{2\pi} = f |\delta v| = \frac{|\delta v|}{\Delta t} = |\delta a|$$

With the preceding Rayleigh (Energy and Power) Theorem based relationships being physically illucidated/explained in terms of *energy*-based equalities, and because the total energy in simple harmonic motion is proportional to amplitude squared, it is (physically) more appropriate to state this equality in terms of the square of each quantity, such that:

$$\Delta a_{\text{field}}^2 = \delta a_{\text{proper}}^2 (= \delta a_{s/c}^2 \text{ if idealised 1D radial}) \quad (11)$$

Note that the cycle time (Δt) is common to both quantities, and by using Equation 11 the specific energy equality (Equation 6) is upheld.

$$\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2 = \frac{1}{2}\delta a^2 \Delta t^2 = \frac{1}{2}\delta v^2$$

This final equality cross-check gives us confidence in the approach we have pursued, although it should be noted that the physical origin and nature of the sinusoidal Δa field undulations, that causes the spacecraft's Δv (sinusoidal) oscillations, has yet to be adequately discussed. Finally, we note (via Equation 7) that: $\Delta e \propto \Delta v^2$, in addition to $\Delta e \propto \Delta a^2$.

3.2.13 The Pioneer anomaly & commenting upon the model's use of 'mass'

The model postulates that when multiple sources for this type of supplementary (acceleration) field effect occur and coexist, the fields act together simultaneously and continuously upon a moving mass. Assuming an (additive) linear system, the superposition principle applies to both the $(\Delta a)_i$ and $(\delta a)_i$ effects⁶⁰.

For the model to fit the (Pioneer anomaly) observational evidence, a (linear) superposition of squared (shortfall) accelerations, that are representative of a superposition of energies, is (necessarily) proposed⁶¹. This leads to a total "shortfall" in expected motion over a given period of time; which is conceivably the Pioneer 10 and 11 spacecraft's (long-term average) anomalous acceleration (a_p). Thus, drawing upon Equations 6 and 11, the model proposes that:

$$\sum \frac{1}{2}(\Delta a_{\text{field}})_i^2 = \sum \frac{1}{2}(\delta a_{\text{proper}})_i^2 = \frac{1}{2}\overline{a_p(t)^2} = \frac{1}{2}a_p^2$$

which is re-expressed as:

$$a_p = \overline{a_p(t)} = \sqrt{\sum (\delta a_{\text{proper}})_i^2} = \sqrt{\sum (\Delta a_{\text{field}})_i^2} \quad (12)$$

Note that due to the clarity of context we've omitted the index of summation (i) and the finite upper bound of summation (n) from the sigma (summation) notation, such that $\sum_{i=1}^n x_i^2$ (for example) is simply expressed as $\sum x_i^2$. Note: use of the 'i' subscript outside of a summation emphasises a variable's plurality.

Equation 12 is the *crucial* result/output of the model; it asserts that (theoretically) the magnitude of the Pioneer anomaly may be conceived as a simple square root of the *summation* of the individual *field* undulation amplitudes (Δa) squared. Fortuitously, the *different* period times, of each summed field element, are included in the individual (spacecraft) acceleration terms ($\delta a = \delta v/\Delta t$). This is further discussed in subsection 6.5.5. We interpret the quantity $\frac{1}{2}(\Delta a)^2$ or

⁶⁰Such that the net response (at a given time and place) caused by multiple 'stimuli' is the sum of the 'responses' which would have been caused by each stimulus individually. The '*i*' subscript indicates that multiple instantiations of the (particular) variable exist/coexist in superposition.

⁶¹Certainly an unorthodox quantity, but in order to relate to both the sinusoidal gravitational/accelerational field *energy* affecting a body, and the (unsteady) kinetic *energy* of that moving body — over the same 'time' period (Δt) — a squared quantity is appropriate (recall Equation 6).

$\frac{1}{2}\Delta a^2$, with units $[\frac{L^2}{T^4}]$, as being representative of the rate of specific energy *transferred* per cycle duration (Δt) for a given single rotating space-warp⁶².

Correct modelling would mean this instantiation of the model's a_p value is our best match to observational a_p . This is largely the case, although further modelling of this (idealised) 'theoretical' a_p value is required. For example, a reduction arising from an offset angle between Pioneer 10's path direction, and the (observational) line-of-sight, is required (see 6.5.10). Later, in subsection 6.5.5, we obtain a 'raw' value for the Pioneer anomaly of $a_p = 8.65 \times 10^{-8} \text{ cm s}^{-2}$, which upon correction (for line-of-sight offset angle) gives a final/'best' value of: $a_p = 8.52 \pm 0.66 \times 10^{-8} \text{ cm s}^{-2}$.

Obviously, not all bodies experience this supplementary effect, because 'high mass'/larger bodies, such as planets and moons, are immune to it. Note that, at this stage, only an equality (Equation 6) concerning *specific* (gravitational/accelerational) field energy and a specific kinetic energy redistribution has been considered. The mass aspect of the body's kinetic energy is obvious, but the field's (distributed or non-condensed) mass distribution and hence energy interaction with the 'condensed' mass⁶³ body/spacecraft is yet to be fully outlined.

Clearly, the nature of mass in the model — particularly its association with the acceleration field — needs to be further considered. Later we shall see that a physical link between a quantum mechanical based energy and this new type of acceleration/'gravitational' field (expressed as an) energy is achievable. This link shall require the introduction of a new type/class of mass at the macroscopic level: "non-local mass" — see subsection 6.4.2 for a proper introduction. Note that the equivalence of *inertial* mass with (active and passive) gravitational mass ($m_i = m_g$) shall remain valid; but we further note that this equality, at the macroscopic/celestial level, has implicitly involved (i.e. been restricted to) *local* bulk matter.

3.2.14 Summary of: celestial simple harmonic motion, and the Pioneer anomaly as a motion shortfall rate

In summary, this subsection (3.2) has examined the retardation effect of a hypothesised supplementary (acceleration/gravitational) undulatory/sinusoidal field phenomenon upon a radially (outward) directed *point* mass — at the point mass, within a systemic (barycentric) reference frame. The retardation direction opposes the motion, mimicking a (flight) path-based drag. Note that this undulatory acceleration field has nothing to do with general relativity's gravitational waves, whose source are large *macroscopic* masses in accelerated motion, e.g. binary star systems.

By way of a (*single* case) 'constant' amplitude field (acceleration/gravitational) sinusoid, expressions for

⁶²Alternatively, $\frac{1}{2}\Delta a^2$ may be thought of as a specific power per cycle. It (also) equals the magnitude of the 'rate' of specific (steady) kinetic energy *loss* per cycle $[\frac{1}{2}(\delta v/\Delta t)^2]$ — from a barycentric perspective and concerning a given (single) rotating space-warp.

⁶³In the sense of 'compact' mass, i.e. solids and liquids.

spacecraft: position, speed and acceleration sinusoids were determined, as well as relationships between their (time and position independent) fixed/constant amplitudes. By way of Rayleigh’s power theorem, an equality involving the (gravitational/accelerational) sinusoid’s energy (Δe) and an unsteady specific kinetic energy is established, with the latter expression containing a motion shortfall term (δv). This motion shortfall (per sinusoidal cycle) is defined relative to predictions that overlook an unsteady component within the spacecraft’s kinetic energy — where steady (non-anomalous) motion is (exclusively) assumed. A concern with the units of angular frequency in celestial simple harmonic motion, as compared to ‘standard’ simple harmonic motion, was also discussed; this affects the magnitude of the speed/velocity amplitude (Δv). This result is especially relevant to section 3.6.

This section culminated (subsection 3.2.13) in a hypothesis/declaration that: the Pioneer anomalous acceleration is determined from a simple linear superposition of motion shortfall rates arising from (and via) *multiple* field sinusoids, (but) with the summed amplitudes having an energy-basis, rather than merely being a simple summation of the sinusoidal/undulation acceleration amplitudes ($\delta a = \delta v/\Delta t$). The contents of this section are pivotal to the explanation and model of the Pioneer anomaly pursued herein.

3.2.15 Interlude: what’s still to come

The previous subsection (3.2.14) gave a brief (overall) summary of this section. In this final subsection, as something of an interlude, we briefly describe aspects of the model that, although not thoroughly discussed as yet, are of relevance — in so much as they provide some degree of wider context to the rather specific (or narrow) aspects of the model ‘presently’ being discussed and formulated.

The (quantum mechanical) energy source of a single (acceleration/gravitational) field undulation remains inadequately explained, and the nature and global distribution of this new field (type) are yet to be established. The field’s three-dimensional (3-D) distribution might alter the conditions at the examined point mass from the preceding discussion’s ‘linear’ scenario — especially if the steady motion involved is not linear-radial, i.e. elliptical, circular, parabolic or hyperbolic motion. Fortunately, the speed retardation *phenomenon* is ubiquitous (i.e. path/direction and speed independent in the outer solar system and beyond); as compared to its line-of-sight *observation* which will vary with the (object’s) velocity vector to (observer’s) line-of-sight angle. Thus, in three dimensions: $a_{\text{proper}} \neq (a_s/c)_{\text{observed}}$.

We shall see that further observational evidence implies that a (2-D planar/3-D cylindrical) rotating space-warp, centered ‘toward’ the far distant barycentre⁶⁴, describes the supplementary *global* field distribution; thus causing a sinusoidal/undulatory field at the moving point mass. Subsequently, the type and/or

direction of a mass’s motion, has no affect upon the (path or velocity vector based) shortfall result established previously; although the same cannot be said for line-of-sight measurements (thereof). The author previously envisaged spherical divergent undulations, but this has been superseded in order to be consistent with the observational evidence (see subsection 3.5.2).

Getting a fair bit ahead of ourselves, note that: the energy of each field is seen to not exceed some ‘wiggle’ room associated with an atomic/molecular-based quantum mechanical energy uncertainty that is shared concurrently by numerous ($\sim 10^{50}$) atoms/molecules in a celestial “third-body” — this being a (‘geometrically’ suitable) spin-orbit ‘coupled’ moon. Physical concepts such as: entanglement, geometric phase, self-interference, and decoherence (or lack thereof, regarding a fractional/virtual geometric phase offset) are also involved⁶⁵. The total *virtual* excess internal energy (below a minimum change in discrete energy levels), is expressed non-locally (and externally) in the form of the hypothesised *real* supplementary acceleration/gravitational field. This quantum mechanical (virtual) energy discrepancy is indirectly induced by curved spacetime effects in a global/systemic reference frame, and it is this (yet to be fully explained) energy source that ‘drives’ each of the (real) “rotating space-warps” — that coexist (together) in superposition.

3.3 Variation in the a_P observations

In this section (and section 3.4) the periodic *temporal variation* of the Pioneer 10 anomalous acceleration values (a_P) is our central focus. We shall step back from the forward reaching aspects of subsection 3.2.15, to simply examine what the temporal variation of the observation data of the Pioneer anomaly (a_P) is indicating/telling us. As such, we seek to approach this aspect of the observational data with a fairly uncommitted mind, (thus) essentially putting to one side aspects of the model introduced and argued for in sections 3.1 and 3.2, and ‘previewed’ in subsection 3.2.15. From this (largely) unbiased perspective (and somewhat of a ‘new beginning’) we seek to support features of the model that have already been ‘introduced’.

Upon completion of this (initial) investigation of temporal variation in the a_P data, section 3.5 furthers the model’s development — or rather this Section’s preliminary version of the model. This (then) paves the way for section 3.6, in which the model’s relationship to various aspects of the Pioneer observational evidence — i.e. the anomaly itself, its temporal variation, and the magnitude of the (post-fit) residuals in the data — are more fully appreciated.

⁶⁵The reader who would balk at, or straight out deny, the ability of quantum mechanical entanglement to persist in (special circumstance) macroscopic physical systems is in need of “updating their world-view” — by way of consulting the (Scientific American) summary paper of Vedral (2011). A proposal for entanglement (to be) acting on a scale of 10^{20} atoms has gained general acceptance, as has recognition of “macroscopic entanglement in materials such as copper carboxylate at room temperature and higher (Vedral 2011, pp.41-42)” — “even though molecular jiggling might be expected to disrupt entanglement (p.39).”

⁶⁴In the case of the Pioneer 10 spacecraft beyond 40AU (approximately Pluto’s orbital radius); i.e. *far* beyond the orbital radii of Jupiter and Saturn.

3.3.1 Introduction

Turyshv, Toth, Kellogg, Lau, & Lee (2006, Section 2.1) state that there is a spatial and temporal variation of the order of 10% for each spacecraft. Turyshv et al. (2006), Olsen (2007), and Levy et al. (2009, Section 3) all stress the *constancy* of the anomaly; (with the first two of these latter citations) implicitly assuming that without the noise of observation, which includes Earth-based diurnal and annual residuals, the data would be free of spatial and (short-term) temporal variance. This is a reasonable assumption, but nevertheless it is an interpretation of the observational data. In this, and the following subsection (3.4), the validity of this “latent constancy” (or “latent steadiness”) interpretation is scrutinised and found wanting — a view somewhat supported by Levy et al. (2009) and now endorsed by Turyshv & Toth (2010).

3.3.2 Spatial variation of a_P

The different average values for Pioneer 10 and 11 may or may not indicate spatial variation of a_P . At the end of Section VI in Anderson et al. (2002, p.25 and reiterated p.34) the experimental (excluding total bias) magnitudes are given as $(7.84 \text{ and } 8.55) \times 10^{-8} \text{ cm s}^{-2}$ respectively. Recalling the total error for the anomaly is $\pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$, the spatial variation of a_P is (inherently) ambiguous.

3.3.3 A comment on observational errors

The unprecedented navigational accuracy delivered by the Pioneer spacecraft Doppler data, and the lack of other similarly precise S/C, makes interpretation involving all, or part, of the error data a “non-exact science”. Anderson et al. (2002, Section X) understandably lump all the experimental and systematic errors together in a least squares uncorrelated manner to obtain the total error.

Markwardt (2002, Table II) finds the RMS residuals of all the non-extreme measurements is approximately 8 mHz ($\sim 0.5 \text{ mm s}^{-1}$), and states that Anderson et al. (2002) assigned a standard error of 15 mHz to their Doppler data processing. Note that for the Pioneer spacecraft’s S-band Doppler, 1 cycle = 0.0652m (Null, Lau, Biller, & Anderson 1981), so $10 \text{ mHz} \Rightarrow 0.652 \text{ mm s}^{-1}$; and Markwardt’s analysis spans 1987-1994 whereas the Anderson et al. (2002) rigorous analysis spans 1987-1998.

Some distinction between error sources and types is useful, especially considering the magnitude of any real temporal variation in a_P , and the total error, are of similar magnitudes.

3.3.4 Temporal variation of a_P

Five types of temporal variation may be distinguished.

1. Pure bias (i.e. no temporal variation) — e.g. radio beam reaction force. Note that RTG heat reflected off the craft is a non-pure bias.
2. Monotonic. The Markwardt (2002) and Olsen (2007) analyses are inconclusive. Monotonic variation is not considered from here on.

3. Effectively stochastic (or random).

4. Effectively discontinuous — e.g. at manoeuvres, and for Pioneer 11’s Saturn flyby (Turyshv, Toth, Kellogg, Lau, & Lee 2006).

5. Periodic. This includes solar plasma effects, and the diurnal and \sim annual⁶⁶ residuals.

The annual residual’s source has been deemed “undetermined” by Markwardt (2002, p.11), and “[some-what] unmodeled” by Olsen (2007, p.397)⁶⁷. Regarding (temporal) variation of the anomaly, and attempts at modelling the anomaly, a good understanding of the ‘annual’ residual is (in all likelihood) crucial.

3.3.5 Non-specific variation in a_P

In the following case both spatial *and* temporal variation may exist. In early 1998 a fairly steeply peaked maximum acceleration value of over $10 \times 10^{-8} \text{ cm s}^{-2}$ for Pioneer 10 (Anderson et al. 2002, Figure 14) occurred. Coincidentally, at this time the (negative) path vector of the S/C is aligned with Saturn, and Jupiter makes its closest approach to the path vector — see Figure 2. Thus, the solar system’s planets (and/or moons) are *possibly* implicated in a physical model.

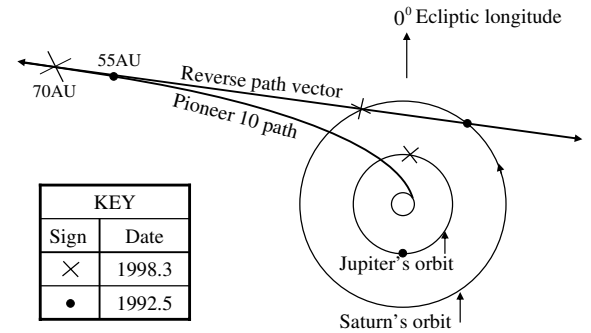


Figure 2: Schematic view down from the north ecliptic pole illustrating where a tangent from the Pioneer 10’s path (at latter times) is in relation to the orbits of Jupiter and Saturn. Saturn’s orbit crosses this line in 1992 and 1998. Jupiter’s orbit lies nearest this (negative) path vector in 1998, but in 1992 this is not the case. Coincidentally, early 1998 produced an all-time maximum in the Pioneer anomaly’s (measured) magnitude.

3.3.6 The diurnal residual

The diurnal residual is a good example of a periodic residual arising from inaccurate parameter constants, rather than observation error or a modelling

⁶⁶Markwardt’s terminology for a residual of not quite one year’s duration (Markwardt 2002).

⁶⁷To be fair, we note that Olsen does not believe there is anything troublesome or irregular in the existence of the \sim annual residual, attributing it primarily to “an artifact of the maneuver estimation algorithm”.

inadequacy⁶⁸. It is an Earth-based effect. Note that the Doppler data only measures the relative Earth to spacecraft speed, and that numerous effects — including the Earth’s movement relative to the solar system’s barycentre — are incorporated in the analysis. Thus, in *raw* data format a significant annual variation necessarily exists.

It appears the periodic nature of the diurnal residual arises primarily from *both* Earth Orientation Parameter (EOP) error and Earth Ephemeris error acting together in the Orbital Determination Program⁶⁹. Anderson et al. (2002, p.38) comment that the period is: “approximately equal to the Earth’s sidereal [spin] rotation period”, which is unfortunately vague.

The diurnal residual’s magnitude is crucial to understanding the \sim annual residual. The noise of the diurnal residual is greatest around solar conjunction, but at solar opposition, near a minimum in the solar cycle⁷⁰, exceptionally good data is available: see Anderson et al. (2002, Figure 18, p.38). Non-periodic noise is evident, arising primarily from interplanetary plasma and the Earth’s ionosphere.

Markwardt (2002) gives a figure of 10 mHz (i.e. 0.65 mm s^{-1}) for the amplitude (on average) of the Pioneers’ diurnal residuals. Anderson et al. (2002, Figure 18, p.38) indicates a Nov–Dec 1996 solar opposition amplitude of 0.1373 mm s^{-1} that may be obtained from: the amplitude relationship $\Delta a = \omega \Delta v$ (Equation 1), the diurnal period $\omega_{d.t.} = 7.292 \times 10^{-5} \text{ rad s}^{-1}$, and $\Delta a_{d.t.} = (100.1 \pm 7.9) \times 10^{-10} \text{ m s}^{-1}$ given by Anderson et al. (2002, p.38). This gives, via $\Delta v = \omega \Delta x$ (i.e. Equation 2, subsection 3.2.5), a *best-case* cyclic diurnal position offset amplitude of approximately ± 1.9 metres (i.e. $\Delta x \approx 1.9 \text{ m}$), and about 9 metres on average. This is consistent with the NEAR⁷¹ spacecraft’s range accuracy of 1 metre, and average position error of 25 metres (Elliott, Hellings, & Miller 1997).

Such a small periodic position error promotes confidence in our belief that the Pioneer spacecraft exhibit exceptional navigational accuracy and precision. The individual “beads on a string” (or catena) placement of most observational points in Figure 18 (Anderson et al. 2002)⁷² involves an accuracy of approximately 0.01 mm s^{-1} , which is ultimately related to the precise and stable frequency produced by a ground-based hydrogen maser atomic clock. With improved

clock precision a best-case Doppler S/C accuracy of 0.0001 mm s^{-1} is now potentially achievable (Asmar, Armstrong, Iess, & Tortora 2005) — although this is not achievable in practice.

3.4 The \sim annual residual

As with the diurnal residual it is the Earth, and not the spacecraft, that is generally accepted as the source of the “annual” residual. This assertion is now challenged by way of the period and amplitude observational data. This subsection shares the sentiments of footnote 125 in Anderson et al. (2002).

We thank E. Myles Standish of JPL, who encouraged us to address in greater detail the nature of the annual-diurnal terms seen in the Pioneer Doppler residuals. . . .

The fact that the Pioneer anomaly (a_P) itself is inconsistent with Standish’s non-problematic ephemerides generation⁷³ results in an inevitable and understandable tension between the Ephemeris and the (real) Pioneer anomaly camps⁷⁴ — necessitating the restriction of a hypothesised “real” anomaly to low mass bodies.

3.4.1 Period mismatch

The period as stated is not 365.25 days. Anderson et al. (2002, Section IX, Part C) gives the angular velocity (over interval III — 1992.5 to 1998.5) as $\omega_{a.t.} = (0.0177 \pm 0.0001) \text{ rad/day}$; this value equates to a cyclic period of 355 ± 2 days.

This stance is supported by Figure 2 in Scherer, Fichtner, Anderson, & Lau (1997) showing the real part of the autocorrelation function of the later Pioneer 10 data (1987–1995). By averaging (from the graph) the clear maximum and minimum range of values, at the half and full year, a period of $\sim 355 \text{ d}$ (i.e. days) is confirmed. The shape of the real part of the autocorrelation function indicates a solitary sinusoidal-like oscillation dominates the spectral aspect of a time series representing the (long-term) Pioneer Doppler data.

The difference in period is too large to be attributed to spacecraft position drift in the celestial sphere over a year.

The only coincidental period to match this is an obscure 356 day *lunar* orbital-based beat duration⁷⁵ (and associated Δa_p amplitude modulation⁷⁶) from the perspective of the Pioneer 10 S/C — see Doppler residuals (in Hz units) Anderson et al. (2002, Figures 9, 13), rather than the more processed 1-day and 5-day batch-sequential (acceleration) data of Figures 17 and 14 respectively. The heliocentric-based orbital periods of Jupiter’s moon Callisto and Saturn’s Titan are respectively: 16.689018 and 15.945421 days. Gravitational

⁷³For the larger (‘high mass’) mass bodies of the solar system at least.

⁷⁴E.g. Standish (2005, p.178).

⁷⁵A beat, in this case, is the interference between two (lunar-based) rotating space-warps of slightly different frequencies, ‘registered’ as a periodic variation in the superposition amplitude of the two $[\Delta a \sin(\omega t - \varphi)]_i$ inputs. Its rate is the difference between the two input frequencies.

⁷⁶See subsection 3.6.2 for more information on Δa_p .

⁶⁸These are the three possible causes of residuals, arising from a method of least squares analysis (Brouwer & Clemence 1961, Ch. 8).

⁶⁹This interpretation is based on correspondence (circa 2003) with E. Myles Standish of JPL, and is analogous to how celestial pole offsets arise. Celestial pole offsets are required because the model (of the Earth’s orientation), that combines precession and nutation, relies on fixed parameters for the Earth’s shape (geodesy) and internal structure; but since these are not fixed the offsets inevitably arise. Other (lesser) effects exist beyond this somewhat idealised account.

⁷⁰There was a broad minimum in the solar cycle around May 1996. The Pioneer 10 S/C was beyond 60 AU.

⁷¹Near Earth asteroid rendezvous mission. Note that this spacecraft uses (the later generation) X-band carrier frequencies whereas the Pioneer 10/11 used S-band.

⁷²Note that only one of NASA’s three Deep Space Network tracking stations is used throughout.

tidal effects ensure that both these moons are in spin-orbit resonance ‘around’ their respective host planets. *Together*, these moons also exhibit a joint spin (or rotational) resonance, in so much that their rotational (and orbital) phases — and their associated rotating space-warp phases — will coincide/resonate every 357.9 d ($m = n + 1$ where $n \approx 21.445$). As observed *from onboard* (and thus at) the Pioneer 10 spacecraft, this period (for years spanning 1992.5 and 1998.5) goes to approximately 356.1 d — by way of making a (synodic-like) correction for the motions of the host planets of these moons⁷⁷. Easy access to spacecraft positions may be found at: <http://cohoweb.gsfc.nasa.gov/helios/heli.html>. Alternatively, the more accurate “JPL’s Horizons” system may be consulted online.

Thus, if a real Pioneer anomaly exists it may possibly be related to the (larger) moons of the solar system.

3.4.2 Amplitude non-compliance

The existence of an Earth-based (exactly) annual residual is *undeniable*. Note that this amplitude is independent of the Earth’s diurnal residual amplitude.

In Anderson et al. (2002, p.38) the amplitude of the periodic Doppler frequency/speed variation (0.1053 mm s^{-1} — in interval III) is at a relative *minimum* compared with its average value between 1987 and 1998 (Turyshev et al. 1999, Figure 1). This amplitude is very similar to the diurnal residual’s (minimum) amplitude (0.1373 mm s^{-1}), but the position ‘error’ of approximately ± 530 metres⁷⁸ is quite different due to the different angular velocities, with $(\omega_{a.t.})_{\text{Earth}} = 1.991 \times 10^{-7} \text{ rad s}^{-1}$ corresponding a 365.256 day sidereal orbit period.

An easily scalable set of annual oscillation amplitude values is: $\Delta x = 1 \text{ km}$, $\Delta v = 0.2 \text{ mm s}^{-1}$ and $\Delta a = 0.4 \times 10^{-8} \text{ cm s}^{-2}$ — via Equations 1 and 2 in subsection 3.2.5, and the (observed) angular velocity value $\omega_{a.t.} = 0.0177 \text{ rad/day}$ given in subsection 3.4.1. Note that: $\omega_{a.t.} = \omega_{\sim\text{annual}} \approx 2.0 \times 10^{-7} \text{ rad/sec}$.

In the literature on the Pioneer anomaly, the \sim annual residual is usually vaguely linked to Earth orientation and position error, and/or spacecraft pointing error. To estimate the magnitude of the \sim annual residual directly is an uncertain exercise; (and) we are basically restricted to a comparative analysis involving pulsar timing measurements and ephemerides accuracy.

If the annual residual’s amplitude, arising from Earth and/or spacecraft position error, is below 100 metres (corresponding to 0.02 mm s^{-1}), then it is effectively invisible to the analysis; and the observed larger amplitude \sim annual residual may *additionally* exist, and be spacecraft based. This is distinctly likely because the Earth’s heliocentric distance, at the time of

DE405⁷⁹ was known to < 20 metres⁸⁰ (Standish 2005). We note that Earth orbit orientation uncertainties of about $\pm 1 \text{ km}^{81}$, implying extremely small angular uncertainty ($\sim \text{nrad}$) in the Pioneer line-of-sight Doppler observations, are of negligible significance.

Millisecond Pulsar timing experiments rely on, and conceivably feedback to ensure, an accurate Earth ephemerides. Their annual residuals act as a check upon the Earth’s orbit orientation accuracy. With long-term millisecond Pulsar timing measurements known to better than 300 nanoseconds ($0.3 \mu\text{s}$) (Splaver, Nice, Stairs, Lommen, & Backer 2005, Section 5.6), the Earth’s ephemeris (DE405) is known to better than ± 90 metres — a view predated by Chandler (1996).

If the aforementioned amplitude residual ($\pm 530 \text{ m}$) is Earth-based, then it is inconsistent with a lack of concern, regarding the magnitude of annual residuals, in Pulsar timing experiments. Actually, *long-term* Doppler data and millisecond Pulsar timing are not accurate enough to feedback data to improve ephemerides accuracy. It is CCD astrometric observations, spacecraft ranging data, and ΔVLBI data of orbiting spacecraft that dominate ephemeris establishment today (Standish 2004).

A comparison of DE405 with the earlier DE200 further supports this stance. Firstly, Pulsar timing data employing DE200 (Kaspi, Taylor, & Ryba 1994, Figure 5 and Section 6.1) had accuracy to $2 \mu\text{s}$ (i.e. 600m), which is nearly an order of magnitude less accurate than the use of DE405 gives. This order of magnitude improvement is confirmed by Standish’s comparison of DE200 with the later DE405 (Standish 2004). Further, Anderson et al. (2002, Section IX, p.36) discounts both Earth orientation parameters and the planetary ephemeris as possible causes of error or significant residuals. Regarding the ephemeris they say: “post-fit residuals to DE405 were virtually unchanged from those using DE200.”

We may conclude: firstly, that the existence of a true-annual residual, arising from parameter inaccuracies similar to those causing the diurnal residual does exist, but its amplitude is *below* the level of the Doppler data’s accuracy. This view is supported by the spectral analyses of Levy et al. (2009, Section 4) and Toth (2009, Section 4.4). Secondly, (we conclude) that the \sim annual residual’s unaccountably large amplitude, by process of elimination, appears to be spacecraft-based and real. Just what this ambiguous amplitude actually represents is becoming a pressing concern.

This second conclusion need not necessarily be ruled out by the aforementioned two references involving spectral analysis, because the sampling intervals are not uniform (Toth 2009, p.20), and the temporal duration of the (\sim annual) signature is quite long cf. total sample time. Further, Levy, Christophe, Métris,

⁷⁷In the 6 years of interval III (1992.5 to 1998.5) Jupiter tracks, with respect to Pioneer 10’s location and trajectory, approximately $+11^\circ$ prograde, whereas Saturn’s position remains essentially unchanged. Callisto’s prograde progression is thus $\approx 1.8^\circ$ (relative to Titan) per 357.9 d resonant cycle. This yields a shortening of the (360°) resonance cycle of $\approx 1.8 \text{ d}$. Note: ‘d’ indicates days.

⁷⁸Determined by way of the “scalable set of annual oscillation amplitude values” given in the following paragraph.

⁷⁹Developmental Ephemeris 405 is used for the Pioneer analysis 1987-1998.

⁸⁰Private communication with E. Myles Standish. The reference cited then adds support to this number. Heliocentric accuracy is ‘now’ i.e. *circa* 2006 (via DE410 and EPM2004) approximately 2 m (Pitjeva 2005, p.183).

⁸¹Standish: further email correspondence in 2004.

Bério, Courty, & Reynaud (2010, Section 5) declare that: “[their] fit⁸² indicates the presence of a significant annual term though this periodicity was not detected by the spectral analysis.” Interestingly, Levy et al. (2010, Section 4) indicate that: “[the] multiple period searching method from Van Dongen et al. as well as the SparSpec method indicate the presence of a semi-annual period which was not detected by the first three methods considered; ...” They declare “a periodicity at (177 ± 3.7) days”, which indicates an \sim annual period (roughly) within the range of 350 to 358 days.

Without our (real) model taking its first small steps, the ambiguity surrounding an explanation of the ‘annual’ signature would be inevitably sustained.

3.4.3 Average amplitude consensus, and discussion of amplitude ambiguity

Removing the constant a_P value from the Pioneer 10 analysis data leaves an \sim annual signature, which unfortunately is of the same order of magnitude as the noise/error of the data; ambiguity abounds. Figures 13 and 17 of Anderson et al. (2002) appear to favour a “stationary” amplitude, within the stochastic noise; whereas Turyshev et al. (1999, Figure 1B) imply a non-stationary (dampened) value. One recent analysis, Olsen (2007, Figures 1,4,5), favours neither and further complicates the situation by also finding apparent stochastic behaviour in the signature. Further, Olsen highlights the amplitude’s sensitivity to the (200 day) correlation time. The stochastic-like appearance of the Olsen residuals loosely supports a hypothesis of (linear) superposition of periodic effects — the dominant one being the \sim annual residual (≈ 355 days, recalling subsection 3.4.1).

Significantly, Olsen (2007, p.396) says there is no clear annual variation for Pioneer 11, which is at odds with Nieto & Anderson (2005, Section 5) and Anderson et al. (2002). Surely, an Earth-based residual should not exhibit such vagaries between analyses? This reinforces our leaning towards multiple periodic contributors, of different amplitudes, in the temporal data. Certainly, other non-periodic factors influence the data, thus “clouding” the veracity of this stance.

Fortunately, a consensus upon the *average* amplitude of the \sim annual residual, in the Pioneer 10 data, can be established.

In the month of data given by Anderson et al. (2002, Figure 18, p.38) for the diurnal residual, covering about 30° of the annual anomaly’s cyclic period, a smooth amplitude increase of about 0.1 mm s^{-1} (to a maximum) over the 30 days is evident. If this implies $[(1 - \cos 30^\circ)^{-1} = (0.134)^{-1} \approx 7.5]$ a speed sinusoid amplitude of $\sim 0.75 \text{ mm s}^{-1}$ then this roughly agrees with the Markwardt (2002) \sim annual value of 10mHz or 0.65 mm s^{-1} . Olsen (2007, Equation 13) concurs with a stated value of $1.5 \times 10^{-8} \text{ cm s}^{-2}$ (corresponding to 0.75 mm s^{-1}).

Of some concern is that the aforementioned data, especially the 1996 diurnal-data based value occur-

ring during interval III (1992-1998), exposes an ambiguity with the significantly smaller average value of interval III, i.e. 0.1053 mm s^{-1} . No single reason can be given with confidence⁸³. A further mission to investigate and clarify such ambiguities is sorely needed.

Nevertheless, an average value of about 0.7 mm s^{-1} , corresponding to undulatory/oscillatory amplitudes of $1.4 \times 10^{-8} \text{ cm s}^{-2}$ and 3.5 km, seems to be the common ground between the various analyses and the figures referred to above. Note that these last two specific values only apply in situation-1 circumstances (recall subsection 3.2.10); i.e. an oscillatory Earth-based Doppler residual — or (alternatively) the mismodelling of solar plasma effects upon the spacecraft’s ‘radio’ signal (Turyshev & Toth 2010, p.125). The (situation-1) derived annual acceleration and position variations (around their equilibrium values) are *not* quantitatively representative of (situation-2) real Pioneer 10 spacecraft motion — i.e. after the anomalous shortfall effect (a_P) has been taken out. This real scenario is examined in subsection 3.6.7.

3.4.4 Concluding remarks on the \sim annual residual

Taken on their own, the period and amplitude analyses are not galvanising in their discrediting of an Earth-based residual — so as to be in favour of a spacecraft-based \sim annual residual. But taken together, and in conjunction with the accuracy of the diurnal residuals (± 1.9 metres), they demand a rethink of the (reasonably well supported) \sim annual residual and the nature of temporal a_P variation. A superposition of various undulation effects is conceivably implicated. Further discussion regarding the quantification, and the physical basis, of the \sim annual residual is presented in subsection 3.6.7.

3.5 Molding the Model

In contrast to Turyshev et al. (2006), who (among others) promoted a constant/static (magnitude) a_P behind the noise of observations, the stance of this article is to accept variance of the Pioneer anomaly data around its long-term *constant* mean as real and indicative of the mechanism underlying the anomaly. It is reassuring that this feature is now supported by Turyshev & Toth (2010, p.121). Additionally, (herein) *long-term* (anomalous acceleration) constancy — as a best fit term/result for the data — is seen to make heat merely a secondary/non-major aspect of an explanation (recall section 2.2).

3.5.1 Aspects of the mechanism

So far in this paper a number of features of a mechanism have been implied; in point form these are:

⁸³Possibly, this amplitude mitigation arises from alterations to the Kalman filter estimation of the data, in conjunction with occasional upgrades to the Deep Space Network System that monitors spacecraft. Why? In order to appease the awkwardly large amplitude of a (wrongly) perceived Earth-based residual.

⁸²Albeit involving a true-annual periodicity.

1. Compound (i.e. steady *and* unsteady) motion in a systemic reference frame, and an Eulerian (point-based⁸⁴) analysis, was instigated.
2. Sinusoidal field undulations (amplitude Δa) lead to (sinusoidal) speed undulations (Δv) in the motion of the Pioneer 10 and 11 spacecraft.
3. The individual field undulations are (each) required to have constant amplitude. To appease GR's need for no preferred location (i.e. positional invariance), this amplitude constancy extends out to "infinity"⁸⁵.
4. For each undulation a shortfall in predicted speed occurs (δv), in conjunction with an oscillatory motion effect (Δv) around a mean value.
5. The moons of the solar system are implicated; by way of subsections: 3.3.5 (position), 3.4.1 (period 'resonance'), and also 3.5.2 (~annual phase difference between Pioneer 10 and 11).
6. Physically, the Pioneer anomaly is considered to be associated with the summation of (specific) kinetic energy ('loss' or) shortfalls $[\frac{1}{2}(\delta v)^2$ or $(\frac{1}{2}\delta v^2)]$ over their various cycle durations (Δt).
7. $a_p = \overline{a_p(t)} = \sqrt{\sum(\delta a_{\text{proper}})_i^2} = \sqrt{\sum(\Delta a_{\text{field}})_i^2}$ where $\delta a = \delta v/\Delta t$, and a_p (as compared to a_P) refers to the *model's* (theoretical) estimation of the (observed) Pioneer anomaly (a_P).
8. Multiple amplitudes and periods of the field undulations — by virtue of different moons producing different field undulations — gives the impression of both periodic and stochastic behaviour in the $a_P(t)$ data⁸⁶.

Further aspects and ramifications, of the mechanism being formulated, are developed and discussed in the following subsections.

3.5.2 Field undulations as planar rotating space-warps, and lunar comment

$a_p = \overline{a_p(t)} = \sqrt{\sum(\delta a_{\text{proper}})_i^2} = \sqrt{\sum(\Delta a_{\text{field}})_i^2}$ (i.e. Equation 12) is applicable for durations much longer than the various (Δt) oscillation periods [or (i.e.) the (Δt)_{*i*} values]. This equation indicates that acceleration/gravitational field undulations coexist with their respective rates of (spacecraft proper) motion shortfall. The nature of the field undulations, and resonances thereof, are constrained by observational evidence.

The difference in phase between the Pioneer 10 and 11 [~annual] waves is 173.2° , similar to the angular separation of the two spacecraft in ecliptic longitude (Nieto & Anderson 2005, Section 5).

⁸⁴Recalling that the solar system's scale is so vast that from a systemic (barycentric) perspective the spacecrafts' *change* in speed and position, over time Δt , is effectively negligible.

⁸⁵Infinity in the sense of *mathematical* infinity; physically, this means an extension to the 'end of the universe' — which itself is a somewhat ambiguous concept.

⁸⁶Albeit, with a fairly small mean deviation from the (constant) mean (rate of speed shortfall) value.

Solar Ecliptic Longitude – first, middle & last data points

Date	Pioneer 10	Pioneer 11
4 Jan 87	70.8	254.8
15 Nov 88		261.0
1 Oct 90		265.6
15 Sept 92	74.4	
22 July 98	76.4	
Average of 1 st & last	73.6	260.2

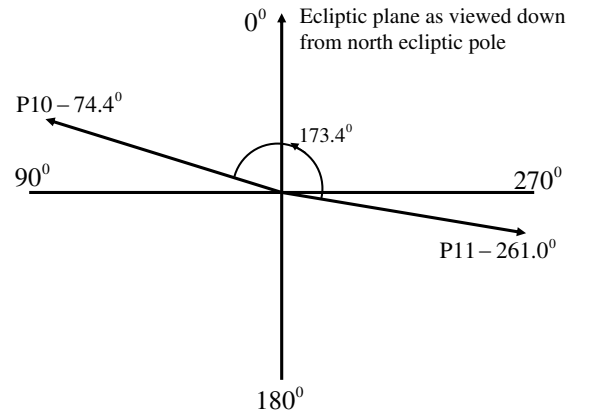


Figure 3: Schematic diagram and table showing the direction (in the ecliptic plane, relative to the barycentre) of the Pioneer 10 and 11 spacecraft for the mid-time of the Anderson et. al. data set (1987-1998). Note that the Pioneer 11 data spans only 3.75 years. The diagram's vectors only indicate *direction* (i.e. solar ecliptic longitude), and not position. The angle of 173.4 degrees is maintained when the average of the first and last data values are used. Ecliptic longitude data was acquired from <http://cohweb.gsfc.nasa.gov/helios/heli.html>

This evidence confines the (constant acceleration amplitude) field undulations to rotating space-warps; rotating at the rate, and in the directional sense, of their associated moon. The angle of 173.2° is prograde⁸⁷, stretching from Pioneer 11 to Pioneer 10 (see Figure 3). We hypothesise that this implies the (lunar 'centered') individual rotating space-warps are disk-like and aligned with (or near to) the plane of the moon's orbit around its host planet⁸⁸ (see Figure 4 to

⁸⁷So as to be compatible with Callisto and Titan's prograde spin motion/direction. These spin directions are/were established by way of: i) the planets' prograde (direction of) orbit, and ii) the formation process that (gravitationally) 'bound' these large moons to their host planets.

⁸⁸The rotating space-warp is similar to a warped rigid (vinyl) record, in that it retains its 'shape' and orientation

Figure 7). Further, spacecraft motion inclined to the space-warp's plane-of-orientation is seen to *also* receive the full motion shortfall (per moon, per cycle). For spacecraft motion, geometrically offset/inclined to the Doppler line-of-sight *observations*, a cosine angle correction is required. For Pioneer 10 and 11 (1987-1998) this correction is very minor (see subsection 6.5.8).

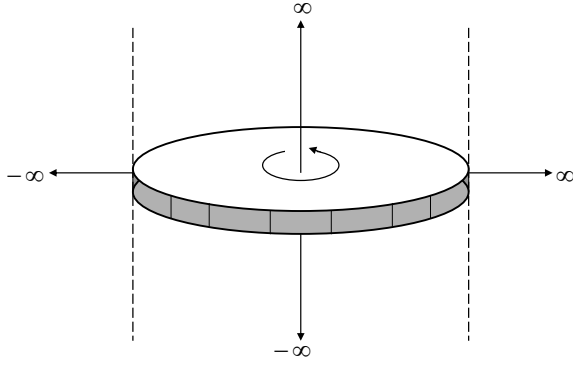


Figure 4: Schematic representation of a *non-warped* thin disk ‘of’ space, that can conceivably be curved or warped; further, this warp (or perturbation) can conceivably rotate. This planar disk extends to the ‘end’ of the universe, with this feature represented by the ‘infinity’ symbol. By way of contrast, Figures 5, 6 and 7 shall illustrate the (curved space) ‘shape’ of the (gravito-quantum) rotating space-warps proposed in this paper, for which the acceleration/gravitational perturbations (Δa) deviate from zero — i.e. deviate from the flat space scenario illustrated above. Note that space itself does not rotate; rather our concern lies with the rotation of a curvature of space — similar (in part) to both: the rotation of a warped vinyl record, and wave propagation upon an ocean.

The point based nature of the Eulerian analysis, and our conceptualisation of gravitation as curved space, favours a purely two-dimensional circumstance — which then needs to be modified to suit three-dimensional space (see subsection 6.2.1). Previously, a (non-rotating) spherical undulation was envisaged by the author (ten Boom 2005), but a perturbed (inherently) planar or disk-like (rotating) phenomenon is demanded by the observational evidence and simple harmonic motion’s (ω -based) mathematical relationships. In three dimensions the phenomenon/mechanism is cylinder-like — although open and of ‘infinite’ extent.

Not all moon-planet-sun systems are suitably configured to “generate” a rotating space-warp⁸⁹. In Section 6 we shall see that Jupiter’s four Galilean moons and Saturn’s Titan dominate the undulated curved

throughout a rotation.

⁸⁹Suitability involves: i) lunar 1:1 spin-orbit resonance ‘around’ its host planet, and ii) a certain relationship between: lunar angular progression around the planet, and the (host) planet’s progression angle around the Sun.

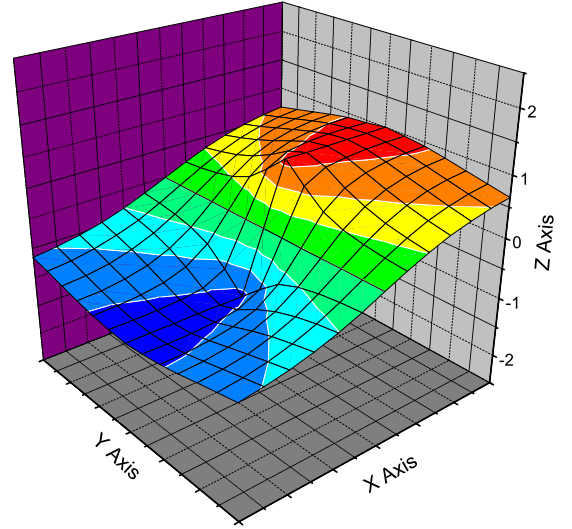


Figure 5: The first of three (‘snapshot’) views of a rotating space-warp shown in a three-dimensional grid-like representation. The Z axis represents acceleration perturbation (away) from ‘standard’ conditions, i.e. additionally curved space. Front and side elevation views (Figures 6 and 7) are preceded by this elevated view lying roughly midway between the front elevation and side elevation views. Note that (beyond the first grid line) points at a constant radius from the central point have a Z (i.e. acceleration/curvature) value that varies sinusoidally around the ‘circumference’; furthermore, the rate of (space-warp) rotation coincides with the rate of rotation and duration (Δt) of a (suitable) moon in spin-orbit ‘resonance’ around its host planet.

space landscape⁹⁰, with Earth’s moon not being a contributor/‘generator’.

Importantly, with Io, Europa, and Ganymede being in a 4:2:1 orbital resonance (with one another), their contribution to the ‘variation’ of $a_p(t)$ about its mean value $\bar{a}_p(t)$ [designated/signified as: $\Delta a_p(t)$], is conceivably one of (additional) attenuation. Hence the (356 day) Callisto-Titan cyclic/‘beat’ repetition, evidenced by the \sim annual residual and ‘sensitive’ to the use of a 200-day correlation time (see subsections 3.6.7 and 3.6.8), is *the* significant characteristic of the temporal variation of a_P (i.e. the Pioneer anomaly).

3.5.3 Anomalous CMB results and lunar spin plane rotating space-warps

Interestingly, anomalous cosmic microwave background (CMB) results — by way of the Wilkinson Microwave Anisotropy Probe (WMAP) — implicate a solar system *ecliptic plane* effect as possi-

⁹⁰The smallest of these, Europa, is “...more massive than all known moons in the Solar System smaller than itself combined [Wikipedia: *Europa* (moon), 2011-12].”

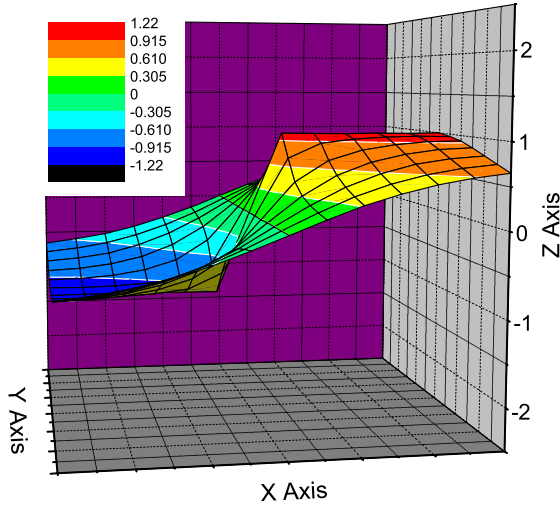


Figure 6: Side elevation view of a (gravito-quantum) rotating space-warp. Note that the colour scale merely illustrates changes in the contour colours on the diagram, with maximum and minimum amplitudes occurring at 1.00 and -1.00 respectively.

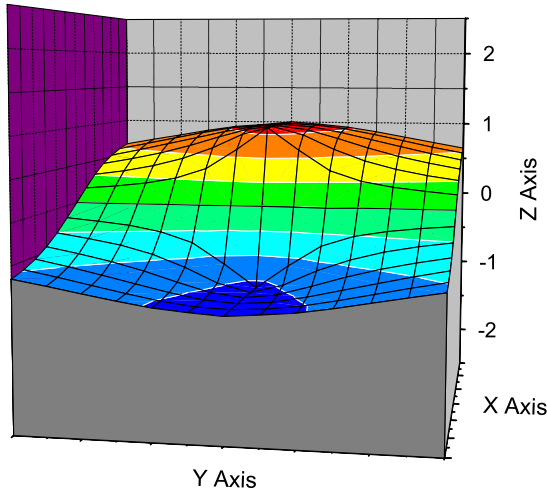


Figure 7: Front elevation view of a (gravito-quantum) rotating space-warp (RSW). Unfortunately, the actual (planar) rotational motion around the central point cannot be shown in this fixed/still image.

bly influencing the otherwise isotropic CMB radiation data (Copi, Huterer, Schwarz, & Starkman 2007; de Oliveira-Costa, Tegmark, Zaldarriaga, & Hamilton 2004; Huterer 2006). For a recent review article

see Copi, Huterer, Schwarz, & Starkman (2010). At present, there is no good reason why maps of temperature anisotropies in the CMB should be correlated with any *geometric* features belonging to our local (solar) system.

In the anisotropic power spectrum the low multipole vectors, quadrupole ($l = 2$) and octopole ($l = 3$), have an unexplained alignment to: each other, and the geometry and direction of motion of the solar system⁹¹. Note that the simple dipole ($l = 1$) is interpreted as resulting from the Doppler shift caused by the solar system's *motion* relative to the nearly isotropic (cosmic microwave) background field.

The physical nature of the (cosmological size) rotating space-warps (RSWs) proposed herein influence all low mass bodies⁹², and conceivably all photons, *equally*. These (gravito-quantum) RSWs are axisymmetric (and rotationally symmetric), 'bound' to, and based in the solar system. Jupiter's Galilean moons dominate the new mechanism, and their inclination angles are offset less than 2° from the *ecliptic* plane.

Interestingly, for each rotating space-warp, the wave-like (specific) energy (Δe) involved in the new mechanism may be expressed (recalling end of subsection 3.2.12) as the *square* of either: a speed undulation (Δv), or an oscillatory acceleration/gravitational field (Δa). A common signature in the octopole and quadrupole, that supplements the dipole ($l = 1$), is conceivable⁹³. With CMB photons arriving (at Earth) from all different directions, it may well be that the presence of the (foreground) influence of a superposition of RSWs affects the *measurement* of CMB photons in a non-isotropic manner⁹⁴.

Section 7 discusses a very different type of foreground influence that (also) may arise from the model.

3.5.4 Steep increase of Pioneer 11 anomaly 'around' Saturn encounter discussed

Observations, concerning Pioneer 11 in particular rather than Pioneer 10, indicate that the (aforementioned) root sum of squares (RSS) approach to $a_P(t)$ appears to *not* apply for Pioneer 11 'between' Jupiter and Saturn. Subsequently, the RSS approach appears to be restricted to the case of all acceleration field undulations, in the form of rotating space-warps, having the same direction of rotation with respect to the spacecraft's direction of motion. This is certainly the case for the Pioneer 10 analysis spanning 1987 to 1998 (40 to 70.5 AU), where Pioneer 10 is beyond the orbital radius of all planets in the solar system.

In the case of the Pioneer 11 S/C between Jupiter and Saturn, the space-warp rotations associated with Saturn and Jupiter's (prograde) moons are opposed at the S/C — see Nieto (2007, p.10 Figure 2) and Anderson et al. (2002, Figure 3) for S/C trajectories.

⁹¹Additionally, the quadrupole's magnitude is significantly less than expected, and the third octopole alignment correlation involves the supergalactic plane (SGP).

⁹²Below a mass cut-off threshold.

⁹³Possibly some vector cross product effect is involved.

⁹⁴Three of the four planes determined by the quadrupole and octopole are orthogonal to the ecliptic plane.

At this stage, we shall assume the spacecraft's *path*-based response (to the different individual rotating space-warps) is one of simple superposition — as is the case with the rigorous Pioneer 10 analysis (beyond the outer solar system).

We treat the individual and different (simple harmonic) speed variations/oscillations acting on the spacecraft [$v = v(t) = -\Delta v \sin(\omega t - \varphi)$], in response to individual acceleration/gravitational field variations/undulations [$a_{\text{field}} = (\Delta a_{\text{field}}) \sin(\omega t - \varphi)$], in the manner of interfering ocean waves — with the different Δa_{field} amplitudes replacing wave height amplitudes. The unequal period times (or angular frequencies) of the (sinusoidal) field undulations results in periods of constructive and destructive interference at the spacecraft. Thus, the superposition of spacecraft speed undulations/oscillations, and consequently (also) the real time (effective/overall rate of) monotonic speed shortfall [$a_p(t)$], can be quite variable — with this occurring over extended periods of time (of the order of days, weeks and months). Later (in Section 6) we shall see that moon size is (indirectly and roughly) indicative of undulation amplitude, and thus the effects of *Saturn's Titan and Jupiter's four Galilean moons dominate the model*.

With Pioneer 11 redirected from Jupiter *across* the solar system to Saturn [see diagram (Nieto 2007, Figure 2)], the line-of-sight correction to a_P is significant — persisting (even) after Saturn encounter (but to a much lesser extent). Subsequently, as Pioneer 11 approaches Saturn, an (assumed) Sun-directed Pioneer anomaly would ‘appear’ smaller; and after the spacecraft has encountered Saturn the anomaly steadily rises to its standard value (Turyshv et al. 2006). Also see Anderson et al. (2002, Figure 7), noting the exceedingly large error bar “around” Saturn encounter.

The *shape* of the best fit line is consistent with the notion of a path to line-of-sight (cosine angle based) correction. Awkwardly, the Doppler-based late 1977 (10-day sample interval) data point (at $\sim 6\text{AU}$) (Anderson et al. 2002, Figure 7) of $< 2 \times 10^{-8} \text{ cm s}^{-2}$ implies a near removal of respective shortfall rates, which is a concern for the model and our (initial approach) to explaining this Pioneer 11 ‘discrepancy’. Even with a near 40% ($\sim 52^\circ$) line-of-sight to path angle reduction⁹⁵ in a_P , and a large variation in effective/overall a_P around its mean value at the time of measurement⁹⁶ — i.e. a *reduction* in a_P equal to the amplitude of the \sim annual residual ($1.4 \times 10^{-8} \text{ cm s}^{-2}$) — the model predicts measured a_P of barely less than $4.0 \pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$.

The mitigating circumstance of this concern is that the data point's magnitude is contentious (in this author's opinion) for a number of reasons. Firstly, solar radiation pressure is significantly greater than a_P [see Anderson et al. (2002, Figure 6)]. Secondly, Doppler tracking data and the orbit determination program's accuracy are somewhat compromised when the direc-

tion of travel is at a large angle away from the line-of-sight. Thirdly, the data point is for the not so rigorous earlier analysis.

Alternatively, and somewhat necessarily, it may be the case that (in late 1977) the *opposing* (rotational) *direction* of Titan's rotating space-warp (later⁹⁷ found to have amplitude/magnitude $3.35 \times 10^{-8} \text{ cm s}^{-2}$), which opposes the ‘spin’ direction of Jupiter's four ‘Galilean’ rotating space-warps (RSWs), introduces a further type of destructive interference⁹⁸ (not previously appreciated). This (other) interference affects the acceleration/gravitation *field* (amplitude Δa_{field}) superposition *prior* to its affect upon spacecraft proper motion and acceleration. This subsection's previous discussion of superposition assumed all the individual acceleration/gravitation fields *fully* participate (i.e. standard linear system addition/superposition by way of their coexistence) when they influence the path-based variation of spacecraft speed around the S/C's mean value (v) — with this implicitly being the case in the outer solar system (and thus/also throughout the rest of this paper). Consequently, an anomalous acceleration $a_P(t)$ (near 6 AU in late 1977) of around $0.7 \times 10^{-8} \text{ cm s}^{-2}$ is conceivable — especially with Jupiter and Saturn on opposing sides of the Pioneer 11 spacecraft at this ‘position’ in space and time, see Nieto (2007, Figure 2).

If this alternative/‘richer’ second scenario is indeed the valid case, the model then conceivably *predicts* that the noise/variation of the $a_P(t)$ data ‘between’ Jupiter and Saturn [i.e. $(\Delta a_P)_{\text{mean}}$] should be at least somewhat muted when compared to the mean variation of $a_P(t)$ in the outer solar system — all other things being equal (*ceteris paribus*).

3.5.5 Spin rate changes may support a real Pioneer anomaly

There were *opposing* variations of spin rate changes for the two Pioneers *between* manoeuvres, Pioneer 10 down and Pioneer 11 up (Anderson et al. 2002, Figures 11,12). A real basis to these quite precise variations is almost unavoidable.

For the Pioneers there were anomalous spin-rate changes that could be correlated with changes of the exact values of the short-term a_P . The correlations between the spin-rate changes and a_P were good to $0.2 \times 10^{-8} \text{ cm s}^{-2}$ and better (Nieto & Turyshv 2004, p.4019).

The Pioneer spacecrafts' solar system motion is hyperbolic orbits in roughly *opposing* directions. The not solely linear-radial (outward) motion of the S/C, in conjunction with the proposed (prograde) rotating space-warp mechanism, may have some bearing upon an explanation of this unusually precise correlation.

⁹⁵By way of using $a_P = 8.74 \pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$ as a reference value.

⁹⁶The short 10-day sample interval for the data point (Anderson et al. 2002, pp.17-18) needs to be at a low point in the (superposition-based) temporal variation of $a_P(t)$.

⁹⁷See Table 5 in section 6.5.

⁹⁸In addition to the *scalar* constructive and destructive (amplitude) interference that arises from the superposition of multiple sinusoidal ‘signals’.

3.6 The model's relationship to the Pioneer observational evidence

In section 3.2 we investigated single/solitary acceleration field undulations and their associated: (proper) speed, proper acceleration, and position undulations — about their respective mean values. We then established/deciphered the overall motion shortfall rate arising from the coexistence of multiple field undulations, with this being the *model's* value for the Pioneer anomalous acceleration (a_p). In this section we turn our attention to further effects arising from *multiple* acceleration/gravitational field undulations affecting the motion of a ('low mass') body or spacecraft.

3.6.1 A two component Doppler residual & the main objective of this section

In addition to residuals arising from measurement noise, the (sinusoidal) motion/speed and acceleration variations arising from the model's (multiple) rotating space-warps shall also be classed as *residuals*. In the absence of this further effect, measurement noise is (simply) the difference between the calculated/expected⁹⁹ and observed/measured Doppler values¹⁰⁰; but herein this difference shall include the model's additional (residual/variation) component.

Unlike the monotonic anomalous acceleration (speed shortfall rate) previously discussed, the mean speed and mean acceleration arising from the model's undulatory/sinusoidal effects — once the monotonic anomalous acceleration is removed (or corrected for) — is zero. Thus, the proper motion and proper acceleration (amplitude) variations around their 'equilibrium' condition shall be referred to as "residual proper motion" and "residual proper acceleration" respectively.

In this section we firstly examine the nature of the model's temporal amplitude 'variation'¹⁰¹ of residual proper acceleration, taking note of its relation to the variation of a_P through time [i.e. $a_P(t)$]; and secondly, variation in the residual proper motion/speed — (in both cases) *after* adjustments/corrections for: the spacecraft's predicted trajectory, systematics, and the constant (per unit time) Pioneer anomalous speed shortfall, have been made. Initially, we are interested in short-term temporal variations, of the order of days and weeks; whereas later in this section, medium and long-term variations are considered, particularly the \sim annual residual.

3.6.2 Superposition of (sinusoidal) proper acceleration and proper velocity

Multiple acceleration/gravitational field 'undulations' $(a_{\text{field}})_i = [\Delta a_{\text{field}} \sin(\omega t - \varphi)]_i$, and/or the spacecraft proper acceleration undulations (represented by) $(a_{\text{proper}})_i = [-\Delta a_{\text{proper}} \cos(\omega t - \varphi)]_i$, are assumed to coexist in, and be described by, a simple superposition.

⁹⁹As 'determined' by (a particular) orbit determination program.

¹⁰⁰Utilising the NASA Deep Space Network.

¹⁰¹As compared to 'variance', as used in probability theory and statistics.

This superposition results in the maximum (possible) amplitude of (residual) proper acceleration being:

$$(\Delta a_p)_{\text{max}} = \sum (\Delta a_{\text{proper}})_i \quad (13)$$

Although involving space curvature/deformation, this superposition is (assumed to be) conceptually and mathematically analogous to a superposition involving ocean wave heights (for example). In subsection 3.6.4, we discuss why this (superposition-based) acceleration summation, from a reconciliation with the observational evidence perspective, is considered secondary to a (superposition-based) summation of *speed* amplitudes:

$$(\Delta v_p)_{\text{max}} = \sum (\Delta v_{\text{proper}})_i \quad (14)$$

where each $v = v(t) = -\Delta v_{\text{proper}} \sin(\omega t - \varphi)$, with each of the speed and acceleration undulatory amplitudes related by (a number of alternative expressions):

$$\Delta a = (\omega_{\text{rad}}/2\pi)\Delta v = \omega_{\text{cyc}}\Delta v = f\Delta v = \Delta v/\Delta t$$

This amplitude relationship is indicative of situation-2 circumstances applying¹⁰² — i.e. real/non-pseudo celestial simple harmonic motion (arising from rotating space-warps). Subsequently, Δv_{cyc} with units $[\frac{\text{m}}{\text{s}} \frac{1}{\text{cycle}}]$ — as compared to Δv_{rad} with units $[\frac{\text{m}}{\text{s}} \frac{1}{\text{rad}}]$ — is implied by the use of Δv [or $(\Delta v)_i$ 'component'] values.

Previously (Equation 12 in subsection 3.2.13), the model quantified (and defined) the Pioneer anomaly's magnitude¹⁰³ as:

$$a_p = \overline{a_p(t)} = \sqrt{\sum (\delta a_{\text{proper}})_i^2} = \sqrt{\sum (\Delta a_{\text{field}})_i^2}$$

This root sum squared approach, based upon the various rate of speed shortfalls (per cycle) ($\delta a = \delta v/\Delta t$), is quite different to the maximum amplitude expressions of superposed acceleration and speed presented above (Equations 13 and 14 respectively) — even though each $|\delta a| = \Delta a$. This distinction is due to $(\delta a)^2 = (\delta v/\Delta t)^2$ being proportional to the rate of (unsteady) *energy* (per cycle), whereas the superposition of individual motion/speed (perturbation) sinusoids and the proper acceleration sinusoids (derived from them) are based upon sinusoidal *motion*. In this section (3.6), it is the undulatory/sinusoidal (residual) proper *motion* (or speed) arising from a superposition of multiple (rotating space-warp based) acceleration/gravitational fields that is being examined.

3.6.3 Background information concerning the (discrete) Pioneer observations

Both Jet Propulsion Laboratory (JPL) and The Aerospace Corporation (Aerospace) assign a standard

¹⁰²Recalling Equation 10 and the distinction between situation-1 and situation-2 established in subsection 3.2.10.

¹⁰³Fortuitously, because $a_p(t)$ has a long-term *constant* mean — primarily by way of constant rotating space-warp $(\Delta a_{\text{field}})_i$ amplitude values and constant $(\omega)_i$ values — long-term (mean) $a_p(t)$ may be written (simply) as a_p . We note that each (Sun-planet-moon system based) set of: δa , Δa_{proper} , Δv , and δv values also have fixed/constant magnitudes — both temporally and spatially.

(1- σ) uncertainty of 1 mm/s over a 60 second count time for the S-band Doppler data (Anderson et al. 2002, p.10) — after calibration for transmission media effects¹⁰⁴. It takes nearly 2 weeks for the anomalous acceleration ($8.74 \times 10^{-10} \text{ m s}^{-2}$) to achieve a 1 mm/s motion anomaly/shortfall. This is some 20,000 times longer than the 60 second count time, and some $13\frac{1}{2}$ times greater than the total anomalous change in speed over a 1-day time scale. Over the substantially longer time scale of ~ 3 months, the uncertainty circumstance (for the Doppler data) is similar in that: the Doppler residuals (from both JPL’s and Aerospace’s analyses) — with the anomalous acceleration taken out — are distributed about zero Doppler velocity with a systematic variation of ~ 3.0 mm/s (Anderson et al. 2002, p.19 & p.37).

The noise of the raw data, especially (thermal or) plasma noise¹⁰⁵, together with the use of just 5 data points per day on average¹⁰⁶, effectively masks any clear, continuous and unambiguous signature in the $a_P(t)$ observational data — other than the constant ‘frequency’/velocity drift (cf. predictions) that is the “Pioneer anomaly” and (to a lesser extent) the \sim annual residual. Upon removing the anomaly, the ‘noise’ in the data and the manner of its determination, makes an accurate ‘real time’ *continuous* assessment of the very short-term (i.e. minutes or hours) temporal behaviour of $v_P(t) = \sum [v(t)]_i$ and $a_P(t) = \sum [a(t)]_i$ an impractical/unrealistic task¹⁰⁷.

3.6.4 The difficulty associated with assessing the temporal variation in $a_P(t)$

The major difficulty associated with relating the temporal *variation* in observational anomalous acceleration $a_P(t)$ to the model’s temporal variation in $a_p(t)$ is that observational $a_P(t)$ is derived from non-continuous Doppler velocity data ‘points’. Even though the raw Doppler data is received *continuously*, this data is (then) integrated over a time interval to give an average for the interval.

With the raw observations directly addressing spacecraft speed, rather than acceleration, and in light of observational and data processing circumstances that impinge upon the Doppler data, our main focus shall be upon a comparison involving how well the variation/range in observed Doppler velocity data

corresponds to the quasi-periodic (or alternatively quasi-stochastic) variation of speed implicated by the model¹⁰⁸, especially $(\Delta v_p)_{\max}$ (as defined by Equation 14 in subsection 3.6.2). This ‘correspondence’ is quantitatively addressed in subsection 3.6.6.

In subsection 3.6.5 we shall give a brief overview of the model’s Δa_p variation/range, noting that there is a *disconnect* between this variation and the variation apparent in the a_P data points, that are determined over the course of the $11\frac{1}{2}$ year long-term (Pioneer 10) rigorous data analysis period (3 Jan 1987 to 22 July 1998) — see Anderson et al. (2002, Figures 13, 14 and 17). This is because the model’s *derived* proper acceleration sinusoid amplitudes: $\Delta a = \Delta v / \Delta t$, do not have a common temporal basis, in that the cycle/period duration (Δt) is different for each Δa . For a summation (rather than a superposition) of the monotonic motion shortfall rates ($\delta a = \delta v / \Delta t$), this “timing issue” is not a concern. Note that these two preceding (and similar) equalities involve the following relationship of units: $[\frac{\text{m}}{\text{s}^2}] = [\frac{\text{m}}{\text{s}} \frac{1}{\text{cycle}}] [\frac{\text{cycle}}{\text{s}}]$. Neither is this timing issue a concern for the model’s Δv values (recall subsection 3.2.10), because these are actual values accessible to ‘real time’ observations — irrespective of their (pedantic) ‘per cycle’ unit basis in the model.

Somewhat restating the above: with discrete (line-of-sight) Doppler velocity data as the primary “means of observational contact”, there is a tension/disconnect between the model’s approach to *sinusoidal* acceleration terms, and *observational* acceleration, with the latter *derived* via a rate of change of discrete velocity measurements (with respect to *time*) — i.e. observation time, which is based upon the standard unit of seconds. In short, the conversion from observational discrete speed/velocity ‘data points’ to acceleration values overlooks the ‘time span’ adjustments required to correctly ascertain the (sinusoid-based) proper acceleration *component* values, and hence the actual (*superposed*) physical variation of (anomalous/residual) proper acceleration [i.e. $a_p(t)$] proposed and formulated by the model. Recall that the spacecraft’s *overall* ‘proper acceleration’ is the acceleration that would be displayed by a (sufficiently sensitive) onboard accelerometer, and thus in principle the ‘residual’ proper acceleration is measurable/determinable — after correcting for the effects of standard (Newtonian and) general relativistic gravitation.

Subsequently, with the actual observational variation in a_P not being indicative/representative of the model’s $a_p(t)$ variation (around a mean value), the model’s relationship to the (Doppler speed/velocity) observational data evidence is restricted to a purely velocity basis. As such, the model’s value of $(\Delta v_p)_{\max}$, i.e. the maximum range of (the superposed resultant) $v_p(t)$, is of vital importance; it supplements the measurement noise/residuals inherent to Doppler data based spacecraft navigation. The model’s $(\Delta v_p)_{\max}$ value is quantified in subsection 3.6.6.

¹⁰⁸With its superposition of multiple (constant and very small amplitude) sinusoidal gravitational/accelerational field effects, all of which have different frequencies, and the associated superposition of (multiple) sinusoidal speed variations/perturbations.

¹⁰⁴The dominant systematic error that can affect S-band tracking data is ionospheric transmission delays (Turyshv & Toth 2010, p.60).

¹⁰⁵Notwithstanding the implementation of a batch-sequential filtering and smoothing algorithm used in conjunction with JPL’s Orbit Determination Program (Anderson et al. 2002, p.16).

¹⁰⁶Based on 20055 data points for Pioneer 10 over an $11\frac{1}{2}$ year period, cf. Pioneer 11 with 10616 data points over its $3\frac{3}{4}$ year analysis period (Anderson et al. 2002, p.20); i.e. approximately 5 and 8 data points per day, respectively.

¹⁰⁷These comments are in no way meant to disparage the data analysis performed by: JPL, the Aerospace Corporation, or other investigators. The intricacy and comprehensiveness of the orbit determination programs are especially awe inspiring. Indeed, the model presented herein, by way of merely dealing with the *anomalous* acceleration and residuals, is decidedly less intricate (in this regard).

3.6.5 The model's $(\Delta a_p)_{\max}$ value and the attenuation of maximum amplitude

By way of Table 5 in subsection 6.5.3 and Equation 13, $(\Delta a_p)_{\max} \approx 18.3 \times 10^{-10} \text{ m s}^{-2}$, which is approximately twice a_P , as compared to the observational evidence (Anderson et al. 2002, Figures 13, 14 and 17) which implies $(\Delta a_p)_{\max} < a_P$. For convenience the individual acceleration amplitudes — that are actually derived much later in the paper — have been included/previewed in Table 1. As previously mentioned (see discussion in subsections 3.6.3 and 3.6.4), the variation/range of $a_P(t)$ is not symptomatic of the model's range/variation in $a_p(t)$. Furthermore, subsection 3.6.2 illustrated the quite different *summation* formulations for the motion shortfall rates (δa), as compared to the sinusoidal motion amplitudes (Δv) and their rate of change $\Delta a (= \Delta v / \Delta t)$. Consequently, we shall (from here on) largely restrict our (temporal variation based) discussion to the validity of the relationship between: the model's $(\Delta v_p)_{\max}$ value, and the range/variation of the observational Doppler velocity/speed data residuals. We also need to take into consideration the typical level/magnitude of noise in the Doppler measurement 'process'.

Attenuation of the maximum possible amplitude variation of the *model's* $v_p(t)$ values [i.e. $(\Delta v_p)_{\max}$] conceivably arises in two main ways. Firstly, with five major moons involved (six including the much lesser role played by Neptune's Triton) and their fairly *short* period durations ranging from Io's 1.77 days to Callisto's 16.69 days, any maximum/minimum value is short lived — being of the order of hours (and minutes) rather than days. With a longest data integration time of 1980 seconds (33 min) and count times (and data integration times) of 10 and 60 seconds quite typical (Turyshev & Toth 2010, p.58) — although 'Aerospace' had a preference for the longer count times of 600 and 1980 seconds (Anderson et al. 2002, p.10) — the attenuation in $(\Delta v_p)_{\max}$ (arising from this short-term feature of the model) is almost negligible¹⁰⁹, and we shall assume it to be $< 3\%$. In other words, the observational data's sampling rate is considered sufficiently dense so as to 'appreciate' (i.e. largely not miss) the shortest-term temporal variation(s) in $v_p(t)$ propounded by the model.

Secondly, attenuation conceivably arises by way of the orbital resonance of Jupiter's Galilean moons: Io, Europa and Ganymede, in that they are in a 4:2:1 resonance; and additionally by way of Ganymede and Callisto which effectively display a (very near to, and subtle) 7:3 resonance. As such, all four (spin-orbit coupled/resonant) moons are never 'aligned' (on the same side of Jupiter), and thus their 'related' rotating space-warps (RSWs) possibly never attain maximum (constructive) or minimum (destructive) interference. This preceding line of thought assumes that the (four) 'initial' phases $[(\varphi)_i]$ of the rotating space-warps (as-

sociated with the *four* Galilean moons) are dependent upon their orbital location, but (it shall become apparent later that) there is no basis for this assumption. Notwithstanding this qualifying remark, Galilean lunar orbital resonance will influence periodicity in the $v_p(t)$ data, but it does *not* follow that their resonant motion involves significant/major attenuation in the range/variation of the Doppler data¹¹⁰. We argue (below) that the attenuation is no greater than 12 percent, by way of one (i.e. the least) of the five major moons being unable to contribute to a maximum superposition magnitude/amplitude.

In determining the Doppler data, a 'batch-sequential' method¹¹¹ was implemented by JPL (utilising a 1-day, 5-day, or 200-day batch duration), and the data is filtered, weighted and smoothed. Although this (data processing) would appear to possibly result in attenuation of the $v_p(t)$ [and $a_p(t)$] data, it is a technique associated with orbit determination; and as such, it *reduces* measurement noise. Thus, the range/variation of the model's $v_p(t)$ [and $a_p(t)$] (residual) values do not incur attenuation as a result of its use.

Consequently, neither of these two possible (model-based) attenuation factors, nor the implementation of batch-sequential processing, are seen to cause any major attenuation (i.e. $> 15\%$) of the model's proposed variation/range in the $v_p(t)$ data and (specifically) the magnitude of $(\Delta v_p)_{\max}$. By way of Table 1 we see that the contributions from different moons are not equal. Of the five moons that dominate the model, Europa is the smallest contributor to maximum *speed* amplitude $[(\Delta v_p)_{\max}]$ ¹¹². With regard to Galilean moon orbital resonance, the removal of Europa's contribution leads to an attenuation effect upon $(\Delta v_p)_{\max}$ of about $5\frac{1}{2}\%$ of the 'overall' total; and 11% if (directly) opposed/negative (which is unlikely). Indeed, it is more likely that the attenuation in maximum speed/velocity amplitude (from Galilean moon orbital resonance) is closer to $5\frac{1}{2}\%$.

Importantly, the model will be rendered invalid if the 'standard' long-term maximum amplitude variation of the observed Doppler velocity data — indepen-

¹¹⁰Later, we shall see that these initial phases — although 'knowable' in principle (but not currently determinable in practice) — have (herein) not been ascertained.

¹¹¹"Though the name may imply otherwise, batch-sequential processing does not involve processing the data in batches. Instead, in this approach any small anomalous forces may be treated as stochastic parameters affecting the spacecraft trajectory. As such, these parameters are also responsible for the stochastic noise in the observational data. To characterize these noise sources, we split the data interval into a number of constant or variable size batches with respect to the stochastic parameters, and make assumptions on the possible statistical properties of the noise factors. We then estimate the mean values of the unknown parameters within the batch and their second statistical moments (Turyshev & Toth 2010, p.81)."

¹¹²Whereas Callisto is the smallest (and Europa the second smallest) contributor to the (theoretical) monotonic anomalous *acceleration* (i.e. speed shortfall rate) — remembering that a_p is based upon a root sum squared (RSS) derivation — and Callisto (necessarily also) makes the smallest contribution to proper acceleration amplitude variation.

¹⁰⁹By way of contrast, the \sim annual (356 day) beat duration of the Titan-Callisto superposition is a relatively much longer-term variation. Indeed, it is the model's longest-term variation, primarily evidenced by its (long-term) amplitude modulation.

Table 1: Lunar orbital periods & frequencies, and component proper acceleration and speed amplitudes.

MOON ^a	Units	Luna ^b	Io	Europa	G'mede ^c	Callisto	Titan	Triton ^d
Moon's orbital period	(days)	27.32	1.769	3.552	7.155	16.69	15.95	5.877
Moon orbit frequency (f)	(10^{-6} s^{-1})	0.424	6.542	3.259	1.618	0.694	0.726	1.969
Accel. amplitude ^e (Δa)	(10^{-8} cm/s^2)	0.00	5.139	2.190	5.354	1.884	3.348	0.416
Speed amplitude ^f (Δv)	(10^{-2} mm/s)	0.00	7.855	6.719	33.10	27.17	46.12	2.112
Speed amplitude ^g (Δv)	(mHz)	0.00	1.205	1.031	5.076	4.167	7.074	0.324

^aThe lunar data is taken from Table 2 in Section 6; with lunar orbital period and frequency rounded off to 3 or 4 significant figures. ^bEarth's moon. ^cGanymede is abbreviated to G'mede. ^dRetrograde motion. ^eAcceleration amplitude data is taken from Table 5 (subsection 6.5.3), where it is called “weighted acceleration” (Δa_w). Note that the square root of the summation of squared (weighted) acceleration is $8.64 \times 10^{-8} \text{ cm s}^{-2}$ without Triton, and $8.65 \times 10^{-8} \text{ cm s}^{-2}$ with Triton's contribution; as compared to the Pioneer anomaly's quoted magnitude of $a_P = 8.74 \pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$. ^fBased on $\Delta v = \Delta a / \omega_{\text{cyc}} = \Delta a / f$. Note that $\Delta v_{\text{max}} = \sum (\Delta v)_i \approx 1.23 \text{ mm s}^{-1}$. ^gVia conversion factor of $10 \text{ mHz} = 0.652 \text{ mm s}^{-1}$ (recalling subsection 3.3.3), with $\Delta v_{\text{max}} = \sum (\Delta v)_i \approx 18.9 \text{ mHz}$.

dent of large variations (and spikes) associated with manoeuvres and other spurious effects — fails to be consistent (i.e. a good ‘match’) with: expected measurement noise, and more importantly, the model's estimated value of $(\Delta v_p)_{\text{max}} = \sum (\Delta v)_i$ (see subsection 3.6.6). Finally, we note that in addition to (mean longer-term) a_P being constant, the model's $(\Delta v_p)_{\text{max}}$ and $(\Delta a_p)_{\text{max}}$ values are also constant — at least as regards the 3 Jan 1987 to 22 July 1998 data period of Pioneer 10 when/where the spacecraft is far beyond the orbits of Jupiter and Saturn.

3.6.6 Comparing the model's velocity range to the Doppler observations

As previously mentioned, Table 1 *previews* results (relevant to this section, i.e. 3.6), that are delivered much later in the paper. From the acceleration/gravitational field amplitude values, and lunar (spin and) orbital ‘frequencies’, we establish the individual (contributing) speed amplitudes¹¹³ (Δv). By way of: Equation 14 (in subsection 3.6.2), noting that: $0.652 \text{ mm/s} = 10 \text{ mHz}$ (recall subsection 3.3.3), and neglecting the (minor) influence of attenuation effects, we have:

$$(\Delta v_p)_{\text{max}} = \sum (\Delta v)_i = 1.23 \text{ mm/s} = 18.9 \text{ mHz}$$

With 10% attenuation, this maximum value (or upper bound) reduces to about 17 mHz, and with (an estimated maximum) 15% attenuation, this upper bound reduces to about 16 mHz.

It is reassuring that the (constant) amplitude range/variation *bound*, established by the model, is consistent (and somewhat less than) the residuals ‘generated’ by: Levy et al. (2010, Figure 7), and Toth (2009, Figure 2 top right diagram) — with effectively all of their (non-spurious) residuals spread between

$\pm 20 \text{ mHz}$ and $\pm 25 \text{ mHz}$, respectively. Levy et al. (2010, Section 5) also found that the standard deviation of their residuals was 9.8 mHz — i.e. about half the model's maximum variation.

We necessarily conjecture/hypothesise that the measured (speed) residuals are the additive ‘resultant’ of *both* measurement noise, *and* the variation inherent in the model's instantiation of multiple and superposed sinusoidal speed variations — around a mean value.

This ± 20 to 25 mHz residual range (derived from the literature) represents a median value (in the literature) in that: Anderson et al. (2002, Figure 13) has slightly smaller residuals¹¹⁴ (in general), whereas the (later data Pioneer 10) residuals in Olsen (2007, Figure 2) are bigger (about $\pm 30 \text{ mHz}$). Olsen (2007, Figure 2) also displays an annual (or \sim annual) amplitude modulation, which is supportive of the Callisto-Titan beat behaviour discussed in subsection 3.5.2 and elaborated upon in subsection 3.6.7. Interestingly, the (shorter sample duration) Pioneer 11 residuals in Olsen (2007, Figure 6) have a (maximum) range of about 20 mHz, which is also (roughly) the maximum amplitude of the (non-spurious) CHASMP-based¹¹⁵ residuals displayed in Anderson et al. (2002, Figure 9).

Importantly, Levy et al. (2010, Section 4) note that: “It should be emphasised that the level of the residuals in [their] Fig. 7 is higher than the measurement noise. It is also clear from the figure that the postfit residuals do not correspond to white gaussian noise.” Further, Turyshev & Toth (2010, Section 4.6, p.80) declare that: “Several, independent efforts to analyze the trajectories of the two [Pioneer] spacecraft have demonstrated that this [i.e. providing a model of as large a segment of the spacecrafts trajectory as possible, using a consistent set of parameters and minimizing the model

¹¹³Note that we have adjusted the model's (pedantic) speed amplitude and frequency units of $[\frac{\text{mm}}{\text{s}} \frac{1}{\text{cycle}}]$ and $[\frac{\text{cycle}}{\text{s}}]$ (respectively), to their observational (and more standard) format. As such, the [cycle] ‘unit’ has been left out.

¹¹⁴Possibly because it is based upon a weighted least-squares (WLS) approach cf. a batch-sequential filter (BSF) algorithmic approach (Anderson et al. 2002, p.23).

¹¹⁵Compact High Accuracy Satellite Motion Program. Note that CHASMP employs a weighted least squares estimation approach.

residual] can be accomplished using multi-year spans of data with a root-mean-square *model residual of no more than a few mHz* [italics added].”

This quantitative circumstantial evidence, although not directly supporting the model, is strongly indirectly supportive of the proposed/hypothesised model. Indeed, Levy et al. (2010, Section 5) found that they could almost *halve* the standard deviation of their model’s residuals (down to 5.5 mHz from 9.8 mHz) simply by introducing (into their model) periodic terms — relating to the Earth’s diurnal and annual periods.

3.6.7 The Callisto-Titan ‘beat’ amplitude dominates the Doppler variation

From Table 1’s previewed results, we see that the amplitude of the 356 day (\sim annual) Callisto-Titan beat period (recall subsection 3.4.1) is 0.733 mm/s or 11.24 mHz, via $(\Delta v)_{\text{Callisto}} + (\Delta v)_{\text{Titan}} = 73.3 \times 10^{-2}$ mm/s. With this value being sizeable, at about 60% of the maximum speed amplitude predicted by the model [$\Delta v_{\text{max}} = (\Delta v_p)_{\text{max}} = 1.231$ mm/s], and because this ‘amplitude modulation’ has (by far) the *longest period* of any of the periodic amplitude variations within the model, it is necessarily the dominant long-term amplitude effect in the observed/measured Doppler residuals (predicted by the model). This (\sim annual/356-day period) amplitude modulation is clearly evident in Olsen (2007, Figure 2).

From the model’s perspective, this ‘beat’ duration/frequency and associated amplitude modulation has been (in the literature) mistakenly understood as an Earth-based annual residual. Markwardt (2002) was sufficiently concerned with its period that he referred to it as the “ \sim annual” residual. This subsection’s (model-based) quantitative result — i.e. the 0.73 mm/s (Δv) magnitude of the ‘annual’ residual — is fully supportive of (and compatible with): the observational evidence for the amplitude of the \sim annual residual; as well as the line of argument, and concluding remarks presented in section 3.4’s fairly extensive discussion of this residual. We recall that in subsection 3.4.3 the observational evidence implied the \sim annual residual had an amplitude of roughly 0.7 mm/s.

3.6.8 On the variation of a_P data around its long-term constant mean value

In this subsection we present a (further) brief discussion relating to the observed acceleration (cf. speed) residuals, and amplitude variation/modulation in the long-term a_P data.

From Anderson et al. (2002, p.37 and p.24), we can compare the 1-day batch-sequential acceleration residuals (Figure 17) to the 5-day batch-sequential acceleration (Figure 14) data — after subtracting the a_P value from Figure 14, and upon adjusting for the [km/s²] units of Figure 17. The 5-day residuals appear to be roughly *one third* the level/amplitude of the 1-day residuals. Further, the much longer 200-day acceleration residuals (obtained using Aerospace’s CHASMP software and included in Figure 14) are in good agree-

ment with the 5-day ODP/SIGMA results¹¹⁶ — with the latter using batch-sequential filtering with a 200-day correlation time. Further, we note that the amplitude range of the 200-day CHASMP data is closely related to (about three-quarters) the range/variation of the much shorter 5-day sample averages.

Assuming the information described by these: two figures, three batch-sequential durations, and two orbit determination programs (ODPs) are comparable¹¹⁷, the implication is that most of the measurement (or observational) ‘noise’ influencing the ODPs is short-term (i.e. less than five days). Of interest, is the amplitude variation (around the long-term constant mean value) that *remains* evident in the a_P data [e.g. Anderson et al. (2002, Figure 14)], or equally, the variation of residual acceleration upon removal of a_P . This situation is consistent with the model (being proposed), in that in addition to the long-term \sim annual residual and short-term amplitude variations, there will be two medium-term amplitude variations (cf. modulations) that are associated with the pairings of: Ganymede with Callisto, and Ganymede with Titan. We note that these medium-term (spacecraft-based) amplitude variations are ‘induced’ by (superposed pairs of) acceleration/gravitational fields, with these fields (in turn) having ‘emanated’ from two *lunar*-based rotating space-warps. Finally, from Table 1 we see that these two medium-term variations have (superposed) amplitudes comparable to that of the (Titan with Callisto) \sim annual residual, because Titan, Ganymede, and Callisto (in that order) are the dominant proper motion/speed residuals.

Interestingly, Iorio (2007, Section 3.1) notes that (with particular regard to the annual residual): “... changes in the [200-day] correlation time lead to quite different results with the Kalman filter.” The model is able to propose an explanation for this circumstance in that: with Ganymede and Callisto in a 7:3 orbital resonance relationship, that spans 50.1 days — and Io, Europa and Ganymede in a 4:2:1 orbital resonance — the use of a 200-day correlation time is in remarkable synchronisation with four full 50-day cycles of this Ganymede-Callisto orbital duration relationship. Subsequently, any deviation from a 200-day correlation time will lessen the distinctiveness of the \sim annual residual, by way of introducing phase offsets and hence additional amplitude effects, that are otherwise (fortuitously) suppressed.

3.6.9 Concluding remarks on the model’s consistency with observational data

The model (at this preliminary stage) is qualitatively and quantitatively consistent with the Pioneer 10 (and Pioneer 11) observational evidence; which in addition to the Pioneer anomalous acceleration (a_P), implies (a number of) short and medium-term periodic ampli-

¹¹⁶ “The latest results from JPL are based on an upgrade, SIGMA, to JPLs ODP software (Anderson et al. 2002, p.22).”

¹¹⁷ Noting that: “The CHASMP 200-day averages suppress the solar conjunction bias inherent in the ODP 5-day averages ... (Anderson et al. 2002, p.24).”

tude variations, and a (long-term) \sim annual amplitude modulation¹¹⁸ of the (post-fit Doppler) speed and acceleration residuals. As such, in the a_P data there is a noticeable temporal variation (i.e. ‘wandering’) around the (very) long-term constant mean (Pioneer anomaly) value — see Anderson et al. (2002, Figure 14). The model’s approach allows this (apparently) stochastic temporal variation in a_P around its mean value, and the (overall and non-spurious data based) amplitude of the Doppler residuals, to ‘exist’ at a level/magnitude that exceeds the (respective) amplitude variations that can simply (and solely) be attributed to measurement noise.

In response to the favourable relationship the model has towards the Pioneer observational evidence, we shall further pursue and build upon this Section’s “first stage modelling of the Pioneer anomaly” — so as to eventually establish (amongst other things) the results that have been partially *previewed* in this section (3.6) — in particular Table 1.

3.7 Summary of the model’s first steps and what’s still to be done

The hypothesis/stipulation of a rotating warp (or rotating ‘fold’) of space curvature (or space deformation) — first mentioned in subsection 3.1 and implied by observational results discussed in section 3.5.2 — is a promising, multi-faceted and provocative first step towards a model of the Pioneer anomalous observations $[a_P(t)]$. Later we shall see that this new mechanism is related to a clash involving a (common) exceedingly small geometric phase offset within (a great many, i.e. $\sim 10^{50}$) *discrete*/quantum mechanical systems (i.e. atoms/molecules) by way of their (“bulk solid” body) motion in *analog* curved spacetime.

Utilising a barycentric systemic reference frame, and applying (a power version of) Rayleigh’s Energy Theorem (also known as Parseval’s theorem) to compound motion (i.e. steady and unsteady motion) in deep space (and curved spacetime), allows the Pioneer anomaly to be conceived as a (path-based, i.e. velocity vector based) shortfall in predicted spacecraft motion — irrespective of the spacecraft’s direction of motion (and speed). Standard general relativistic gravitation determines the (temporally) ‘steady’ motion component, whereas the unsteady component embodies the new/supplementary physical mechanism presented (and espoused) herein.

Each ‘constant’ (i.e. the same everywhere) amplitude “rotating space-warp” (RSW) leads to a low mass body (such as a spacecraft) experiencing an (unsteady) sinusoidal variation in acceleration/gravitational field strength, which in turn causes a sinusoidal variation in speed around an ‘equilibrium’ speed — all other things being equal. From a barycentric (or Earth-based) perspective, this oscillatory variation in speed (and kinetic energy) results in a constant rate of motion shortfall (per cycle), cf. the (predicted) motion in the absence of the unsteady motion. The individual RSWs and

their effects coexist in a wave-like (linear) superposition, although the effect of this RSW superposition upon spacecraft proper motion and proper acceleration, as compared to the (monotonic) rate of motion shortfall (a_p), are quite distinct (see subsection 3.6.2).

Later we shall see that five major rotating space-warps account (almost completely) for the Pioneer anomaly, specifically involving Jupiter’s four Galilean moons and Saturn’s large moon Titan — i.e. large moons that are part of a spin-orbit coupled three-body (Sun-planet-moon) celestial system. For geometric suitability reasons (explained in subsection 5.6.7), the Earth’s moon is *not* a RSW ‘generator’. It should be noted that direct variations in gravitational field strength arising simply from lunar motion is *not* (at all) what is being proposed! Furthermore, subsection 3.5.3 briefly discussed how these RSWs could be non-isotropically affecting the measurement of CMB (radiation) photons, e.g. ‘producing’ a ‘foreground’ solar ecliptic plane signature upon the CMB data.

Significantly outstanding at this stage, is an understanding of the restriction of the new mechanism’s effects to low mass bodies; (in so much that) the affected (spacecraft) mass is below the ‘non-local’ cut-off masses associated with each (individual) rotating space-warp. This was the first of subsection 2.4’s three primary observational constraints. Only by going beyond the scope of General Relativity’s explanatory domain and standard contemporary physics (a process begun in Section 4) can this crucial constraint be circumvented.

We have adhered to sound scientific methodology, in that the establishment of the model’s ‘form’ is being dictated by the Pioneer anomaly’s observational evidence — especially its more subtle (and awkward) aspects. Section 3.5.1 outlined the proposed mechanism’s *essential characteristics*. Compatibility with the fine detail of the observations is both a primary motivator, and the distinguishing feature, of the model; e.g. the low a_P reading of Pioneer 11 *en route* to and ‘around’ Saturn encounter (subsection 3.5.4). Additionally, the temporal variation of the a_P data (*around* its constant long-term mean) — which is quite distinct from the (anomalous) monotonic shortfall aspect — and the (rotating space-warp ‘driven’) temporal variation of the model’s (proper) speed/motion and acceleration ‘residuals’ were investigated (section 3.6). Prior to this, the diurnal and annual residuals were closely examined (subsection 3.3.6 and section 3.4 respectively), and the (sinusoidal) ‘annual’ residual has been recast as an \sim annual (356 day) non-Earth-based amplitude modulation — that arises from a long-term ‘beat’ duration involving two RSW’s of similar (spin and) orbital frequencies/periods (i.e. Jupiter’s Callisto and Saturn’s Titan). Further, the model’s amplitude (of 0.73 mm/s) for this Titan-Callisto beat frequency is a good match to the observed (~ 0.7 mm/s) amplitude of the \sim annual residual — see subsection 3.6.7.

The intractability of comparing the amplitude variation of the model’s $a_p(t)$ proper acceleration value with the observational a_P data (through time) is discussed (subsection 3.6.5). Subsequently, the Doppler speed/velocity residuals become a primary indicator

¹¹⁸Noting that this (beat-based) amplitude modulation ‘contains’ numerous short-term amplitude variations.

of the model's veracity. These observable residuals are comprised of (i.e. indicative of) *both* measurement noise and the model's proposed sinusoidal/undulatory proper motion effects (around an equilibrium speed). Additionally, we discuss the quite minor attenuation effect (upon the model's proper motion/speed residuals) arising from the (1:2:4) orbital resonance of Ganymede, Europa and Io, and the (3:7) orbital resonance of Callisto and Ganymede (subsection 3.6.5).

By way of previewing results determined later in the paper (Table 1), it was shown (firstly) that the model's long-term (constant and monotonic) *mean* rate of motion shortfall value a_p is consistent with the observation-based Pioneer anomalous acceleration value a_P (subsection 3.2.13). Secondly, the combined magnitude/amplitude of a (superposition-based) maximum amplitude summation of the model's proper motion/speed residuals, on top of typical (non-spurious) measurement noise-based residuals inherent in the Doppler method, is consistent with the observed/measured spacecraft Doppler residuals determined by various investigators (subsection 3.6.6).

Certainly, there is still much more that a robust explanation needs to cover and examine, including:

1. Ascertaining how the model can theoretically co-exist with General Relativity.
2. Establishing the amplitude and energy of the (individual) acceleration/gravitational field undulations (Δa), which equal and correspond to a rate of spacecraft speed shortfall (δa). These δa (components) together (in superposition) then largely quantify the model's Pioneer anomaly estimation (a_p) — recall subsection 3.2.13.
3. Understanding how a *virtual* spin phase offset — relative to the spin phase (of all elementary fermion/matter particles) within numerous individual lunar atoms and molecules (exhibiting spin-orbit coupling) — that is below a minimum (quantum) energy level, yields a non-inertial (excess) spin energy. This virtual energy, is recognised globally (i.e. systemically) but it is locally (i.e. at the QM 'level') inexpressible. Thus, this (*total* QM) energy — pertaining to a great number of atoms/molecules — is necessarily released externally as a: 'single', *real*, supplementary acceleration/gravitational field — so as to preserve conservation of energy *globally*.
4. Appreciating the non-local nature of the mass aspect of the (total) virtual quantum mechanical *energy* — with this energy necessarily having the same magnitude as the externally expressed *supplementary* ('acceleration undulation/warp' and 'mass' based) 'gravitational energy'. This (latter) real energy is comprised of both a macroscopic "rotating space-warp" (RSW) and its (associated) non-local mass 'distribution'.
5. Establishing the qualitative and quantitative distribution of *non-local* mass (cf. standard inertial or gravitational mass) in the field.

A useful, yet overly simplified, *encapsulation* of the new model is that: in standard general relativity, mass

and (*macroscopic*) motion tell space how to curve, which then tells mass how to move — and let us herein consider the ensuing motion to be *steady*. In contrast, and supplementary to this, is the (as yet uncorroborated) proposal that: in special circumstances yet to be fully outlined, (an angular momentum-based energy associated with) *microscopic* mass 'motion' ('within' a great many atomic and molecular 'systems') can also tell space how to curve (i.e. sinusoidal-like undulations upon the gravitational field at a point mass, or RSWs more generally), which then tells (a moving) mass how to *vary* its movement. The latter — with regard to a single rotating space-warp — involves an oscillatory variation of speed about a mean motion/speed i.e. *unsteady* motion, that leads to (or rather co-exists with) a very small (incremental) loss/decay of a body's 'given' kinetic energy (per cycle). In this supplementary 'gravitational' case a global/systemic basis to energy (quantification and 'transfer') is necessary, incorporating both the macroscopic and microscopic 'domains' (or realms) together. Once again we note that only the motion of 'low mass' bodies are affected.

4 Theoretical concerns and philosophical aspects of the proposed mechanism

The awkward observational constraints discussed in subsection 2.4 have been addressed and largely satisfied (recall section 3.7) by the model preliminaries introduced in Section 3. Unfortunately, giving the reliable observational evidence full priority, so as to make the Pioneer anomaly real, has lead to a quite bizarre preliminary model. The challenge of this Section is to show that this model, although restricted and supplementary to current theory, is not incompatible with accepted theory — once certain assumptions (metaphysical in some cases) influencing physicists' current understanding are appreciated, and certain ontological¹¹⁹ ramifications of quantum non-locality are appreciated.

This Section effectively forms a conceptual bridge between the awkward implications of the observational evidence and the model's mathematical quantification (given partly in Section 5 and mainly in Section 6). After a series of preliminary remarks, the model's tensions in relation to assumptions underlying contemporary physics are addressed; e.g. *theoretical* reductionism¹²⁰. Further conceptual features of the model¹²¹ are presented in Section 5, and prior to that where it is considered appropriate.

This Section is necessary because the conceptual,

¹¹⁹Philosophy may be divided into three main branches: ethics/morality, the theory of knowledge (epistemology), and ontology; the latter being concerned with the nature of being, existence, and reality.

¹²⁰Theoretical reduction is the reduction of one explanation or theory to another — that is, it is the absorption of one of our ideas about a particular thing into another idea; for example Newtonian gravitation into general relativity.

¹²¹Especially quantum mechanical aspects.

ontological, and mathematical foundations of: QMs, ‘gravitation’, mass, space, time, and energy are being touched upon. Subsequently, the explanation to follow needs to address ‘philosophical’ issues beyond the usual scope of today’s “physics”.

4.1 Preliminary remarks

In order to enrich the model being developed, a number of preliminary remarks need to be presented. The remarks are wide ranging involving: philosophy, philosophy of science, limitations of GR, a (decoherence) interpretation of QMs; and issues such as: Ehrenfest’s theorem, quantum-to-classical transition, quantum condensate behaviour, and angular momentum addition. Together, they can be thought of as building blocks for the new model, and the write-up that follows.

4.1.1 Societal consensus in physics

Unlike philosophy which (to the best of its ability) pursues and holds alternative interpretations with rigour and relish, the mathematical nature of physics, with its singular solutions, appears (in practice) to also extend to explanations of anomalous physical phenomena — e.g. dark matter, dark energy. Generally, a dominant conceptual “party line” is favoured; and subsequently, alternatives face a daunting task, compounded by the interrelated nature of conventional physical modelling (e.g. cosmology’s ‘concordance model’). Due to the interrelated nature of this (Pioneer anomaly motivated) unconventional model, the reader is asked to refrain from drawing firm conclusions until the *end* of this Section (i.e. Section 4) is reached.

4.1.2 Philosophy of science comment

Borrowing terminology from the philosopher of science Imre Lakatos (1922-1974), it may be said that an alternative “research programme” (RP) is to be presented; and thus alternatives to beliefs in the “hard core” of the current research programme shall be considered¹²². The combined alternative views to be presented, assemble to form, in the author’s opinion, a ‘progressive’ research program.

Interestingly, some of the alternatives to be pursued are external to the curriculum of today’s physicists; and yet well-known to philosophers of science and physics. One need only remember the surprise with which an expanding universe [post Vesto Slipher (1917) and Hubble’s observations] was greeted to appreciate that: a lack of doubt may be a physicist’s worst enemy¹²³. The assurance with which both: GR fully/correctly describes the solar system’s (curved spacetime based) mechanics, and that theoretical “reductionism” (e.g. GR to QMs) is currently favoured;

¹²²An increase in the number of ‘anomalies’ is a feature of a “degenerative” research program. For example, see: “Do we really understand the solar system? (Lämmerzahl, Preuss, & Dittus 2006)”

¹²³Before Hubble’s Law (1929), the *assumption* of a ‘static’ universe was the generally accepted view.

are the primary accepted beliefs to be confronted herein — at least in the low energy, mass era of the universe.

4.1.3 Physics as possibly one step ontologically removed from “deep-reality”

Physics relies ultimately on observations to understand the world. It is worth mentioning at the outset that mass, spacetime¹²⁴, and the meaning of intrinsic curvature remain somewhat mysterious beyond their role in physical law. These are philosophical (rather than scientific) concerns, that may be a waste of time to some physicists; but while anomalous phenomena exist, philosophy of physics issues should not lie beyond consideration.

The observations of physicists, regarding gravitation beyond Earth, utilise electromagnetic (EM) radiation. It is not clear whether “measurements exhaust reality” or whether a physical Kantian-like¹²⁵ distinction between phenomena (measurement) and noumena (a reality beyond direct measurements) is valid. Note that we have stated “a Kantian-like distinction” as compared to explicitly employing Kant’s version of the distinction. Nevertheless, the terminology shall be retained herein with the *new* (or rather modified) meanings of ‘phenomena’ and ‘noumena’ gradually outlined.

If this distinction is valid, then local measurements might be one step removed from a further reality, i.e. a reality with non-local (i.e. global or systemic) hidden variables. What is certain is that physicists generally reject such a Kantian distinction¹²⁶, and that *local* hidden variables cannot exist. Yet, physics concerns what we can say about the world or nature, not what nature is¹²⁷; and the distinction cannot be easily resolved, nor easily removed.

4.1.4 A wider sense to use of the word “gravitation”

Herein the expression/word “gravitation” is used in a wider sense than normal, since we are entertaining the fact that (predominantly macroscopic) mass, momentum, and energy (comprising GR’s stress-energy-momentum tensor) may not be the only contributors to non-Euclidean geometry. Microscopic-based energy, by way of a virtual¹²⁸ intrinsic angular momentum offset per specified cycle time (i.e. a virtual spin energy), is seen to also contribute to non-Euclidean geometry — in the very special circumstances to be described in

¹²⁴Of which we shall restrict ourselves to just three dimensions of space, and one of time, interlinked *observationally* by the demands of Special Relativity (SR), in conjunction with electromagnetism, to form “spacetime”.

¹²⁵Immanuel Kant (1724-1804), a highly influential German philosopher.

¹²⁶Were it not for the Pioneer and other anomalies, Occam’s razor would strongly support their view.

¹²⁷These sentiments are often accredited to Niels Bohr and Werner Heisenberg.

¹²⁸‘Virtual’ as in below the first energy level or a minimum change in energy levels, and necessarily implying non-local systemic (i.e. global) hidden variables.

Section 5. Later, it shall become clear why this virtual/hidden energy contribution may not be included in the stress-energy tensor¹²⁹. Rather, we need to treat it as an independent and supplementary contribution to non-Euclidean geometry; necessarily expressed as a rotating space-warp, and arising from the ‘experience’ of quantum mechanical (QM) ‘systems’, specifically atoms and molecules, moving in curved spacetime. Importantly, this virtual effect/experience, arising via *analog* curved spacetime, causes no measurable change in the (*digital*/quantum) atomic and molecular (systems) themselves — by way of EM forces being dominant within an atom/molecule.

The term “gravitation” continues to be coextensive with “non-Euclidean geometry”; and the path of any mass in ‘unforced’ motion (in a non-Euclidean geometry) continues to be a ‘geodesic’, in the sense that it is governed by something akin to a “principle of least action”. Although, it can no longer be said that: “matter responds to the geometry of spacetime, as based upon a stress-energy tensor, and nothing else.” Thus, the statement: “*only* a metric theory can describe gravitation,” is necessarily being cast into doubt.

4.1.5 Beyond the scope of GR’s formalism

The model being developed requires the existence of moon-planet *macroscopic* spin-orbit resonance, also known as: synchronous rotation, tidal locking, and phase-lock. This arises from tidal effects acting upon a (non-rigid-body) planet-moon system. Later, we see that a moon-planet-sun (three body) system is important in the new model. It is worth noting that GR is ‘set-up’ to see neither of these circumstances as in any way significant — and certainly not related to (a supplementary) non-Euclidean geometry. Note that the expression “non-Euclidean geometry” is not meant to (in any way) imply the existence of absolute space.

General Relativity itself has issues. It is problematic at small distances and very high energies/temperatures; additionally, the reality of spacetime singularities is an ongoing issue. GR’s foundation stone, the principle of equivalence (Pr. of Eq.), is restricted to local effects, i.e. very small volumes and/or small bodies. For example, in large bodies, tidal effects lead to non-uniformity in field strength, which means the Pr. of Eq. cannot be applied throughout. The point being made is that, solutions to the equations of GR are restricted to quite simple circumstances; but the model herein is based upon a (quite different) complicated set of circumstances, particularly involving three-body celestial motion.

4.1.6 The equivalence principle and the uncertainty principle

Concerning the relationship between GR and QMs, Raymond Chiao dispels any concern regarding the conceptual tension between Heisenberg’s uncertainty principle and the locality of the equivalence principle.

¹²⁹This is due to the non-local role (quantum mechanics based) mass plays in the energy concerned.

... , whenever the correspondence principle holds, the *centre of mass* of a quantum wave packet (for a single particle or for an entire quantum object) moves according to Ehrenfest’s theorem along a classical trajectory, and *then* it is possible to reconcile the two principles (Chiao 2004, p.267).

The tracks of subatomic particles in a bubble chamber illustrate this circumstance.

In Section 5 we shall examine the subtle ramifications of: three-body macroscopic/celestial orbital motion in curved spacetime affecting the geometric/Berry phase of atomic/molecular intrinsic spin — especially the relationship between orbital phase and spin phase in curved spacetime.

4.1.7 Interpretative and ontological stance for quantum mechanics

The idealisation to a classical (and thus simple) approach utilised in this write-up, favours the acceptance of both: quantum non-locality and a realist stance on QM atomic and/or molecular ‘particle’ position — *but only as regards their centre of mass* (recall subsection 4.1.6). The closest interpretation of QMs to this is the de Broglie-Bohm (pilot wave) theory, which incorporates non-local hidden variables. This philosophical stance is *not* being explicitly promoted; rather its ontological preferences overlap somewhat with the model that is to be presented. Unlike Bohmian Mechanics the model denies (QM) particles having definite positions at all times. Additionally, we embrace Decoherence Theory — albeit applied with somewhat non-standard background philosophical assumptions.

4.1.8 Transition from quantum to ‘classical’ behaviour — via decoherence

“Quantum decoherence” is the mechanism whereby quantum systems interact (irreversibly) with their environment, to give the *appearance* of wave function collapse, and hence classical behaviour. Herein we deviate from a strict (reductive) decoherence agenda that seeks to:

...eliminate primary classical concepts, thus neither relying on an axiomatic concept of observables nor on a probability interpretation of the wave function in terms of classical concepts (Joos, Zeh, Kiefer, Giulini, Kupsch, & Stamatescu 2003, Preface to 2nd ed.)

We shall retain the classical conception of the physical world as a ‘complementary adjunct’ to post-decoherence behaviour. A classical formalism, although a secondary approach, is seen as an indispensable aspect of physical description¹³⁰. Thus, ‘decoherence’ herein is primarily associated with a disappearance of *observable* quantum (superposition and entan-

¹³⁰Can the concept of a non-Euclidean geometry exist without the existence, conceptually at least, of a Euclidean geometry?

glement) effects, and (importantly) a suppression of interference effects.

In this [decoherence] process, the quantum superposition is turned into a statistical mixture, for which all the information on the system can be described in classical terms, so our usual perception of the world is recovered (Davidovich, Brune, Raimond, & Haroche 1996, p.1295).

For everyday macroscopic objects the “decoherence time” is extremely short. Thus, bulk matter is considered to be (always) free of (directly measurable at least) quantum superposition (and entanglement) effects. Herein, we shall explore an exception to this rule arising from analog curved spacetime effects upon a QM system — with the effect being *below* a lowest energy level and/or increment.

In the spirit of the decoherence (many-worlds) interpretation of QMs, the QM aspects of macroscopic bodies upon measurement are considered to *not* be obliterated out of existence — as compared to the case with the wave function collapse of the Copenhagen interpretation¹³¹.

4.1.9 On quantum condensate behaviour

Herein, we need to entertain the idea that there is a second way to attain quantum condensate behaviour. In standard QMs, ‘macroscopic’ quantum condensate behaviour is restricted to very low temperature circumstances¹³². Alternatively, very low *energy* circumstances (below a minimum or first energy level), although *hidden* from any direct observation, will starve off decoherence; and may well exhibit coherence over a large number of such atomic/molecular (QM) systems — i.e. a bulk mass. Subsequently, this situation could facilitate a (new) non-standard macroscopic condensate behaviour. Quantum entanglement is an important auxiliary condition for attaining this behaviour.

The origin of such a common (virtual) QM energy, and its relation to the rotating gravitational field perturbations (of Section 3) is the major concern of Sections 5 and 6. Later we see that this energy is indicative of a relative (and virtual) angular momentum offset achieved (through self-interference) over a certain (finite) ‘process’ time interval (Δt).

4.1.10 On the addition of (rate of change of) angular momentum

Feynman (1965, p.80) ruminates upon the fact that, even though (classically) angular momentum depends upon a ‘projection’ angle, quantum mechanical angular momentum always involves integer multiples of some quantity — regardless of the axis about which we measure. Certainly, within a quantum *system* we have an inability to ‘count up’ angular momentum, as

with (say) electric charge; but this additive restraint need not be extended to duplicate or multiple quantum systems (i.e. atoms and molecules) exhibiting a common (i.e. equal and shared) effect. For the model, the effect in question is a common *rate* of (virtual) angular momentum offset (effectively) extending over an entire macroscopic body — in our case a moon.

4.2 Tensions of the model regarding reduction, unification, and GR

As was the case in section 4.1, this section constructs further building blocks for the new model, but more importantly it begins the process of ‘deconstructing’ certain presumptions that stand in the way of the new model.

4.2.1 Celestial “gravitational” motion as a dissipative process

General Relativity is based upon a perceived conservative or non-dissipative process; and thus the useable (kinetic) energy of a body in motion cannot be ‘lost’ in the manner argued herein for the Pioneer spacecraft. Loosely speaking, this paper has argued that cyclic variation in acceleration/gravitational field strength (at a point), arising from (new) acceleration undulations, detract from ‘useable’ kinetic energy (recall subsection 3.2.9). Thus, by way of this supplementary space-warping/curvature, the motion (of low mass bodies) is seen (from an Earth-based or barycentric perspective) to be depreciated or ‘dissipated’, and hence the motion is time-asymmetric. Our common sense notion of time-irreversibility for macroscopic phenomena is reinforced; but *not* the standard belief that all gravitational theorisation is inherently time-symmetric.

Section 2 and 3’s hypothesising of a non-systematic based (real) Pioneer anomaly implies that: the astrophysical/celestial motion of low mass bodies in the solar system joins the ranks of other macroscopic phenomena *always* acted upon by some type of dissipative effect, such as: air resistance, friction, and increasing entropy for example¹³³. It should be noted that there is a distinction between microscopic/quantum and macroscopic phenomena regarding the basis for dissipation. The uncertainty principle ensures time-asymmetry at quantum scales¹³⁴, whereas a variety of processes, including the second law of thermodynamics, are responsible at ‘classical’ scales. We shall see that the uncertainty principle (applied to energy, expressed as a rate of angular momentum) is the ‘facilitator’ of the new supplementary space curvature being proposed.

¹³¹Obviously, experiments do disrupt a system’s wave function but ‘reality’, the ‘subject’ of the measurements, necessarily persists — albeit quite differently post-measurement or post-decoherence.

¹³²There is also decoherence by thermal emission of radiation.

¹³³Note that this comment/declaration ignores the well known effects of (various) celestial radiative forces, and is thus restricted to our newly proposed non-Euclidean space-time effect.

¹³⁴For example, quantum vacuum fluctuations provide disturbances that may result in radioactive decay.

4.2.2 Linking a very large number and a very small (energy) number

Herein (loosely speaking), the quantum mechanical energy-time uncertainty principle ($\Delta E \Delta t \geq \hbar/2$) is given a new interpretative twist. We exploit the fact that (unlike Δx , Δp , and ΔE) Δt is not an operator belonging to a particle (see subsection 6.3.2 for further discussion). Energy-time uncertainty can also be seen to give an *upper* bound to the “no change” condition of an *external* effect imposing itself upon a QM system’s internal (spin) energy over a given (cyclic/process) time (i.e. its intrinsic angular momentum). If $\Delta E \Delta t < \hbar/2$, then *no* quantum mechanical influence upon the system can be expressed within the (microscopic) system itself — with such effects considered to be “below the first (or minimum) energy level”, and hence below a conceivable decoherence threshold. Thus, the state of the (microscopic) quantum system is *not changed*. Note that regarding this new externally driven effect — dependent upon motion in curved spacetime — an exact QM energy (albeit virtual) is assumed to exist in this interpretation of the energy-time uncertainty relation.

A (virtual) energy ‘difference’ (ΔE) conceivably exists. For the moment let us simply say that this energy is only meaningful at a ‘universal systemic’ level. We assume that by way of curved spacetime, the global inertial conditions for a QM system can differ from the local QM equilibrium conditions — when a QM system is taken around a closed loop (see Section 5). Later, we argue that this (virtual) energy difference quantifies the QM system’s non-inertial status — from a universal systemic perspective.

With specific regard to the model, the (maximum) non-inertial energy “not internally incorporated”¹³⁵, that can be associated with Dirac’s constant¹³⁶ (minimum angular momentum) per (typical) lunar spin-orbital period: $\Delta E \sim \frac{1}{2} \hbar \Delta t^{-1}$, is of the order of 10^{-40} Joules. With typically 10^{50} atoms/molecules in a large (solar system) moon¹³⁷, an energy effect *commonly* shared by all atoms/molecules could (conceivably) involve (up to) approximately 10 gigajoules of energy.

Later we shall see that this commonality, i.e. space curvature affecting neighbouring lunar QM (atomic/molecular) systems in effectively the same way, is possible (in part) because of the small diameter of a (celestial) moon, relative to the distance from a planet to its moon. In the model, it is the *total* (virtual) energy that gets re-expressed *externally* to appease a universal (systemic) conservation of energy constraint — arguably as (one of) the rotating space-warps discussed in Section 3. The manner of the external curved spacetime (input) effect that influences QM internal energy is discussed very briefly in the following two subsections 4.2.3 and 4.2.4.

4.2.3 The basis for (quantum) condensate behaviour, and QM spin

Landau and Lifshitz use the example of a dielectric, where a non conductor gets a displacement charge, to argue that:

the separation of the first energy levels of a macroscopic body may even be independent of the size of a body, as for example in the electronic spectrum of a dielectric (Landau & Lifshitz 1958, p.14 fn.).

This independence from the size of the body is an important concept in this paper.

Moving away from electromagnetic phenomena, we seek to show that condensate behaviour may arise in relation to the same (i.e. a common) effect, below the first (or a minimum) energy level, acting upon every atom/molecule’s (elementary fermion/matter particle-based) intrinsic angular momentum (over a given process time), i.e. the spin energy. This common effect is simply additive, yielding a total virtual energy.

Note that in QMs, total energy is (also) a function of QM spin.

Historically, the application of an external magnetic field to electrically neutral particles (or atoms), led to the ‘discovery’ of the quantised nature of intrinsic/spin angular momentum¹³⁸. Herein, we seek to show that the presence of curved spacetime acts upon QM (spin) angular momentum, resulting in something new — albeit (only) if (and when) non-decoherence behaviour of a many atomed/moleculed body is ‘active’.

4.2.4 The basis for the energy uncertainty

Assuming a systemic reference frame datum and incorporating moon-planet orbital resonance, looped motion in curved spacetime is seen to slightly affect (i.e. $\leq 1/2$ a wavelength¹³⁹) the wave phase of *spin* angular momentum (per cycle time) but *not* orbital angular momentum ‘within’ an atom or molecule. To maintain a multitude of electromagnetic internal/QM angular momentum coupling relationships as well as conservation of energy, as they would be in a flat spacetime, the QM system(s) involved simply export any imbalance arising from motion in curved spacetime (S/T). This scenario is restricted to a *virtual* imbalance, i.e. below a first or minimum change in energy level.

Importantly, this virtuality permits a common spin “precession” *direction* and magnitude, *relative* to inertial frame conditions, to (conceivably) apply to *all* elementary fermion/matter particles within *all* atoms/molecules regardless of their ‘internal’ spin orientation. With a great many atoms/molecules similarly involved, the (real) rotating space-warps previously hypothesised are (conceivably) the *singular* (condensate) expression of this energy exportation/deferral process. Thus, internally/microscopically it is as if this

¹³⁵Note that we drop the term “energy uncertainty”.

¹³⁶More commonly referred to as the “reduced Planck constant”.

¹³⁷See Table 4 at the end of section 6.3.

¹³⁸In retrospect, the first direct experimental evidence of particle spin (e.g. the electron spin) was the Stern-Gerlach experiment of 1922, named after Otto Stern and Walther Gerlach.

¹³⁹Circumstances where the effect is $>1/2$ a wavelength exist but they are not relevant to this discussion.

(new) curved spacetime based effect is not there; but from a ‘universal’ systemic perspective things are quite different. The process illustrates nature ensuring stability in a simple, elegant and economic manner.

4.2.5 Questioning physicists’ objective — regarding reduction and unification

Upon removal of dissipative effects the laws of mechanics are considered to be time reversal invariant. ‘Conservative’ systems are described by a Lagrangian and/or Hamiltonian formulation, e.g. Hilbert’s variational approach to GR’s field equations. Time symmetry (in this sense) supports the reductive inclinations of contemporary physicists — and draws confirmation from the previously successful reduction of thermodynamics to statistical mechanics. The generalisation of reduction to all macroscopic phenomena needs to be recognised as a goal or agenda and not ‘a given’.

The current unification agenda/goal of physicists; i.e. weak, strong, electromagnetic, and gravitational ‘forces’, is ostensibly ‘force’ based and involves the exchange of particles. Other macroscopic phenomena such as: elasticity/deformation of materials, springs (Hooke’s Law), and rotor/propeller behaviour in fluid mechanics¹⁴⁰ are assumed to ‘fall in line’. Is this (to date unsuccessful¹⁴¹) metaphysical¹⁴² reductive agenda too simplistic? Anecdotal evidence suggests that both Niels Bohr and Erwin Schrödinger doubted a grand unified theory was possible.

In this paper the emphasis is upon an interaction *between* the microscopic and macroscopic realms — in the form of an energy *re-expression*. The word ‘re-expression’ is preferred to transfer, because the latter has particle connotations. The nature of their *co-existence*, from a global/systemic perspective, is considered to lead a new (“emergent”) phenomenon, i.e. (gravito-quantum) rotating space-warps¹⁴³ — this being the external energy expression of unaccounted-for (virtual) (and non-inertial) QM energy. Possibly “non-reductive” is a better word than ‘emergent’. We cite quantum mechanical non-local entanglement as the mechanism/‘vehicle’ by which this re-expression is achieved.

Alternatively, if we assume the correctness of GR and QMs, and that the current reduction and unification agendas/objectives are correct; then a ‘real’ Pioneer anomaly is at odds with accepted theory and these

related agendas. Subsequently, a systematic explanation (involving heat) is experiencing favour within the Pioneer anomaly community; essentially to make the anomaly “go away”.

4.2.6 The differences that divide microscopic and macroscopic physics

The existence of a divide between the microscopic (i.e. up to atoms/molecules) and macroscopic/classical realms, and the laws describing them, is undeniable. Aspects of this circumstance include:

1. The inherent incompatibility of GR and QMs.
2. The absence of electron radiation within atoms.
3. The nature of dissipation (recall 4.2.1).
4. The ‘exact’ symmetries (and hence quantised gauge theories) displayed by microscopic systems cf. the approximate symmetries displayed by macroscopic systems.
5. The role of special relativity is primary in QMs, whereas in a variety of celestial circumstances, e.g. the solar system and galaxies, SR plays a minor role.

On the other hand, an aspect of commonality between the micro and the macro realms is waves and wave superposition; simply because wave behaviour is ubiquitous.

To proceed with a real explanation of the Pioneer anomaly, the *divide* between micro- and macroscopic physics needs to be emphasised. In the following subsection (4.2.7) we examine if a force and particle platform, as used in microscopic physics, is sufficient for the description of a significant (non-gravitational) macroscopic physical phenomena.

4.2.7 The role of energy and power in the Vortex Theory of Propellers

In general physics, energy is considered to be the capacity (of a system) to do work; with work defined as force applied over a distance. This implicitly assumes that force (usually in conjunction with energy) is *necessarily* a primary quantity in the analysis of major physical interactions.

Theodore Theodorsen’s Theory of (Aircraft) Propellers (Theodorsen 1948) primarily examines the physics of a ‘frictionless’ propeller/airscrew, i.e. drag effects are ignored. Minor corrections may then be made for drag effects. A vortex/circulation¹⁴⁴ approach is employed, with this approach being uniquely applicable to (low solidity¹⁴⁵) airscrews and (horizontal axis) wind turbines cf. other types of (high so-

¹⁴⁰Which are described “exactly” by (an idealised) continuum mechanics approach, or (alternatively) approximately by finite element methods.

¹⁴¹The failure of string theory and quantum loop gravity to deliver a successful unification, or a falsification criteria, makes them currently non-rigidly scientific. This may change, but the recent advent of a perceived accelerating universe expansion was not even upon the radar of these “cutting-edge” approaches. Both of these approaches are particle based, and hence assume reduction is viable. For more information see Cartwright & Frigg (2007), and George Ellis’ review (Ellis 2006) of Lee Smolin’s “The Trouble with Physics” (Smolin 2006b).

¹⁴²In the sense of: based on speculative or abstract reasoning.

¹⁴³Together with a “non-local mass” distribution.

¹⁴⁴“Circulation” is a fluid mechanical term. It is essentially a macroscopic continuum mechanics equivalent to angular momentum, and closely related to the (velocity) potential in a “potential flow” analysis. In fluid dynamics, potential flow describes the velocity field as the gradient of a (scalar) velocity potential — if the flow is irrotational.

¹⁴⁵Solidity refers to the amount of (swept) disc area occupied by the (solid) material blades themselves.

lidity) *open* turbomachinery¹⁴⁶ such as ship propellers and (household) fans. It is worth noting that:

It was not until 1929 that [Sydney] Goldstein solved the potential flow problem completely for a lightly loaded single-rotation [aircraft] propeller of small advance ratio. This was unquestionably the greatest single step in the evolution of the propeller theory (Theodorsen 1948, p.1).

Equally significant, Theodorsen extended the theory, with its ability to determine optimum (i.e. minimised energy loss) circulation distributions, to incorporate non-light/heavy propeller blade loadings and pragmatic advance ratios.

Utilising a control volume, dimensionless expressions for: shaft power (P), blade thrust (T), and energy lost to the wake (E) are derived (pp.28-29), which may then be converted back to their dimensional quantities¹⁴⁷. The point being made is that a force basis fails to *accurately* represent (i.e. model) the less than 100% efficiency of converting (mechanical) rotational energy/power into linear (aerodynamic) thrust work. A power based expression $P = TV + dE/dt$ is the equality describing the system's behaviour, where V is (freestream) airspeed. In physics, rather than aeronautical (or aerospace) engineering, a force basis to the power expressions is preferred, in conjunction with an efficiency (η) factor¹⁴⁸; usually written as $\frac{1}{2}\rho AV^3\eta$ — the emphasis being the kinetic energy of the airflow relative to an aircraft's propellers. This is an inferior explanatory approach because the efficiency factor is not theoretically determinable as is the case with the vortex method¹⁴⁹.

In the theory (Theodorsen 1948, Ch.4) the dimensionless power coefficient $c_p = 2\chi\bar{w}(1 + \bar{w})(1 + \frac{\epsilon}{\chi}\bar{w})$ [Equation 37, p.29] can be (with some algebraic effort) re-expressed as a *dimensional* power equation $P = \rho AVn\Gamma_e$ which then may be reduced to a new type of (potential) energy equation/expression:

$$E_p = \rho A L \Gamma_e n = m \Gamma_e n \quad \text{where} \quad \Gamma_e = B \bar{\Gamma} (1 + \frac{\epsilon}{\chi} \bar{w})$$

or alternatively (and more appropriately herein)

$$E_p = m \Gamma_e f \quad (15)$$

¹⁴⁶Examples of *closed* turbomachinery are numerous, including: centrifugal pumps, water turbines (e.g. Pelton wheel and Kaplan turbine, steam turbines and (jet aircraft) gas turbines.)

¹⁴⁷Theodorsen's theory remains (in a sense) the pinnacle of classical propeller theory. Its development during the late 1930s/early 1940s coincided with the dawn of the jet age. The theory is just as effectively applied to propeller-style wind turbines. Some iteration is required for optimisation, and it is necessary to optimise for a single 'design' airspeed.

¹⁴⁸Similarly, for an airfoil's lift, a simple action-reaction force basis is given preference by physicists. The circulatory airflow, where the 'work' is done, is given either secondary emphasis or no emphasis.

¹⁴⁹Finite elements methods are preferred today because the effects of: engine nacelles, aircraft fuselage, etc., may be incorporated. Vortex theory treats the rigid vortex sheet, with its surface of discontinuity, in isolation from *all* interference effects — including drag arising from fluid viscosity.

where: m ('air' mass) is the fluid density (ρ) multiplied by cylindrical volume (blade swept area 'times' the axial length/distance — associated with one revolution), $n = f = (\omega)(2\pi)^{-1}$ is the number of (complete) revolutions per second, and Γ_e is (newly) defined as the effective circulation around the blades. Γ_e (with 'dimensions' $[\frac{L^2}{T}]$) is essentially the number of blades times the average circulation (across each blade) multiplied by the (Theodorsen) efficiency factor of an *optimum* propeller — i.e. one where the trailing flow, in a frictionless fluid, is shed into the wake as a "rigid" vortex sheet, i.e. a (three-dimensional) 'solid' helix or helicoid¹⁵⁰ — which together with a (two-dimensional) plane is the only other surface of revolution that is also a *ruled* minimal surface¹⁵¹.

Of interest here is firstly, the fact that this new (fluid mechanical circulation based) energy expression is proportional to the rate of (cyclic) rotation (n or f), so there is (also) a *unit energy* corresponding to a *single rotation* of the blades; and secondly, that a solely force and/or kinetic energy basis fails to be sufficient to (theoretically) best represent this linear *and* rotational system — which can only be realistically modelled in *three* spatial 'dimensions'¹⁵².

Thus, we have a further difference between macro- and microscopic systems; in terms of the most dimensionally "rich" physical quantity (i.e. power $[\frac{ML^2}{T^3}]$ cf. energy $[\frac{ML^2}{T^2}]$) needed to 'best' model the essence of a system's physical behaviour — albeit that the case of 'vortex' propeller theory is somewhat idealised and "old school" in its approach.

4.2.8 Energy to frequency proportionality

An alternative physical expression of energy, other than work (a force applied over a distance), is wave energy; where the wave's energy is proportional to wave amplitude squared. A further alternative route to energy is the representation of electromagnetic wave/radiation energy as proportional to a frequency; i.e. Planck's formula¹⁵³ $E = hf$. In general, frequency can be conceived as either (and sometimes both) indicating: a rotational process that occurs over a finite time¹⁵⁴, or simply the number of (wave) oscillations

¹⁵⁰At optimisation the (dimensional) displacement velocity ($w = \bar{w}V$) of the airflow, for all radial locations along the blade span, are equal; it is positive for propellers, and negative for wind turbines. In the nomenclature of Theodorsen $\Gamma_e = wH\chi(1 + \frac{\epsilon}{\chi}\bar{w})$ where $wH\chi = \bar{\Gamma}B$ with $\bar{\Gamma}$ being the average circulation 'over' each blade, B is number of blades, ϵ is the axial energy loss factor, and χ is the (all important) "mass coefficient" — see Theodorsen (1948) for further details. H is the pitch of the rigid helix, with $Hn = V + w$. A further (minor) correction for propeller wake contraction, or wind turbine wake expansion, has not been included.

¹⁵¹Such that: for any point on the surface there is a line on the surface passing through it; and with a zero mean, the surface area is minimised. Note that a 'catenoid' only exhibits the latter characteristic.

¹⁵²The far wake (also) plays a pivotal role in the model.

¹⁵³Also called the Planck relation or the Planck-Einstein equation, and alternatively written as: $E = h\nu$.

¹⁵⁴Philosophically, time in-itself, can be conceived as process based; this being an unorthodox (although well estab-

over a given time — with the role of a time standard, i.e. the second, essential. Note that precision clocks are frequency based.

The vortex theory energy (Equation 15) discussed in subsection 4.2.7, with mass as fluid density multiplied by a cylindrical volume (and thus non-solid), is a macroscopic instantiation of an energy to ‘frequency’ proportionality. When energy is proportional to frequency (i.e. revolutions per second or cycles per second), the importance of angular momentum is paramount; whether (expressed) as a macroscopic fluid circulation or by way of Planck’s constant (re: electromagnetic radiation/waves).

Later, we see that the energy of interest herein is a geometric phase based intrinsic/spin angular momentum (offset¹⁵⁵) *per cycle time*, or angular momentum multiplied by frequency; i.e. spin energy — with this energy being below the first and/or minimum energy level, or below a minimum energy uncertainty ‘level’, in/of an atomic/molecular ‘quantum’ system. This energy, and the energy of vortex propeller theory, may be contrasted with the standard *position*-based potential energy (P.E.) at a given time (of Classical Mechanics). Conventional P.E. has no role to play here, although for our purposes we may still speak of the supplementary field energy *over a given unit cycle time* Δt , at a given distance from the (rotating space-warp’s) energy source¹⁵⁶ — and similarly, when a moving body’s positional change over Δt is considered insignificant (when compared to the scale of the system).

The aim of this subsection has been to show that requiring potential energy to be exclusively position (i.e. location) based is somewhat misguided; because classical vortex theory (of a *three*-dimensional propeller¹⁵⁷) and to a lesser extent Planck’s formula in electromagnetism provide counter examples. Non-kinetic energy, i.e. ‘potential’ energy¹⁵⁸, is conceivably also frequency-based, and/or alternatively cyclic process time-based. Recall $\Delta e = \frac{1}{2}\Delta a^2\Delta t^2 = \frac{1}{2}\delta v^2$ from subsection 3.2.8; proposing that the *specific* ‘potential’ energy of the proposed supplementary oscillatory gravitational field (per cycle¹⁵⁹) equals the *unsteady* specific kinetic energy component of a body in otherwise steady (i.e. non-oscillatory) motion (per cycle).

4.2.9 Is macroscopic gravitation mediated by the graviton particle?

Returning to a discussion of four force unification, we accept that the three micro forces are particle based¹⁶⁰.

lished) stance.

¹⁵⁵The description of what is involved in this offset, and what this offset is relative to, is far from trivial — see subsection 5.4.5.

¹⁵⁶Nothing we are (at present) restricting ourselves to position in a plane when discussing (gravito-quantum) rotating space-warps (GQ-RSWs).

¹⁵⁷The trailing flow far downstream of the blades is fundamental to the theory.

¹⁵⁸With ‘potential’ now used in a wider sense, in that it is not exclusively a function of position.

¹⁵⁹Per cycle, or alternatively, over the course of a full cycle.

¹⁶⁰As evidenced by the success of the Standard Model of particle physics.

On the contrary, GR is primarily a geometric theory that is considered to be mediated by the (to date undetected) graviton particle. Our proposed supplementary space curvature, on the other hand, cannot be a standard particle based phenomenon, because it is (partly) QM-based and exhibits a non-local ‘influence’ aspect — although changes to the strength/amplitude of undulations conceivably propagate at the speed of light. Awkwardly and inevitably, some form of very-fast hidden variable theory appears to be required in the conjectured model — to convey the impression of either a non-local ‘correlation’ or ‘action’. The later is inconsistent with the particle based (reductive) theory agenda of modern physicists that employs the speed of light (c) upper limit (to information exchange); whereas a correlation-based understanding is *not* inconsistent with experiments illustrating the presence of quantum non-locality. In this sense, non-local QM (spin) effects only *appear* to involve an instantaneous causal ‘action’. Contemporary physics cites entanglement as a precedent (condition) for non-locality.

The explanatory waters are muddled (so to speak) by the fact that the existence of the graviton particle remains an open issue. The non-detection of a graviton particle means that: possibly GR’s spacetime geometry is ‘enforced’ by a non-particle (non-graviton) mechanism — albeit with changes propagating at the speed of light.

4.2.10 Hypothesising a non-local mechanism

The following proposal of a ‘hidden’ supplemental mechanism for ‘explaining’ non-local correlations is non-rigorous. Time (and mass) even today retain a certain mystery, and particle based interactions in (macroscopic) ‘spacetime’ *may* not exhaust reality. Conceivably, the universe’s evolution might only *appear* analog; such that from a wider (temporal¹⁶¹) perspective it may well evolve in a staccato manner, that is inherently imperceptible to physical observations — (conceivably) by way of the limits imposed by Heisenberg’s uncertainty principle (HUP). Thus, physical reality would effectively ‘occur’ in a digital (go-pause-go-pause...) manner, with a new type of ‘process’ occurring between the instants/moments (or states) of measurable time, i.e. in the ‘pause’; thus complementing reality as is currently understood.

Since this process lies outside of (or beyond) observational time, and within limits imposed by HUP, it is effectively invisible to (i.e. *hidden* from) measured reality. Some type of (unknown) cosmological size/scale *wave* effect(s) is one conceivable, yet vague, option for what this process involves¹⁶². This cyclic wave process facilitates a form of subliminal whole-universe communication and interaction, that is quite conceivably ge-

¹⁶¹Immediately, upon their use in (general) discussion, the meaning of the words “time” or “temporal” becomes inexact and somewhat ambiguous.

¹⁶²If rotational, the direction of (possibly two orthogonal) rotation(s) could leave a signature that results in the laws of physics being slightly different for matter and anti-matter.

ometric and topological¹⁶³. Subsequently, non-locality can regain a (new type of) ‘cause and effect’ explanatory narrative. Additionally, invariance and conservation demands may be appeased; particularly regarding (non-local) QM intrinsic angular momentum. This hypothesis is consistent with the phenomena–noumena distinction introduced/discussed in subsection 4.1.3.

Finally, this cosmological process allows the universe to evolve in a temporally harmonious/coordinated (and stable) manner. Due to the imperceptible brevity between (digital) ‘moments’, we effectively have a hidden mechanism that ensures a universal *background* time simultaneity — even though clocks may run at different rates within the observable universe. Section 4.4 shall elaborate upon this (admittedly sketchy) hypothesis.

4.2.11 Interim conclusion and subtleties of the model

Neither the non-particle, nor the non-reductive aspects, of our proposed supplementation to non-Euclidean geometry, is sufficient (as yet) to scuttle the model being pursued. Further, the differences between the micro- and macroscopic domains (and their theoretical descriptions) has been emphasised. The non-simultaneity of special and general relativity (SR and GR) presents a far more challenging denial of the model, and this shall begin to be addressed in subsection 4.3.

The model’s need to externalise a (coherent) virtual quantum spin energy offset as a large scale (rotating) acceleration/gravitational field perturbation implies a global nature to the model, and hence global co-ordination between the microscopic (QM) and macroscopic (curved spacetime) domains. This in turn seems to require a coordination that effectively exhibits temporal simultaneity, reminiscent of (historical) classical mechanics; and thus the model would benefit from interpreting non-locality as implying some form of *apparent* instantaneous ‘causation’ (recall subsection 4.2.10). Non-locality regarding QM spin is undeniable, and remains open to interpretation. Appropriately, QM (intrinsic) angular momentum is central to the ‘global’ model being hypothesised. It is insufficient to simply cite GR’s $v < c$ information speed limit and then reject the model, because the model involves *both* curved spacetime and QMs acting in concert.

From an (idealised) classical conception, we can (with some imagination) visualise a *constant amplitude* rotating space-warp, with the acceleration amplitude representable in a field based manner¹⁶⁴ (recall Figures 5 to 7). As a first stage idealised approach, the use of classical mechanics involving: (a QM process based) energy, a continuum of space, rotating acceleration perturbations, and utilising (clas-

sical) time simultaneity, appears feasible. Further, if we regard the accelerations (equating to gravitational field strength) proposed in Section 3 as ‘true’ accelerations, in the sense of reference frame independent ‘proper’ accelerations; then (qualitatively) any *ensuing* speed perturbations (of low mass bodies) also need not be frame dependent. Note that even though the rotating space-warps (RSWs) rotate about a central point/region, and the associated acceleration is considered a ‘true’ (cyclically) varying acceleration, the motion of any body ‘receiving’ this influence is inevitably described using a reference frame — e.g. the Pioneer spacecraft relative to the solar system’s barycentre.

4.3 Tackling fortress General Relativity and its non-simultaneity

The aim of this and the next section (4.4) is to cast doubt upon the ontological commitments of GR. By assuming a ‘real’ (i.e. non-systematic based) Pioneer anomaly we necessarily take issue with GR’s confident assertion that: there are no (indeed never) global reference frames, by way of an underlying acceptance of relativism. This leads into subsection 4.4, where an alternative interpretation of: time and length dilation is examined, so as to illustrate that space and time’s co-existence or ‘union’ may be something other than *only* SR’s and GR’s ‘spacetime’. It would be so much easier to simply declare the model non-special-relativistic, and indeed the sceptical reader should keep this in mind; but the model appears to demand simultaneity of time (throughout the field), by way of quantum non-locality being involved. This is compatible with a solar system (systemic) time standard; one of whose roles is to quantify (a RSW’s) rotation duration (Δt).

4.3.1 Physicists and philosophical thought

Science involves both concepts and mathematics, and usually physics has a mathematical emphasis because the concepts involved are stable. Here the emphasis is strongly conceptual, because we are attempting to establish *new* physics, in the sense of a supplement involving an alternative conceptual structure, or “disciplinary matrix” as philosophers of science would call it. We do this not in order to unnecessarily attack physics, but merely to resolve at least one (Pioneer) anomaly, and possibly further our understanding of how curved S/T and QMs coexist in a celestial material *system*.

The huge success of physics from about 1930, when Paul Dirac’s *Principles of Quantum Mechanics* was published, has led to today’s theoretical physicists under-valuing conceptual discussion. About 1975 the standard model was ensconced, and 1975 is when Vera Rubin and Kent Ford’s observations of spiral galaxy flat rotation curves appeared, marking the beginning of significant “unfinished physics business”; e.g. dark matter and dark energy. From a (Thomas) Kuhnian perspective (Kuhn 1970), the years 1930 to 1975 typifies a period of *normal science*, and 1975 (possibly) heralds the beginning of a rise in conspicuous scientific issues and anomalies — which may require rethinking our conceptualisation of the physical world.

¹⁶³Note that there is no *observational* difference between the digital ‘reality’ moments occurring “all at once”, or sequentially ‘around’ the universe — by way of an observationally subliminal ‘scanning’ process. The latter scenario seems more appropriate and viable.

¹⁶⁴Varying cyclically (and sinusoidally) at a ‘fixed’ point in a barycentric reference frame.

[Indeed at] ... the beginning of the 20th century — Einstein, Bohr, Mach, Boltzmann, Poincare, Schrödinger, Heisenberg — thought of theoretical physics as a philosophical endeavor. They were motivated by philosophical problems, and they often discussed their scientific problems in the light of a philosophical tradition in which they were all at home. For them, calculations were secondary to a deepening of their conceptual understanding of nature (Smolin 2006a).

Assuming a real Pioneer anomaly implies something of a (minor) crisis in physics, and necessitates a return to this early 20th century conceptual approach. If the model went straight to the mathematics, we would be implicitly assuming that any associated conceptualisation was non-problematic — i.e. normal science or “business as usual”.

4.3.2 Looking for cracks in the General Relativity fortress

It is worth mentioning that herein we accept without question the standard model’s unification of the three (microscopic) forces/interactions (weak, strong, electromagnetic¹⁶⁵) that incorporates SR. It is the extension of this to (large-scale macroscopic) gravitational effects that is our concern.

Loosely speaking, GR describes the space between *points*. Aspects of GR include:

[The Relativity Principle:] The laws of nature are merely statements about spacetime coincidences; they therefore find their only natural expression in generally covariant equations (Barbour & Pfister 1995); [and] the distance between adjacent points is expressed by a metric equation ... The metric coefficients in the metric equation depend on the arbitrarily co-ordinates chosen and geometric properties of the space are expressed in the form of differential equations showing how the metric coefficients vary from place to place (Harrison 2000, p.198).

Since GR is a generalisation of SR, the theory necessarily builds upon the mathematical structure of SR, more so than SR’s conceptual structure. Central to GR are Einstein’s equations, which link the geometry of a four-dimensional semi-Riemannian manifold representing spacetime with the energy-momentum contained in that spacetime.

Further, since GR is a generalisation of SR, it has an inherently localised (theoretical) starting-‘point’, providing no conceivable avenue with which one could express the (as yet to be derived) excess QM *energy* arising from the closed loop (finite time) motion of (bulk) QM matter in curved spacetime upon a geodesic. The supplementary space curvature (proposed herein), although not explicitly expressed as a

metric theory, could possibly be supplemented to GR’s formalism as a pure energy supplementation — but the need for an apparent simultaneity/non-locality feature in the new model remains in direct conflict with the *pairing* of: SR’s non-simultaneity and GR’s general principle of relativity¹⁶⁶. These two aspects of relativity ensure that GR does not permit/contain any invariant geometric background structures. A solution of the Einstein equations consists of a semi-Riemannian manifold (usually defined by giving the metric in specific coordinates) on which are defined matter fields.

Each solution of Einstein’s equations describes a whole history of a universe; i.e. we have a “blocktime” basis to the physical explanation. A prominent and persistent philosophical concern of “blocktime” is that it effectively denies an individual’s freewill. This inconsistency with our own perceptions is a central issue in philosophy, and is reason enough to examine the bedrock upon which GR is constructed. Additionally, GR’s non-linear partial differential equations are very difficult to solve, and only a few exact solutions have direct physical application¹⁶⁷.

4.3.3 The ‘principle’ of general covariance: the logic behind it and its generality

Special relativity eliminated absolute space and absolute motion. The theory of general relativity is an extraordinary theory of gravitation, and has been an unmitigated success. With GR, Einstein sought to extend SR’s *relativity* of motion to acceleration, so as to generalise relativity; and with the possible exception of the Pioneer anomaly, this conclusion appears quite reasonable. If we accept that acceleration is (also) relative, then a number of implications follow:

1. Most importantly, an acceptance of a *principle* of general covariance.
2. Matter, momentum and energy distribution are the sole determiners of spacetime curvature.
3. Gravitational theorisation must utilise a metric, because the structure of spacetime is encoded in the metric field tensor, with the curvature encoding ‘gravity’ at the same time.
4. The principle of equivalence applies to all forms of accelerated motion.
5. There can be no privileged reference frames.

Accepting the strict relativism of Einstein’s GR then strongly supports a reductive stance to physics, especially four force unification.

The first of these listed points is the most important, with the other points logically dependent upon it. Our concern is not with GR as concerns celestial matter, rather it is with a quantum mechanical (QM) system moving in curved spacetime¹⁶⁸.

¹⁶⁶Not forgetting GR’s use of the (Einstein) Equivalence Principle as effectively a ‘foundation stone’ for generalizing special relativistic physics to include gravitational effects.

¹⁶⁷Parts of this subsection paraphrase Wikipedia: *General Relativity*; as modified on 16 May 08, at 02.24.

¹⁶⁸Support is possibly lent to this by way of the black hole information paradox (BHIP). Stephen Hawking concludes

¹⁶⁵Note that electromagnetism has both microscopic and macroscopic aspects.

In the spirit of (Sir) Arthur Stanley Eddington we see (he saw) (QMs and) GR “...as fundamentally ‘epistemological’ in character, meaning that they provided insight into *how* we see the world, rather than *what* the world is (Stanley 2005, p.37).” In this subsection we shall argue, in line with certain philosophers of physics, that general covariance (GCoV) should be regarded as simply a new mathematical technique, and not necessarily an expression of physical content (i.e. an ontological aspect) — especially since the physical implications of SR’s invariance of the interval (under Lorentz transformations) remain far from obvious (or fully settled).

Only if GCoV loses its (general) ‘principle’ status, i.e. beyond GR’s formalism, can our model be viable; because it has features such as: acceleration/gravitation encoding curvature (cf. *vice versa*); no need for a metric nor an equation of (‘point’ mass) geodesic motion; and (crucially) a situation that effectively violates GR’s (weak) principle of equivalence — with this last feature not fully addressed until subsection 6.4.1.

In the philosophy of physics, and for some physicists, the following is accepted.

Einstein offered the principle of general covariance as the fundamental physical principle of his general theory of relativity and as responsible for extending the principle of relativity to accelerated motion. This view was disputed almost immediately with the counterclaim that the principle was no relativity principle and was physically vacuous. The disagreement persists today (Norton 1993, Abstract).

Certainly, GR is a generalisation of SR and subsequently, this greatly restricts any supplementary alternative (rather than modified) gravitational theorisation; but the mathematical technique of general covariance is not necessarily ‘active’ beyond GR (i.e. gaining “principle” status). Thus, the various restrictions to gravitational theorisation (listed above) that (logically) follow from a *principle* of GCoV are not validly deduced. Admittedly, it is one thing to say the principle of GCoV is logically restricted, and another (far more difficult) thing to explain why. Section 6.7 addresses this issue in some depth.

Recalling the exclusions and idealised circumstances associated with general relativity outlined in subsection 4.1.5, GR’s ‘completeness’ credentials are clearly non-‘water-tight’¹⁶⁹. To simply use GR’s relativism (in particular) to deny the supplementation of non-Euclidean geometry proposed herein is totally inappropriate (although understandable) — especially since both curved spacetime *and* (non-inertial aspects of) QM atomic/molecular systems are involved in our model. Note that QM atomic/molecular systems are

the paradox implies the usual rules of QMs cannot apply in *all* situations. Note, this situation (also, like the BHIP) involves QMs *and* gravitation.

¹⁶⁹For example, concerning GR’s *restricted* coverage to non-complicated physical phenomena; in that exact solutions to N-body systems elude (NMs and) GR. Admittedly, numerical methods are promising in this regard.

physically dominated by electromagnetic concerns cf. the inertial (and *mass*-based) concerns of the model.

4.3.4 Non-relativistic aspects of gravitation in the model

Gravitational theorisation involves a clear distinction between accelerating and non-accelerating motion (i.e. non-inertial and inertial motion). The model utilises both: the concept of a rotating space-warp, and a (constant amplitude) *acceleration* (perturbation) that is related to (and induces) the anomalous motion of the Pioneer spacecraft; but the (rotating) acceleration/gravitational field is only indirectly related to (the driving energy of) ‘*motion/movement*’ within (a great many) atoms and molecules. The *specific* potential energy of the proposed supplementary oscillatory gravitational field (per cycle) $\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2$ (recall subsection 3.2.8) is a surprisingly simple law-like expression, with Δt representing the duration time for a full rotation. The need for a systemic time standard is clear.

Regarding gravitation, philosophers of science have argued that it is (*only*) acceleration and rotation that (in at least one sense) effectively resist a completely relativistic interpretation¹⁷⁰; e.g. (hypothetical) whole universe rotation. Hence they retain, not an absolute aspect, but rather in the case of acceleration a ‘true’ aspect (independent of relative motion); and in the case of rotation a sense of being global (or systemic). Fortunately, only these two concepts are involved in the model’s quantification of Δe , and the proposed existence of RSWs (of cosmological extent). Further, with the model’s cyclic acceleration amplitude (necessarily) constant ‘everywhere’, the simplification of the model’s quantification could not be greater¹⁷¹; providing a compelling reason to (further) advance the model. In section 4.4 we do this by way of a continued re-evaluation of SR’s and GR’s ontological commitments.

4.4 Evaluating SR’s and GR’s ontological commitment

In this section we seek to further expose the not so rigid ontological underbelly upon which GR is founded. Indirectly we are examining the scope of GR and questioning ontological assumptions behind SR, that are subsumed into GR. We also outline a variety of ‘invariance’ aspects required for the supplementary ontological perspective to be presented in section 4.5, which (amongst other things) are consistent with the model’s use of a single time variable to describe the whole system. Eventually, it shall be seen that this secondary perspective is not in defiance of special and general relativity (SR and GR), rather it is supplementary; suiting the very different ingredients associated

¹⁷⁰The issues involved herein are very deep and are necessarily muted in their presentation. This and other parts of section 4.3 have drawn somewhat upon an article called “Absolute and Relational Theories of Space and Motion” in the (online) Stanford Encyclopedia of Philosophy.

¹⁷¹No metric, no equation of motion, no stress-energy tensor, etc.

with the model — e.g. virtual QM energy, QM non-locality, and a three-body celestial system. Specifics of GR (on its own), such as the relativistic energy-momentum tensor are incompatible with the model's 'ingredients'. This and the next section are philosophical/conceptual rather than mathematically physical, and concentrate upon re-examining space, time, their coexistence; as well as mass, motion and energy. Our motivation is to eventually explain how a new contribution to gravitation (in its widest sense) can co-exist with the standard approach of general relativity — with the latter (by far) the dominant approach to gravitation.

4.4.1 Concerns regarding an ontological commitment to spacetime

Special Relativity demands that space and time be fused together into spacetime. This arises in part from the 'operational' approach employed in Einstein's SR¹⁷²; this being almost always associated with an additional ontological commitment to the idea that: measurements dictate reality (without any remainder); indeed this is the essence of the scientific approach. Spacetime is seen by physicists as the correct understanding of (now obsolete) classical space and time; successfully rebutting all alternative (in the sense of different) interpretations of the Lorentz transformations that have been proposed. The existence of the latter is indicative of a suspicion that SR's spacetime and GR's (physical) ontology may be somewhat restricted, i.e. not quite the whole 'story'.

Supporting this broad concern are a number of specific issues or concerns. Firstly, locality was assumed 'universal' in the days prior to QMs' full establishment in the 1920s and beyond. Historically, SR resolved the tension between electromagnetism (EM) and classical mechanics, that had become apparent at the end of the nineteenth century. Quantum mechanics' incorporation of special-relativistic effects has been amazingly successful, but non-locality remains a vexing issue. Secondly, in the *formalism* used to describe physics' two realms, the macro- and microscopic, the relationship between space and time is quite different. A locally classical/flat space *background* is (almost always) implicit in (micro) QMs, whereas curved spacetime is the essence of (macro) GR; and further:

... in quantum mechanics, time retains its Newtonian aloofness, providing the stage against which matter dances but never being affected by its presence. These two conceptions of time don't gel (Merali 2009, p.18).

Thirdly, time still remains mysterious outside of physics' (possibly incomplete) accounts of 'reality' in

¹⁷²Operationalization is the process of defining a concept as the operations that will measure the concept (variables) through specific observations. That even the most basic concepts in science, like "length," are defined solely through the operations by which we measure them, is the discovery of Percy Williams Bridgman, whose methodological position is called operationalism (source: Wikipedia, 2011).

its widest sense¹⁷³. Finally, quoting from a philosophical article:

... the conventional view of space-time as the ground of physics is [for some people] increasingly being called into question (Clarke 1993, p.53).

By way of an assumed unification of reality, 'space-time' is standardly seen to be common to the micro- and the macroscopic realms. The new approach pursued herein pursues the coexistence of a secondary *global* "neo-classical" notion of time (and space), that cannot be directly 'grasped' by local observations. This supplementation, by way of two different idealisations, is necessary; so as to physically link (in the model) an energy (magnitude) between both realms seamlessly (and simply). From this perspective, we begin to conjecture that GR's need for general covariance is a direct consequence of (further) adding gravitational effects to the (already additional) conformal structure demanded by SR's Lorentz symmetry.

4.4.2 The failure of alternative approaches to special relativity

S. J. Prokhovnik's "The Logic of Special Relativity" (Prokhovnik 1978) sought to highlight (p.84) that at different levels of description, and/or describing different sets of phenomena, SR's Lorentz transformations can be associated with a secondary (rather than alternative) interpretation, and hence further ontological aspects. We shall not pursue a detailed discussion here. The reader is simply asked to recognise a historical pursuit of (unsuccessful) alternative interpretations of SR's Lorentz transformations, pursued by scientists and philosophers who perceive logical contradictions in SR and 'spacetime', as (possibly indicative of) an incomplete 'grasp' of physical reality — in particular the twins paradox¹⁷⁴. Indeed, Prokhovnik and others have struggled to justify their alternative interpretations; and we shall not champion any of these approaches.

4.4.3 Other indicators that GR may be incomplete and/or restricted

A real Pioneer anomaly demands something additional to GR rather than a simple modification. The elegance of GR tends to imply that it completes gravitational theorisation. There is no assurance of GR's completeness, and subsequently we cannot assume that unification of GR and QMs is the next (and possibly final) major step in theoretical physics. Alternatively, the elegance of GR may be associated with restrictiveness in some sense.

¹⁷³See Martin Heidegger's "Being and Time" for example.

¹⁷⁴For example: consider a two observer, figure eight symmetric version of the twins paradox, involving geodesic motion in a suitably configured asteroid belt. Who's clock runs (relatively) slow when the two S/C, moving in opposing rotational directions, *periodically* observe each other's clocks at the nexus of the 'eight'? Note that asymmetrical acceleration is *not* present in this curved spacetime (*geodesic* motion, i.e. non-force) scenario.

The extrapolation of GR's local gravitational curvature to the global/cosmological case is neither supported, nor denied, by observations indicating that (cosmologically) space is flat/uncurved ($k = 0$). Thus, we may question whether whole universe/global curvature ($k = \pm 1$), implied by GR's perceived *exclusive* role in describing gravitation, is capable of being physically 'realised'. In other words, global (whole universe) curvature remains an unproven conjecture, supported by an assumed 'totality' of GR's scope. It is quite conceivable that GR's domain of application might always be restricted to a subset of the whole universe, applying to the largest of galactic clusters but not necessarily to the universe as a 'whole'. The model needs to enforce this (sub-global to global) distinction.

4.4.4 Three levels or scales of mass by way of the amount of binding energy

To establish the model it is necessary to elaborate upon our previous distinction between micro- and macroscopic matter/mass (subsection 4.2.6), and distinguish *three* 'levels' (or scales) of matter/mass.

1. Sub-atomic particles in which binding energy, as a proportion of total mass/energy, can play a *major* role.
2. Atomic and molecular matter in which binding energy, as a proportion of total mass/energy, plays a *minor* role.
3. Macroscopic matter in which binding energy, as a proportion of total mass/energy, plays a *negligible* role.

Note that the expression "microscopic matter" (or mass) shall imply either or both levels one and two. Also note that vast quantities of gas or plasma, e.g. vast celestial gas clouds, are considered under this criteria to be matter on a 'macroscopic' *scale*.

The value of this distinction is that standard gravitation (and the use of GR) pertains (in practice) to macroscopic matter as defined above; whereas the model involves (non-stationary) mass at (and 'below') the level of atoms and molecules — notwithstanding the fact that these atoms and molecules are encased in a very large macroscopic (lunar) body. In particular, the model involves a virtual energy discrepancy¹⁷⁵ arising from (macroscopic) celestial (geodesic) motion of (second level) atomic/molecular matter; with the energy, or rather a rate of angular momentum, necessarily expressed externally as a rotating acceleration/gravitational field perturbation. Thus, the second type of non-Euclidean geometry proposed herein clearly has its basis (in part) at a different (non-macroscopic) level of matter (in motion).

The new model's acceleration/gravitational field is a global (whole universe) scenario, and it also exhibits global homogeneity (re: its sinusoidal amplitude). If GR is seen as restricted to sub-universe macroscopic scenarios, then these two types of gravitational influence are clearly distinct and non-overlapping.

¹⁷⁵Later we see that this atomic/molecular virtual energy, from a global/systemic (whole universe) perspective, is actually a real inertial energy discrepancy.

4.4.5 Reintroducing the notion of relativistic mass

Observations indicate contraction or dilation effects for length, time and relativistic/apparent mass, but not electric charge. "The concept of relativistic mass has gradually fallen into disuse in physics since 1950, when particle physics showed the relevance of invariant [or rest] mass"¹⁷⁶. Unsaid in this quote is the fact that: invariance in physical laws, a principle based approach to physics, and gauge theories have been on the rise since about then; with this emphasis harbouring a particle-based unification agenda, along with a sense that ontological interpretation of SR's Lorentz transformation *is* complete, or even unnecessary. In line with our distinction between the micro- and macroscopic realms (subsections 4.2.6 and 4.4.4) we shall distinguish and utilise two approaches to mass in SR: microscopically we embrace the emphasis given to (particle) energy and momentum, but at the macroscopic/cosmological level, and for the discussion of ontological concerns, we shall revert back to the *simplicity* of relativistic/apparent mass (Sandin 1991).

4.4.6 Extending the scope of invariance, and 'time'

The distinction between relativistic mass and invariant (or rest) mass outlined in subsection 4.4.5 shall be supplemented in the following two subsections by arguments for an existence of invariant aspects of time and length/space in idealised, yet ultimately practical, global circumstances. The motive for doing this is to eventually get around the restraint that SR's invariance of the interval has upon the conceptualisation of gravitation; (and we do this) by way of seeking out new and further conceivable instances of invariance. In section 4.5 we shall see that (in idealised circumstances) the distinction between relativistic mass and invariant mass can be extended to length and time; but prior to this extension certain idealised physical circumstances need to be established by way of conjecture.

Often in physics the expression "invariance" means: unaffected by a transformation of coordinates. In this section invariance shall also be used (somewhat trivially) in the sense of: a homogeneity or uniformity throughout the global distribution, or occurrence, of a particular quantity.

The consideration of a less simple approach to the 'time' of physical observations appears to be required by way of the ability of QM systems to 'act' non-locally i.e. effectively instantaneously, in terms of *observational* time (recall subsection 4.2.10) — at least as regards correlations between properties of distant systems, and (we shall conjecture) how these correlations are *maintained*. This alone is reason enough to elaborate upon time's ontology/reality¹⁷⁷; otherwise we are somewhat in denial of a startling physical obser-

¹⁷⁶Wikipedia: *Mass in special relativity*, August 2007.

¹⁷⁷Paul Davies, from personal observation, strongly objects to more than one time being propounded; but this (completely appropriate) *scientific* stance unavoidably involves a philosophical stance upon what time *isn't*.

vation — albeit with non-locality (probably) restricted to the QM *spin* of particles/atoms/molecules, and collections thereof. Albert & Galchen (2009) provide a good overview of this recently rejuvenated concern.

4.4.7 Idealised invariant cosmological time — the universe’s fastest ticking clock

Atomic clock based Terrestrial time (TT) differs from (atomic clock based) Geocentric Coordinate Time (TCG) by a constant rate¹⁷⁸. The unit of time interval in TT is defined as the SI second at mean sea level. The reference frame for TCG is not rotating with the surface of the Earth and not in the gravitational potential of the Earth; and thus TCG ticks slightly faster than TT. Barycentric Coordinate Time (TCB) is the equivalent of TCG for calculations relating to the solar system beyond Earth orbit. TCB ticks slightly quicker than TCG. The difference between TCB and TCG involves a four-dimensional spacetime transformation. We note that all these times: TT, TCG and TCB are theoretical ideals; but their ‘constancy’ allows them to have practical applications.

We may conceivably extend this idealisation of time rates to a galactic centre coordinate time, then the centre of the local galactic group, and then the local supercluster centre; with the rate of clock ticking being slightly faster for each four-dimensional spacetime transformation. Assuming that we could observe the whole universe, and know the positions, motions, and gravitational potentials of other superclusters; there is a conceivable and final conceptual extrapolation that results in an (idealised) ‘Local’ Cosmological Coordinate Time (TCC) located at the local supercluster centre. This would be a clock for practical applications involving the *whole* universe; and it is a quicker/faster (ticking) clock than all conceivable others.

4.4.8 Non-locality and an idealised invariant cosmological time (rate)

Time can be understood as either the interval between two events or as a sequential arrangement of events. To account for QM non-locality, subsection 4.2.10 raised the possibility of the universe’s evolution having a step-like aspect, which is ‘separated’ and coordinated/harmonised by a hidden cosmological/global (cyclical) *process* of non-measurable duration. Observational reality, which is the basis of the scientific method, is necessarily oblivious to this supplementary ontological process. This cyclic process ensures that a background coordination of sequential (cosmological) “moments” is conceivable, which introduces a major shift in our conceptualisation of what observational time may involve (and be coexistent with). Briefly entertaining this broader temporal attitude/perspective, it is conceivable that these (available to observation) ‘reality moments’ — when compared

to the ‘duration’ of an accompanying hidden non-measurable (background ‘scanning’) process — (would most likely) have an extremely short/‘infinitesimal’ duration; i.e. the hidden process is a *comparatively* much ‘longer process’. This (moment-to-moment) situation would (in all likelihood) also involve incremental local¹⁷⁹ changes throughout the universe.

We may conceive that this cyclic (go-pause-go-pause-... or pivot-pause-pivot-pause-...) universal occurrence can be broken up into fractional parts. By definition: any measurement of time is ultimately based on counting the cycles of some regularly recurring phenomenon and accurately measuring fractions of that cycle. Although this hidden (i.e. noumenal) and cyclic process is beyond ‘measurement’, hypothesising it allows observational (i.e. relativistic) time to latently possess a supplementary hidden aspect that is effectively (temporally) *digital* and cosmologically uniform in ‘nature’ — in addition to the usual (solely) *analog* perspective of observational (and relativistic) time. Note that the scientific merit of this hypothesised scenario is based upon the *observation* of QM non-locality, as well as discontent with simply citing entanglement when ‘explaining’ the ‘occurrence’ of non-locality in experiments.

In subsection 4.4.7 we hypothesised an idealised cosmological (coordinate) time (TCC). Such a ‘global’ approach to time is fully consistent with a universe that evolves in a (digital and hence) *globally* ‘coordinated’ manner. Note that an idealised cosmological time *rate* is *not* a reprise of classical or absolute time in an absolute space reference frame, although it is well suited to a systemic approach. Clearly there is a distinction between idealised “cosmological time” and “observational time”, with the latter incorporating the relativity and temporal variances (i.e. non-simultaneity) associated with observations of a real, dynamic and non-homogeneous universe. It is worth noting that historically, before international atomic time was given preference, the observational time of an ‘Earth day’ or ‘Earth year’ was a sufficiently exact (reference) of *cyclic* duration.

The important feature of this discussion is the *invariance* of the TCG, TCB, and (the new) TCC (idealised) ‘time rates’; and their applicability throughout a given system — something that clearly eludes GR. Satellite and spacecraft navigation illustrate the necessity of establishing an idealised systemic time rate in practical and scientific astro-dynamical experiments. There will be a vast finite (indefinite) number of ‘noumenal’ cycles/increments occurring within any SI second. The existence of the model proposed (herein), to explain a real Pioneer anomaly, appears to demand this global coordination of events. Indeed, from a universal/global perspective, a coordinating/harmonising (noumenal or hidden background) process appears to be a very good way to ensure that all parts (of a vast system) can be maintained in stable ‘coordination’.

¹⁷⁸See, for example, IERS (International Earth rotation and Reference systems Service) Technical Note No. 32, Section 10: General Relativistic Models for Space-time Coordinates and Equations of Motion.

¹⁷⁹Where “local” also implies extremely small ‘regions’ of the universe.

4.4.9 Benefits of appreciating a broader scope to the concept of invariance

The concept of time dilation depends upon one's stance upon what time is. Ontologically speaking, time and time dilation are considered to be analog (cf. digital), such that time, relative to a stationary observer, simply runs *slow* at relativistic speeds. Alternatively (and hypothetically), if we accept the introduction of a second/further fixed (yet hidden) aspect of cosmological time, then some *measured* time could simply be *lost* at the level of observations, as a result of motion — in the manner that relativistic/apparent mass is *gained*. This notion is the major outcome to be established in section 4.5; it is a subtle concept and requires the preliminaries discussed throughout this section (4.4).

Regarding mass, we inevitably examine the *same* rest/intrinsic mass, but as (relative) speed increases from zero the *magnitude* of mass (in physical formulae) is modified. Could the same be true with time (and length)? Does motion alter the *measurement* of 'time', thus giving the impression, via *time lost*, of time dilation; but not altering a deeper (non-observable) true *intrinsic* invariant aspect of time (recall subsection 4.4.8)? Or should we refuse to seek progress by way of new hypotheses; trusting in general principles and invariance, and accepting that mass is (always and only reducible to) *just* energy. Thus, (effectively) dispensing with questions that seek a (potentially) richer explanation of: (observed) time dilation, energy's increase with momentum, the origin of mass, and so forth. Accepting a real Pioneer anomaly demands we must strive towards something different.

Our alternative view results in a distinction between: the standard relativistic appreciation of time¹⁸⁰; and a (new) scenario that has *both* a globally fixed (and hidden) time rate¹⁸¹, *and* a non-fixed time rate for relative motion, various physical systems, etc. — with only the latter being "measurable". In section 4.5 we see that this new scenario allows an appreciation of observed time (and length) similar to (old-fashioned) relativistic mass; in that there is both a rest/intrinsic value (albeit in idealised circumstances) *and* a relativistic/apparent (measured) value.

4.4.10 Substantivalism without a cosmologically global reference frame

We now turn our attention to the notion of universal background space. Physicists such as: E. A. Milne, H. W. McCrea, V. Fock, and S. J. Prokhovnik, have all entertained the idea of a cosmological substratum; i.e. a fundamental/global reference frame acting as a

background (or stage) to physical interactions, events, and processes. Not conflicting with this notion is:

1. space's (vacuum) impedance of 377 ohms,
2. the astronomical aberration of light, for observations from an Earth that orbits the Sun, and
3. a dipole effect associated with Earth's motion through the microwave background radiation.

The idea of a cosmological substratum is certainly not a consensus (nor popular) view. Indeed, all explanations of SR and GR incorporating absolute motion and/or absolute space *should* be rejected. Opposition to such proposals have been strongly stated, e.g. Clifford Will; and quite rightly, since nearly all experimental evidence, *excluding a real Pioneer anomaly* and non-locality, supports an implementation of Ockham's razor — thus, excluding any 'hidden' (metaphysical) aspects beyond direct observational evidence.

A question that remains is: must all discussion of a *global* perspective to space and/or time *always* be strongly rejected? Certainly, regarding observational physics employing light rays, the answer is yes; but a cosmological substratum need *not* be always localised (nor 'real' cf. virtual), so as to necessitate an accompanying absolute reference frame.

The (or an) exception to this rule would be a globally homogeneous cosmological substratum, with all points/places the same; thus, it would exhibit spatial invariance by way of global homogeneity. This substratum could only exist in the *absence* of any mass, motion, momentum or physical field energy; and thus it does not 'exist' *per se*; rather it is necessarily a conceptual idealisation. This includes 'taking out' the motion of the solar system barycentre with respect to cosmic microwave background (CMB) radiation (i.e. the CMB dipole); and then 'taking out' the energy field associated with the CMB's (very nearly) homogeneous temperature. Effectively establishing an 'absolute rest frame'¹⁸², at a temperature of absolute zero. One interpretation would be to see this (idealised) globally homogeneous substratum as effectively the empty mold (form, or die) upon which physical reality (all that exists) is 'cast'. If so, it has a latent physical relevance.

In philosophical terms we are skirting around the debate between relativism and substantivalism. The substantial 'something' being proposed is not SR's spacetime *per se*; rather it is an idealised (beyond physical reality) *background* continuum of uncurved (i.e. flat) space involving three dimensions. Only conceptually can this empty universe coexist with a non-empty universe¹⁸³; yielding a relativistic substantivalism. Additionally, this idealised background also has

¹⁸⁰Figuratively, time (or rather the rate of time) is elastic, stretching and shrinking *with* the clock that measures it.

¹⁸¹In the sense of a globally coordinated/harmonious reality, i.e. state of the universe, that (unbeknownst to observational physics) pauses before re-expressing the next cosmological state of the universe; one of a long sequence of observable (cosmological scale) 'reality' states (or steps). Both perceptually and observationally, (our and scientific) reality appears 'to be' continuous; with the additional (noumenal) aspect always hidden from experienced and observational (i.e. 'phenomenal') reality.

¹⁸²The dipole is a frame-dependent quantity, and one can thus determine the 'absolute rest frame' as that in which the CMB dipole would be zero (Scott & Smoot 2010, section 23.2.2).

¹⁸³Note that dark (or vacuum) energy is not being addressed in this dichotomy. Actually, the RSWs (rotating space-warps) proposed by the model, conceivably affect redshift measurements of EM radiation (cf. alter its speed); and thus, dark energy may not exist, once the existence of RSWs is recognised — see subsection 6.2.6 and Section 7.

an ability to coexist with the (idealised) form of (staccato) time ‘simultaneity’ discussed in subsection 4.4.8 (also recall subsection 4.2.10).

Clearly, an ontological open frame of mind is required to entertain this ontological ‘supplementation’; involving a *combination* (or coexistence) of measured reality exhibiting relativity, together with the additional conceptual ‘existence’ of an (idealised and hidden) background/platform of space (and time).

4.4.11 On idealised invariant cosmological empty space

What is the purpose of proposing an (idealised) ‘substantial nothing’ background in addition to the measured/observed world? How can the existence of an (ongoing) homogeneous nothingness (throughout and) of space, in the absence of gravitational (and all other) effects, possibly be related to the relativism of special and general relativity? Although special and general relativity do not explicitly involve/require this supplementary (physical) ontology — notwithstanding Ockham’s razor — they cannot deny it.

The use of a (flat) space background, together with the (noumenal/background) time simultaneity discussed in subsections 4.4.8 and 4.4.9, indirectly supports the additional (perturbative) non-Euclidean geometry of the model to be mathematically formalised by way of (simple) ‘neo-classical’ mechanics¹⁸⁴. The conceptual arguments permitting such a simple approach, in defiance of special and general relativity’s usual implications, are far from trivial and require further discussion, as well as the subtle conceptual move outlined in section 4.5.

4.4.12 Supplementary invariant ontology & the model’s invariant attributes

The hypothesising of: a hidden cosmological evolution process, a latent universal time coordination, and a latent homogeneous space background, are all anathema to GR with its metric based approach. With quantification in the new model only requiring (several instantiations of): a systemic (i.e. barycentric) cyclic time¹⁸⁵ (Δt), a (field based) cyclic variation of a ‘true’ acceleration/gravitation (Δa) that is of *fixed* magnitude everywhere¹⁸⁶, and a *constant* (non-local) mass (m^*) to (celestial space) enclosed volume (V) relationship¹⁸⁷; a simple neo-classical *formalism* is sufficient to account for the Pioneer anomaly. Note the *global invariance* (i.e. homogeneity) associated with the latter two quantities: Δa and (m^*V).

¹⁸⁴The attendant conceptualisation, that allows this simple “neo-classical” formalism, is anything but ‘classical’ and far from simple.

¹⁸⁵Note that this (sidereal) rotation period/time takes into consideration all (minor) general relativistic effects such as geodetic precession.

¹⁸⁶Later we see that an (inner) central ‘hole’ is required because the rotating space-warp is the externalisation of an inexpressible QM energy arising from atomic/molecular mass in a non-inertial configuration.

¹⁸⁷Non-local mass is a new concept unique to the model. It is more elaborately ‘defined’/introduced in subsection 6.4.2.

4.4.13 Another kind of ‘uniform’ acceleration that can be visualised

Other than the product of non-local mass and volume enclosed, the model has only one (type of) field value (sinusoidal Δa) (which is) of fixed value/amplitude throughout the universal system. If we allow the acceleration/gravitation to be represented by space curvature (cf. spacetime curvature), it becomes possible to visualise each (gravito-quantum) rotating space-warp (in two dimensions) as a rotating ‘rigid’ warped disk (recall Figures 4 to 7). This ability to visualise is more in keeping with a (neo-classical) fluid mechanical treatment incorporating a vast *space* continuum and time simultaneity; rather than anything remotely resembling general relativity’s formalism. Expressing the model’s features as a supplementation of GR, in the language of GR, is not feasible; nor is a modified Newtonian mechanics (MOND) viable¹⁸⁸.

It is important to note that with: the Δa amplitude (at any given point) being *non-uniform*, in the sense of sinusoidally or *cyclically* variable; and uniform or homogeneous, in the sense of being the same magnitude/amplitude everywhere; compatibility with GR (by way of the latter sense) is trivially assured. Supporting this new type of field is the fact that GR’s conceptual foundations are restricted to *uniform* (i.e. locally homogeneous) gravitational fields. In GR non-uniformity is both expressed as, and restricted to, curved spacetime cf. curved space in time. Pragmatically, each instantiation of the model’s (very small) supplementary acceleration/gravitational field is seen to sit (evenly) on top of any existing curved spacetime field¹⁸⁹; but conceptually and mathematically this co-existence is an uneasy alliance (see subsection 4.4.14).

4.4.14 The model’s separate gravitational relationship cf. general relativity

The implicit contention herein is that general relativity is “not all of (gravitational) reality”; and that a (solely) relativistic approach is restrictive in some negative sense. Indirectly supporting a notion of incompleteness and restrictiveness is GR’s inability to incorporate/ascertain total energy (and total mass). Primarily, SR and GR are restrictive in a positive sense, in that the *laws* of physics are invariant with respect to coordinate transformations.

Whereas SR utilises local frame-to-frame relative relationships in conjunction with ‘spacetime interval’ invariance, our global approach has highlighted a system-upon-idealised background relationship in conjunction with the amplitude invariance of sinusoidal Δa . These relationships are distinctly different. This further confirms that the systemic/global nature of the model is completely foreign to what a relativistic theory can encompass.

Fortunately, the three levels/scales of mass systems, outlined in subsection 4.4.4, provides some clar-

¹⁸⁸MOND tends to alter the mathematics of gravitation without any supportive conceptual justification.

¹⁸⁹Over extended periods of time, i.e. a fair bit longer than the longest Δt .

ity in distinguishing the basis of GR's predominantly (macroscopic) gravitation from the model's (ultimate) basis in atomic/molecular 'mass'. More specifically, this basis is an atomic/molecular (process-based) rate of angular momentum; relating to an asymmetrical relationship between the QM spin and orbital values arising from closed loop celestial (geodesic) motion of the atoms/molecules in curved spacetime, and also involving the *self*-interference¹⁹⁰ and QM fermion 'wave' aspects of these atoms/molecules over the course of each complete orbit (of their macroscopic 'host' body).

Furthermore, *scalar* magnitudes such as: total and specific energy, (constant amplitude) acceleration, and (non-local) mass (albeit referenced to an enclosed volume), are all well suited to the model's global perspective. Later (section 6.3) we see that the energy of each rotating space-warp — together with its initial non-local mass (m_1^*) — is $\Delta E_w = \frac{1}{2}m_1^*\Delta a_w^2\Delta t^2$.

4.4.15 Invariant light speed and the limits of observational physics

Electromagnetic radiation's constancy of propagation speed is pivotal to relativity's mathematical formalism; and subsequently how space and time are seen to go together (at least observationally). Further:

It is a striking fact that *all* the established departures from the Newtonian picture have been, in some way, associated with the behaviour of *light* (Penrose 1990, p.285, Note 1 Chapter 5).

Responding to this quote of Roger Penrose regarding light, a sceptic of pure-relativism can hypothesise upon additional ontological aspects excluded by SR and GR. Particularly, whether one's observational (ontological) framework in some way *guarantees* the (measured) constancy of light (EM radiation) speed in a vacuum (c) in all relative motion circumstances. This implies that the outcome of an act or process of observation utilising electromagnetic radiation, and reality itself, may be deeply entwined — but not in a way that conflicts with the SR view that c (the speed of light) is a fundamental feature of the way space and time are unified as spacetime. Once again the finality of (direct) observational evidence (alone) is being questioned and deeper ontological circumstances entertained.

Associated with this EM radiation speed invariance is whether (and if so, why) the specific energy (E/m) of all phenomena is limited by a c^2 upper limit? This is further examined in section 4.5.

Heisenberg's uncertainty principle and the Planck (or reduced Planck) constant support both: the notion that observational precision is limited in a quantifiable manner, and that hidden features (cf. variables) of the physical universe may exist, especially where (QM) uncertainty prevails — e.g. below a minimum (quantum) energy level, or change in *energy* levels, in an atomic or molecular *system*.

¹⁹⁰By way of a difference between pre- and post-orbital-cycle wave phase conditions.

4.4.16 Section overview and summary

Although a real Pioneer anomaly is a very minor gravitational/accelerational effect¹⁹¹, its existence (and associated constancy) requires significant modification of our accepted understanding of reality. Its existence is in defiance of GR's (assumed) 'completion' of gravitational theorisation.

In reply to the strong dependence of SR upon invariance (of the interval) this section (4.4) has sought to establish further invariances — in the widest (less technical) sense of the term, i.e. including homogeneity, constancy and uniformity¹⁹². We may divide these into three types.

1. Special relativity has the invariance of the interval under Lorentz transformations (Lorentz invariance) and the constancy of the velocity of light (in a vacuum).
2. In the model to be presented there are two major types of (observational) invariants: the amplitude of the cyclic acceleration in the field (Δa), and the product of non-local mass and volume enclosed (m^*V).
3. Two new idealised 'invariances' have been introduced. By way of non-locality we conjectured a hidden (background) cosmological coordination of time, i.e. a (type of) frequency invariance. Also, by way of space being (observed as) cosmologically 'flat' ($k = 0$), we inferred that empty space — i.e. in the absence of matter and energy — is homogeneous; and thus this (idealised background) empty space displays spatial invariance.

What is the point of this proliferation of invariance, both overt and covert? Firstly, *hidden* background aspects 'support' the classical-like formalism in the model's account of the Pioneer anomaly¹⁹³; especially since the field perturbation/curvature arises from a *virtual* asymmetrical energy offset, that although inexpressible at/within the atomic/molecular level of matter¹⁹⁴, it is witnessed/registered at the global background 'level'. Secondly, we note that the four new invariances introduced, all have a global or systemic application.

More importantly, we may distinguish two distinct types of 'relative' effects (especially involving

¹⁹¹Note that the proposed mechanism to explain the Pioneer anomaly is of cosmological scale; and its influence may (conceivably) become more and more significant as the periods of time involved are extended.

¹⁹²In physics, invariance of angular momentum and energy are directly related to conservation *laws*.

¹⁹³Note that the model requires standard gravitation to be conceptualised as curved spacetime, rather than as a gravitational *force*.

¹⁹⁴'Virtual' energy; because the dominant electromagnetic forces, at this level of matter, overwhelm any non-inertial energy effects that could arise from atomic/molecular (i.e. mass) motion in (celestial scale) *curved* spacetime. Actually, it is the intrinsic spin of *all* elementary fermion/matter particles 'within' each (composite/whole) atom/molecule that is 'influenced' in a virtual manner.

motion): the standard object-to-object (‘phenomenal’) relativity, and a *new* relationship between the observable world and the empty space and coordinated/harmonious time of a (hidden background) ‘noumenal’ idealised ‘world’. This latter, local-phenomenal to global background/noumenal physical relationship, argues for the coexistence of relativity’s invariance of spacetime with both homogeneity of (empty) space, and uniformity of time in this empty space. Thus, mass (for example) can be seen to curve an otherwise flat space, *and* cause a deviation from ‘the’ (empty space) rate of time. Whereas, in SR’s *formalising* of space and time, it is the invariance of the spacetime interval that is paramount.

The reader should be aware that we are *not* seeking to downplay the importance and exclusivity of the Minkowski metric, nor are we promoting a “disguised” Euclidean metric. Although similar, but with a subtle difference, we shall promote the notion of global reality as a “distortion” of an idealised Euclidean metric — that would exist in a universe bereft of (mass, momentum and) energy.

By way of highlighting: certain shortcomings of special and general relativity; the limits of observational physics; and conceivable restrictions upon the capacities of the process of observation; this subsection has sought to undermine confidence in the *completeness* of (GR’s) existing gravitational theorisation. Note that within GR’s scope of application, i.e. effectively everything except the model’s specific/unique case, GR’s correctness and/or accuracy is *not* doubted.

A primary aim of this section has been to “set the scene” for the decisive conceptual and theoretical move to follow (section 4.5); whereby SR’s Lorentz transformation is given a new/further and physically supplementary/different interpretation — involving conditions in a real/measured universe *relative* to inertial circumstances, with the latter being associated with a new (cosmological scale) specific energy based *invariance*. This new conceptual landscape results from our assumption/hypothesis that the Pioneer anomaly is a real and new physical effect.

4.5 Local relativistic effects within a globally invariant universe

Section 4.4 supplemented the local and (and as measured) invariances of SR, i.e. light speed and the laws of physics in constant relative motion, with the establishment of *idealised global* invariances — i.e. homogeneous space and time on the largest scale possible. Regarding the latter, note that globally (and beyond observations) space and time are treated *separately*. This, together with the weaknesses in the exclusivity of special and general relativity previously outlined, forms the platform for this section. This section (4.5) discusses the specifics of how the new model, with its noumenal-time simultaneity and (further) incorporation of (measurable) global (invariant) quantities, can *coexist* with special and general relativity’s account of gravitation.

4.5.1 Preliminaries

Loosely speaking, we are countenancing two aspects of reality: one measured; and one beyond measurement, yet also (in a certain sense) the platform ‘behind’ measurement¹⁹⁵. For observations/measurements we retain standard or ‘proper’ physics; e.g. the relativistic energy-momentum equation $E^2 - (pc)^2 = (mc^2)^2$ with its relativistic (observer dependent) values of energy (E) and momentum (p). In what follows, it is necessary to move away from a purely observational basis that gives light rays and coordinate systems (i.e. operationalism) exclusive priority.

Moving from the standard “phenomenal-to-phenomenal” [*sic*¹⁹⁶] approach of relativity, to a phenomenal-to-background (i.e. a phenomenal-to-noumenal) approach, we recognise a (flat) background space continuum with a (latent) global time simultaneity; but not a universal time *coordinate*, nor a background reference frame that could be used with *observations*. Only this latter approach can facilitate the additional global gravitational/accelerational effect proposed herein. Additionally, GR’s domain of application is perceived as always a subset of the whole universe, applying up to the largest of galactic clusters. Relativity’s curved spacetime is systemic, but herein we need to deny its (seamless) extrapolation to universally (i.e. globally) systemic conditions.

The digital global sequenced temporality discussed in section 4.4, allows us to appreciate one (idealised) global ‘now’ existing with (or in amongst) many different local time rates. The temporal symmetry of most laws of physics tends to support this picture of reality (i.e. hidden simultaneity); but note that the model/mechanism to be presented is unlike this and relies upon an irreversible asymmetry (and/or offset) of QM *energy* ‘achieved’ (and externally re-expressed) over a (system-based) cyclic time period.

4.5.2 Inertial frames and: nothing, something, and everything

In Newtonian Mechanics inertial frame circumstances apply to bodies in uniform motion or at rest (in an attendant inertial frame). The concept of an idealised global/universal empty space is (herein) considered to be a deeper (i.e. more fundamental) inertial circumstance than uniform *motion* conditions — albeit not ‘realistically’ (i.e. measurably or phenomenally) viable.

If ‘the phenomenal’ is seen to always involve ‘something’, then the idealised (noumenal) stance pursued herein involves both: ‘nothing’ as in truly empty space; but somewhat by way of invariance, the idealised (noumenal) stance also involves ‘everything’ (spatial) — albeit in another sense. Pursuing this idea, a response to the philosophical question: “Why is there

¹⁹⁵Note that this stance is based upon anomalous *observational* evidence in need of an explanation.

¹⁹⁶We shall intentionally use the adjective ‘phenomenal’ also as a noun, in order to highlight its new/specific meaning herein; as well as to indicate its juxtaposition with/alongside ‘noumenal’, which has usage as either an adjective or a noun.

something rather than nothing?” should necessarily *also* include/involve the notion of ‘everything’.

When dealing with finite quantitative circumstances, we are necessarily dealing with ‘something’. A (coexistent) nothing-everything dichotomy lies (either at, or) beyond the bounds of (the something within) a minimum-maximum dichotomy. Quantitatively, any minimum ‘amount’ would appear to also require a conceivable maximum as the other ‘book-end’. For example, regarding velocity we have rest and light speed.

In subsection 4.5.3 the something of physical reality is considered to lie within the coexistent (quantitative) limiting ‘book-ends’ of the empty and the full — corresponding to the (aforementioned) nothing-everything dichotomy. This coexistent dichotomy has an attendant global invariance; i.e. one existing everywhere (and) at all ‘times’. Our objective is to look at (hitherto non-phenomenal or noumenal) ‘empty space’, i.e. energy-less space, in a second coexistent way; as energy-full — with this different/reversed (i.e. *noumenal*) perspective being then understood as the more appropriate noumenal perspective.

Initially, we restrict ourselves to a discussion of SR’s uniform relative motion circumstances. Later in this section (4.5) gravitational circumstances are examined.

4.5.3 A Copernican-like reversal regarding inertia and specific energy invariance

By way of: envisaging restriction in one’s own observational capacity (subsection 4.4.15); homogeneous background length/space and time, light speed invariance in SR; and $E_{\text{rest}} = m_0 c^2$; it is feasible that a ‘richer’ invariant (background/noumenal) global quantity may exist. All that we can be sure of is that the background must be invariant, at all places and times in some sense; and if a magnitude is involved, it most probably involves c or c^2 — with the latter having the dimensions/units of specific energy (E/m) i.e. $[L^2/T^2]$. Thus, c^2 conceivably quantifies an invariant physical (or phenomenal) maximum in some way.

To appease this situation, it appears that we need to also conjecture a Copernican-like reversal regarding inertial motion (in SR) and (specific) energy; where zero energy (and hence zero force, zero angular momentum, etc.) at a ‘point’ is *actually*, from a reversed perspective, maximum possible specific energy (c^2). From this reversed perspective, i.e. a noumenal supplementation (or ‘complementation’) to the phenomenal/observational, an inertial frame or inertial motion is ‘full’ of *latent* ‘potential’ (specific) energy. Subsequently and additionally, what appears (observationally) to be an increase in specific energy (from zero) is, from this *reversed perspective*, actually a *drawing down* from an (everywhere and always) constant universal-wide latent-*potential* specific-energy *source*. Left completely alone this (background) potential would be spatially homogeneous at c^2 , at all times — but the phenomenal universe is anything but fully empty.

As was the case in subsection 4.5.2 regarding inertial circumstances, this reversed (noumenal) perspective, that coexists with an observational/phenomenal

perspective, is seen to be the deeper (or more fundamental) of the two perspectives. Some discussion of mass is required before we examine the ramifications of this reversed (and supplementary/complementary) perspective (in subsection 4.5.4 and beyond).

In reality, mass cannot be said to ever truly exist at a point, because this implies an infinite density. Further, stable macroscopic mass, and QM mass especially, always occupy a volume of space, and thus their (self) energy is distributed over a finite region — with the electron being the most ‘point-like’ of all masses. Observationally, it is considered that: “atoms are largely comprised of ‘empty’ space”. Herein, we also need to recognise the volume of space that an object’s mass encompasses, with various mass-to-volume relationships existing for different types of mass — i.e. particles, atoms, molecules, and bulk matter.

Consider the tremendous expansion the universe has undergone subsequent to the highly ‘dense’/energetic conditions existing just after the “big bang”. Thus, when mass is in atomic and molecular form, the mass to volume ratio is negligible compared with what was (and is) capable of being achieved. In other words, the existence of an actual atom/molecule in space is seen to involve only a very minor proportion of the maximum (noumenal) ‘specific’ energy possible/available, cf. the case where all of an atom’s space (i.e. every point in its volume) has the maximum specific energy possible (i.e. c^2) — notwithstanding the huge energies associated with nuclear fission or fusion effects. Thus, the existence of a *single* atom/molecule is seen to only involve a very minor “drawing-down” from the maximum possible specific energy available/possible throughout space.

4.5.4 Ramifications of this reversal regarding specific energy

There are a number of ramifications of utilising the ‘reverse’ stance conjectured in subsection 4.5.3. Firstly, we hypothesise that (observations made from a frame in uniform) relative motion involves a ‘drawing-down’ from this (reversed/noumenal) ‘potential hill’ (as compared to a ‘potential well’). In subsection 4.5.6 we shall argue that this (also) gives rise to the same type of Lorentz transformations that occur in SR. Significantly, gravitational effects also draw down, in a *different* way, from the *same* ‘hill’. It is the double-draw, (for example) of a body’s mass and its motion in a field, that is seen to necessitate the use of general covariance (GCoV) in the mathematical formalism of GR; thus giving GR its uniqueness amongst physical theories.

Secondly, and on a lesser note, regarding high speed relativistic motion; no rocket-ship (of non-zero mass) could ever get to the speed of light (c) because the (noumenal) latent specific energy is just ‘not available’ for this (combination of mass and motion at light speed) to be achievable — the noumenal potential hill is “only so high”. By way of contrast, the ‘compulsion’ of *massless* photons to always be observed as moving at light speed makes them unique amongst particles¹⁹⁷.

¹⁹⁷ Assuming neutrinos have some mass, they (then) travel at very *close* to the speed of light.

Finally, in order to proceed with this discussion, it is proposed that the maximum specific (latent) ‘potential’ energy (in a non-point-like region of spatial volume) c^2 is ‘complex’, literally in the sense of being able to possess real and imaginary components.

4.5.5 An alternative magnitude for the invariant specific energy source/‘hill’

Regarding *relative motion* it appears that $\frac{1}{2}c^2$ is a better choice (cf. c^2) for maximum latent specific energy available, i.e. the amount that physical reality can be ‘drawn upon’. Four reasons are given to account for this preferred magnitude (or ‘potential hilltop’ value); none entirely compelling (individually or collectively).

1. Neo-classically this is maximum conceivable specific kinetic energy, in that $\frac{1}{2}v^2 \rightarrow \frac{1}{2}c^2$ at the limit.
2. With relative motion involving two bodies, the specific energy of an ‘examined’ body can only (in some sense) be half of the total maximum value.
3. By way of time symmetry in electromagnetic theory, we might embrace Huw Price’s philosophical argument that a (secondary) non-observational (–ve) time may exist, permitting subliminal ‘backwards’ causation¹⁹⁸; thus halving any latent-potential for real (+ve) time motion (Price 1996).
4. The virial theorem, for rotating gravitational systems, yields $\frac{1}{2}P.E. = K.E.$ and a cosmological instantiation of it may exist for maximum specific latent-potential energy and maximum specific kinetic energy.

In what follows, the choice of $\frac{1}{2}c^2$ or c^2 as an invariant maximum specific energy has no major impact upon the discussion. Indeed, depending on the circumstances both of these maximums may have a different role to play.

4.5.6 Relative motion and background invariant specific energy

This subsection begins the process of showing how SR’s Lorentz transformations can also be related to the phenomenal-to-background/noumenal distinction previously discussed (subsections 4.5.1 and 4.5.2). Accepting the reversed perspective on ‘reality’ argued for in subsection 4.5.3, the Lorentz transformations appear to (also) quantify the proportion of *globally* invariant specific energy that is no longer available to (observable) physical reality — when two bodies in uniform relative motion observe each other. Compared to inertial circumstances, there is an equal loss in the (square of) “available to measurement” time rate and length (in the direction of motion). We shall see how this alters our standard ‘phenomenal’ understanding

¹⁹⁸The negative frequency (–f) of subsection 3.2.8, that was deemed non-physical, could be indicative of a (–ve) time (direction).

of: a clock’s *local* ‘time dilation’ relative to an inertial frame, and (especially) relative to other clocks in motion.

If we consider the maximum latent specific energy *available* to relative motion as a complex ‘quantity’ (i.e. magnitude and unit), then $\frac{1}{2}c^2$ is the invariant square of the hypotenuse of a right triangle, and this allows a break down of total (invariant) specific energy into real and imaginary components. Further, if we consider relative motion at v as contributing (or involving) a $\frac{1}{2}v^2$ imaginary (number based) component of specific energy, then the real component of specific energy remaining is $\frac{1}{2}c^2 - \frac{1}{2}v^2$ with ‘dimensions’ $[L^2/T^2]$. As a *proportion* of maximum available specific energy this is $(c^2 - v^2)/c^2$ or $1 - v^2/c^2$. Note that the same result is obtained if c^2 and v^2 are (alternatively) taken as the maximum specific energy, and total kinetic energy respectively. Thus, for motion, which has dimensions $[L/T]$; a (non-dimensional) proportionality factor $\sqrt{1 - v^2/c^2}$ is required to represent the proportion of time and length availability *lost*; cf. time dilation and length contraction.

Alternatively, $\sqrt{1 - v^2/c^2}$ is simply a/c , if a is the third side of a (complex) right triangle with sides: imaginary v , real a and complex hypotenuse c . The same would be true for a similar triangle with sides: $v/\sqrt{2}$, $a/\sqrt{2}$ and $c/\sqrt{2}$. Subsequently, an inertial (rest) frame has an *idealised* background specific energy ‘potential’ (i.e. availability) that is ‘actually’ a fully *non-imaginary* quantity. Importantly, we note that, from an observer’s perspective, cf. a universe-based (noumenal) perspective on things, motion (at v) is considered ‘real’ — whereas from the universe’s perspective motion (at v) is considered ‘imaginary’.

From observational evidence, and an observational perspective, we make the (new) interpretation that real physical *phenomena*, and hence measurements thereof, may only obtain the real component of specific energy. By way of the reversal we have considered, we may say that: relative motion ‘draws’ down from a latent and invariant specific energy ‘potential hill’, with relative motion giving rise to an imaginary component that can no longer contribute to ‘reality’, in the sense of “be made available” to reality. Further, to satisfy dimensional constraints, *both* the ‘units’ of length and time (together) must be equally reduced from their ‘inertial’ (fully real) magnitudes. In SR, this speed based influence is quantified as the time dilation, and length contraction, involving $\gamma = (\sqrt{1 - v^2/c^2})^{-1}$. Mass dilation is discussed in subsection 4.5.10.

4.5.7 A different interpretation of SR’s time dilation and length contraction

In special relativity, relative to an *observer* in an inertial or stationary system, a clock (t') moving with velocity v appears to run *slow* according to the relation: $(t'_2 - t'_1) = (t_2 - t_1)\sqrt{1 - v^2/c^2}$. In addition to this, the length contraction of a rod, in a rest frame, as observed from a moving frame is: $(x'_2 - x'_1) = (x_2 - x_1)\sqrt{1 - v^2/c^2}$.

The relationships of SR’s description of time dila-

tion and length contraction are not the same; they are conceptual opposites, i.e. a dilation vs. a contraction. In contrast, from our ‘universal’ perspective, they are seen to describe the loss in time (rate) and loss in (measurable) length separation respectively. The difference lies in one’s observational ‘perspective’, with SR’s results using light ray propagation and coordinate differences. SR also has a different (and simpler) ontological stance upon what time ‘is’ and how time ‘passes’, with this grounded in an ‘operationalism’ approach. In SR time itself slows, whereas from the new perspective, we say that: in a non-inertial frame temporal *processes* simply take ‘longer’ than in the local inertial frame. Note that the ticking of a clock is a ‘process’.

To maintain the spirit of (uniform relative motion) special relativity, the time loss/dilation scenario discussed above must only apply (relatively) between two observers. Thus, all relative inertial frames are seen to involve maximum invariant specific energy¹⁹⁹.

4.5.8 Further discussion upon relativistically altered time and length

If we accept that (from a beyond just a measurement-based perspective) a digital universal pivot-pause-pivot-pause... temporal evolution exists²⁰⁰, and subsequently that the universe retains a form of global co-ordination as it ‘evolves’; then the non-inertial frame is seen to miss a bit/proportion of each (inertial) time increment. Total time loss/dilation is therefore an accumulation of very small time losses. Length loss/contraction is ‘slaved’ to this time loss so that: in the new ontology a (material) rod *appears* contracted because the full length of space containing it cannot be fully perceived by a viewer in relative motion. A given process in a stationary frame requires fewer ‘sequential moments’, i.e. digital instances of (phenomenal/measured) reality, than the same process in a moving frame.

A second (non-standard) interpretation of the same measurements is possible. Consider: clocks upon an aircraft, moving *relative* to the Earth’s (effectively/essentially) inertial frame — noting that the Earth’s motion relative to other bodies is irrelevant. Interestingly, the time *lost* off the clock on the aircraft during its flight remains after the experiment, whereas the length lost (off a rod) is reversed once relative motion ceases. This arises because the *rate* of time was only slower during the flight. This highlights a primacy of time loss over ‘ensuing’ length loss. Length contraction is ‘real’ only in the sense of being an apparent observational artifact.

As a final step in this new (and complementary) interpretation let us declare that: uniform (inertial)

relative motion leads to an alteration in the magnitude of the standard *unit* or scale of measurement of length and/or time. This unit contraction results in a change to the *magnitude* of measured length or time — time actually slows and length appears contracted. By way of background ontological aspects of the universe, our standards of measurement are adjusted or compromised, rather than time (itself) dilating and an object’s length (itself) contracting.

Finally, we note that at a deeper ontological level, uniform relative motion is (conceptually) seen as a less than optimal idealised-‘inertial’ system. For a fully *idealised*-‘inertial’ system we need to have an empty universe devoid of all relative motion. When considering gravitational effects (see subsection 4.5.14) this notion is conceptually useful, although in practice/reality (due to the relativity of observational reality) it is not required in special relativity (SR).

4.5.9 Briefly on the “clock paradox”

In SR, the twins clock paradox is resolved by using a Minkowski spacetime diagram and appreciating the asymmetry of the situation²⁰¹. By way of the ontological supplementation pursued herein, Minkowski spacetime is not the only ontological perspective of space and time. In the model’s new/complementary (background and global) perspective two clocks in the same motion (relative to an inertial frame) will ‘slow’ by an equal amount/proportion.

This is made particularly evident in a figure eight symmetric version of the twins paradox, involving geodesic motion, discussed in a footnote within subsection 4.4.2. The paradoxical aspect of SR’s (dual) time dilation dissolves, once and for all; and the dreaded third observer (at rest), so easily conceptualised, need not be derided — as it must be in SR. The reason for this conceptual ‘enrichment’ is that the model seeks to incorporate a global perspective in addition to the (dominant) relative perspective of observations. When ‘going global’ the model must be very careful to not “step on SR’s toes”, and this is why global invariance is so important — both for physical quantities in the model, and the (hidden) background ontological ‘limitation’ of specific energy introduced in this section (4.5).

4.5.10 Mass dilation and clarifying the intended meaning of the word “real”

The apparent increase or dilation of mass in the measurements of high-speed elementary particles is seen to arise by way of a mass ‘experiencing’ less real space, as compared to inertial conditions. By way of relative motion the ‘weighting’ of mass in the ‘real world’ (left available to it) is proportionally *increased*. Note that from the model’s new perspective, it is the time loss that is ultimately responsible for the dilation of mass; because the space lost is in equal proportion to, and ensues from, the amount of time lost.

¹⁹⁹There are issues that need to be pursued, especially regarding the relationship between a global background and nested reference frames moving relative to each other — e.g. Earth in solar system, in galaxy, in galactic cluster, etc. SR demands that this hierarchy is irrelevant to local relative observations, which further mystifies just what reality is ‘in itself’ when it is not being observed.

²⁰⁰Without an associated universal (absolute) time coordinate (recall subsection 4.4.8).

²⁰¹Once and for all, in the minds of most physicists. This ‘party line’ is championed by Paul Davies in: “About Time: Einstein’s unfinished revolution” (Davies 1995).

Two aspects of mass dilation, by way of space loss/contraction, need to be distinguished. Firstly, a mass exists in a space that is predominantly external to it; and its high-speed motion ‘encounters’ less of this space. Similarly, yet more importantly, the ‘internal’ space of a mass is also reduced — effectively increasing its density. It is this latter case that pertains to the mass dilation of SR, or mass *gain* from our new perspective. It has been confirmed experimentally²⁰². As with length and time, the dilation/gain effect (exists and) is quantified in a purely relative way.

Note that from a noumenal perspective, magnitudes at the phenomenal level vary with respect to the noumenal’s invariant ‘nature’; but equal adjustments in the units of the Length and Time dimensions ensure that (the observational/phenomenal value of) the speed of light is invariable — both spatially and temporally.

The word “real” in this section (4.5) retains its common usage, as in: (1) having objective existence, (2) not imaginary, and (for our purposes) (3) being ‘phenomenally’ existent in the sense of being measurable. Note that relative motion *is* real, but it also has a further unmeasurable effect in that it generates an ‘imaginary’ component of latent specific energy [say $\text{Im}(e)$], thereby reducing the maximum potential specific energy available that can become ‘real’.

4.5.11 A brief interim summary; & replies to: “Can everything be relative?”

In subsections 4.5.6 through 4.5.10, we have argued that: special relativistic time dilation, length contraction effects, and mass dilation effects can be traced back to (or grounded in) a hidden supplementary (deeper) ontology. Subsequently, the Lorentz transformations can be seen to describe *both*: how space-time intervals remain invariant, so as to be compatible with electromagnetism; as well as how *uniform relative motion* coexists with (and ‘bites’ into) a maximum ‘available’ specific energy — that remains invariant everywhere throughout the universe, for all sequential ‘moments of observable time’. From the observer’s perspective, the time describing physical phenomena is analog.

The Lorentz transformations effectively build a bridge between a *global* (i.e. universal) specific energy invariance, and SR’s (and EM’s) *local* invariance of two spacetime intervals. Physics is arguably stymied by way of lacking an appreciation of this ‘new’ coexistence; preferring to pursue a deeper understanding of both special and general relativity through the study of the Minkowski metric.

Relativity implicitly answers the ontological question: “Can everything be relative?” in the affirmative, in line with its stance on the formalistic representation of physical reality, whereas the contents and sentiments of the preceding discussion requires that we

answer this question — which is a borderline oxymoron — in the negative.

4.5.12 Justifying a constant specific energy value throughout the universe

The aim of this subsection is to argue that reasonable justification for a homogeneous background specific energy throughout all of space, at all ‘times’ — upon which reality ‘draws-down’ — can be found within mechanical classical mechanics (i.e. excluding electromagnetic effects). To achieve this we examine the laws describing a range of classical or everyday (macroscopic) mechanisms or phenomena.

Subsections 4.2.6 and 4.2.7 argued for a qualitative distinction between the physical laws of the macro- and microscopic realms. Subsequently, four ‘force’ unification is not endorsed. This subsection pursues the (non-reductive) ramifications of this distinction; particularly the unsuitability of ‘force’ in describing gravitational effects. Note that gravitation is the only *mechanical* phenomenon that (inevitably) exists on a universal (or cosmological) scale. Also note that at the microscopic level, i.e. up to the ‘level’ of atoms and molecules, the standard account of reality in terms of three (unified) forces is fully embraced.

In non-chemical, non-thermal and non-electromagnetic circumstances, macroscopic energy equations commonly have the form $E \propto m$; e.g. $E = mgz$, $E = \frac{1}{2}m\omega^2x^2$ (linked to $E = \frac{1}{2}Kx^2$), $E = \frac{1}{2}mv^2$, and $E = m\Gamma_e f$ (recall subsection 4.2.7) concerning (respectively): positional change in a constant gravitational field, springs, kinetic energy, and wind turbines/‘airscrews’. In these ‘mechanical’ systems, or aspects thereof, the establishment of specific energy ($E/m = e$), from the kinetic or potential energy expression, is easily obtained. Clearly, force plays a vital role in the first two energy expressions, but not the latter two.

Regarding electromagnetism, the Planck relation — between the energy of a photon and its electromagnetic wave, $E = hf$ — retains the prominence of energy; and regarding thermal physics, the ideal gas law $pV = nRT$ retains the dimensionality of energy, i.e. $[\frac{ML^2}{T^2}]$. Neither of these expressions/laws is simply an integral of force over distance. Energy (cf. force), or at least “dimensions to that effect”, appears to be more general and useful at the macroscopic level of physical formalism (i.e. representation).

No expressions from the mechanics of solids have been presented, but the concepts of deformation and curvature have some ‘overlap’ with the formalism of general relativity. Accepting universal ‘coordinated time’ (see subsection 4.4.8), the notions of a cosmological space continuum and space curvature has validity — albeit at an idealised level of conceptualisation.

In fluid mechanics, we utilise the concept of a mass continuum, and (necessarily) examine force per unit volume rather than force. By way of utilising conservation of mass (i.e. continuity) and conservation of momentum, we may obtain equations describing the motion throughout a flow field. In the case of an inviscid flow we obtain Euler’s equation. If density (ρ) is

²⁰²Walter Kaufmann was the first to confirm (1901-03) the velocity dependence of electromagnetic mass by analyzing the ratio e/m (where m is the mass and e is the [constant] charge) of cathode rays. Paraphrased from Wikipedia: *History of Special Relativity*, 2011.

constant (i.e. incompressible flow), then integration of Euler's equation throughout an irrotational (i.e. zero vorticity) flow, yields

$$\frac{\partial \phi}{\partial t} + \frac{v^2}{2} + \frac{p}{\rho} - gz = f(t)$$

where ϕ is velocity potential and p is pressure (Kuethe & Chow 1976, p.69). This is the same as (a special form of) Bernoulli's equation $\frac{1}{2}\rho v^2 + p - \rho g \Delta z = p_0$ (Kuethe & Chow 1976, p.62), except for the inclusion of the unsteady (flow) term and adjustment for the integration constant. Importantly, the dimensions are those of *specific energy* [$\frac{L^2}{T^2}$]; and in steady inviscid incompressible flow, the stagnation pressure (p_0) is *constant throughout* an irrotational flow.

It is now argued that the magnitude of our proposed (maximum) latent/noumenal potential (specific) energy 'source', at c^2 , is similar to the constant stagnation pressure (divided by constant density) in (an idealised) fluid mechanics; in that it is constant throughout (the 'whole' of) space. Background space itself can be seen as a continuum; but in regard to physical reality with different local rates of time, it cannot be a systemic frame of reference for observations. Nevertheless, this conceptualisation permits the existence of a *second* (and quite different) type of gravitational 'draw-down', that serves to indirectly facilitate an explanation/model for the Pioneer anomaly — the crucial step in permitting a *real* Pioneer anomaly.

In section 6.5 we shall see that the model's value of (rotating space-warp) specific energy, $\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2$, is (also) constant throughout space.

4.5.13 A new reason for general covariance in General Relativity's formalism

Subsection 4.5.12 illustrated that in fluid mechanics there is (often) a representational necessity to work with specific energies. GR's principle of equivalence means that a (test) body will 'experience' the same acceleration independent (or regardless) of its mass. Historically, general relativity moved us away from an analysis based upon force; although, in somewhat classical terms, we might (nowadays) think of acceleration as a specific force (F/m).

Conventionally total energy is an *addition* of energies (from zero *up*). From our reversed ontological perspective we now (also) think of conventional *macroscopic* (specific) energy increases as actually a reduction in, or drawing-down upon, a (very large magnitude) invariant specific energy 'noumenal-source'²⁰³.

²⁰³The reader may be wondering whether the 'mechanical' specific energy (E/m) discussed herein also applies in the case of decidedly electromagnetic quantities and processes, where "the mass" in question is not at all obvious. Since our concern herein only involves the inertial/mass aspects of QM atomic/molecular systems, electromagnetism shall not directly concern us; but it *may* be that electromagnetic *quantities* (cf. their effects) need to be treated quite differently at a deeper ontological level — especially if specific energy is a very important quantity. The following may possibly be of relevance. Reducing the units/dimensions of electromagnetic quantities to the

Recall from subsection 4.5.12, that the physical representation of energy in macroscopic (mechanical) mechanisms easily yields a specific energy. Further, mass itself and its motion, are considered to also 'draw-down' upon a "background availability"; and at a macroscopic level this circumstance is (now) conjectured to be associated with conventional gravitational effects. From such a perspective, the (standard) representation of various effects within general relativity is complicated by the numerous ways in which such draw-downs can occur; e.g. mass, momentum, and energy; as well as temporal rate changes that accompany these effects. Subsequently, it can be argued that only a theoretical approach that employs general covariance is capable of coping with this 'multiplicity' of (draw-down) effects.

4.5.14 Three gravitational 'draw-downs' upon invariant specific energy

SR's relative motion, GR's gravitation, and the model's rotating space-warps (RSWs) can all be understood as various ways of "drawing-down" from a (hidden) uniform specific energy background. Subsequently, the two types of non-Euclidean geometry associated with GR and the new model's RSWs, although quite different, can coexist — without any conflict.

Firstly, we have the simple draw down of high-speed (relativistic) relative motion, as discussed in subsection 4.5.6 through 4.5.10, giving rise to the idiosyncrasies of special relativity — and expressed by way

fundamental units/dimensions of (kg, m, sec, C) and (M, L, T, and Q) respectively, it can be seen that the ratio M/Q appears in the dimensions of most electromagnetic quantities. Note that C is coulombs and Q is (electrical) charge. For example, the inverse of capacitance has the unit $[\text{Farad}]^{-1}$ which can be expressed (with some effort) in fundamental units as $[\frac{\text{kg} \cdot \text{m}^2}{\text{C}^2 \cdot \text{sec}^2}]$; which in fundamental dimension notation is $[\frac{ML^2}{Q^2 T^2}]$. The extra $[M/Q]$ ratio arises from physics giving primacy to *mechanical* energy, which has dimensions $[\frac{ML^2}{T^2}]$. Note that there is no Q dimension in work/energy, even if the system doing the work is electrical and/or magnetic. Since the charge to (rest) mass ratio of an electron $(q/m)_e$ is a fixed quantity and has dimensions $[Q/M]$, we can use this to alter the dimensions of (inverse) Farads to $[\frac{L^2}{Q T^2}]$, which is now expressed in a Q, L, T system of units (without M). The Q dimension is in the denominator rather than the numerator (as M is with mechanical energy). Hence, it appears that the specific energy, i.e. $[\frac{L^2}{T^2}]$, associated with this electromagnetic quantity (capacitance⁻¹), in the new 'units', involves inverse capacitance multiplied by the (stored) electrical charge, which is actually *electrical potential*. In other words, in this new non-mechanical (Q, L, T) unit/dimension system especially suited to electrical quantities, specific energy $[\frac{L^2}{T^2}]$ is simply (electrical) potential. In the traditional system of units/dimensions, potential energy per unit charge has the unit of 'Volts' and dimensions: $[\frac{ML^2}{Q T^2}]$. The primacy of electrical potential in electromagnetism makes it well suited to playing an important role at a deeper ontological level, *if*: specific energy is important, and the use of the M, Q, L, T unit/dimension system for describing electromagnetic quantities is replaced at a deeper level with the simpler (and conceivably more natural) Q, L, T system.

of Lorentz transformations. Herein, this has been additionally understood as relative motion's (specific energy) introducing an imaginary component of specific energy, i.e. a drawing-down from the maximum background/noumenal specific energy field/potential-hill; thus leaving less specific energy available to 'reality'.

Secondly, as mentioned in subsection 4.5.13, a plethora of effects result in (GR's) gravitational effects. Further, by embracing the alternative ontology outlined in this section, especially that universal (noumenal/global) sequential evolution involves a hidden (beyond observation) simultaneity; spacetime effects may then be re-interpreted as a spatial separation change accompanied by a rate of time change. Note that this alternative (and 'neo-classical') conceptualisation has no (direct) observational relevance. This interpretation allows GR's gravitation to be visualised (solely) as space curvature, in conjunction with both: a local time rate and a (hidden/background) global time rate. This intrinsic curvature of space²⁰⁴, as compared with extrinsic curvature, is seen to occur with the assistance of imaginary numbers in the three space dimensions. Conceivably, and neo-classically, these imaginary (number based) spatial components are then related to a (specific) deformation energy, which in turn equates to the (specific energy) 'draw-down' associated with (GR's) gravitation. The mathematical quantification of the specific energy associated with deformation into (imaginary number based) space shall not concern us²⁰⁵.

Thirdly, the model represents the specific energy of each rotating space-warp (RSW) as $\Delta e = \frac{1}{2}\Delta a^2\Delta t^2$. This value is constant throughout (the whole of) space. Additionally, it is *process* based, occurring over a time (Δt). This duration is additionally a cyclic loop time primarily determined by (lunar) *motion*²⁰⁶; and subsequently, relativistic effects upon this motion are accommodated. It is quantified by way of a *systemic* clock rate; most conveniently a (solar system) barycentric time rate. In the case of RSWs, the specific energy 'draw-down' is simply $\frac{1}{2}\Delta a^2\Delta t^2$, which is quite distinct from 'draw-downs' associated with GR's gravitation — although an acceleration amplitude (Δa), i.e. *gravitational* space-warp magnitude, can also be envisaged in terms of curvature.

Importantly, the constancy of the model's specific energy appears to be necessary, so as to allow GR's 'gravitation' and the model's new aspect (or supplement) of gravitation to be compatible and *coexist*. These two 'gravitational' effects are quite different; e.g. having their 'basis' in different 'levels' of matter — i.e. macroscopic mass in motion vs. atomic/molecular mass in motion. Sections 5 and 6 shall explain how the total *energy* associated with RSWs, incorporating

a non-local mass, equals the total virtual and unexpressed non-inertial energy (at the atomic/molecular level) of a (suitable) moon-planet system. Outstanding at this stage, is a discussion concerning the model's reliance upon the principle of energy conservation (see section 4.6).

4.5.15 Summary and concluding remarks

The primary goal in this section has been to outline how a second type of non-Euclidean geometry (i.e. gravitational field) can arise, in addition to the macroscopic-based gravitational effects of general relativity. This has involved the introduction of a supplementary ontology involving a reversed perspective upon (observational) reality. These two different (and complementary) ontological perspectives are seen to be required for a full description of reality — if the Pioneer anomaly is a real gravitationally based phenomenon. Respectively, one is sub-global and physical, while the other is global and 'metaphysical'; i.e. they are phenomenal and noumenal respectively.

This "dual-ontology" allows us to simply posit that measurements in spacetime do not exhaust 'reality'. Measurements can only 'apprehend' the 'real', i.e. non-imaginary, components of a physical phenomena that in their (formalistic) entirety conceivably also involve complex numbers. Aspects of the supplementary ontology include: recognition of a special type of hidden background/global time simultaneity, and a background Euclidean 'flat' space continuum (in idealised circumstances). The latter exists (conceptually) in the absence of all mass, momentum and energy (fields). Thus, we may (also) ascribe gravitational effects to both a loss in *measured* time rate cf. a 'universal' time rate, *and* a change in measured spatial separation cf. an (idealised) flat space separation.

A crucial aspect of the model (to come) is that a *virtual* QM energy is re-expressed as a (non-virtual) physically real rotating curved/warped space (together with a non-local mass distribution). In section 4.6 we shall see, somewhat by way of our new supplementary/complementary (and reversed) perspective on 'reality', that *conservation of energy*:

1. has applicability to the 'gravitational' (i.e. the space perturbation) aspect of the model;
2. is grandly systemic in that it is not necessarily restricted to one 'level' of 'material' reality; and
3. can involve a virtual energy magnitude, and thus it is not merely restricted to directly measured (i.e. 'phenomenal' and real) quantities.

4.6 Symmetry, Noether's Theorem, and energy conservation

Our model, requires we allow a qualitative divide to exist between the micro- and macroscopic realms (also see section 5.1), and permit a non-reductive (i.e. not to a graviton particle) gravitational phenomenon at the macroscopic level. It may or may not be that GR is reducible to a graviton *particle*, but with our proposed

²⁰⁴Relative to what *would* be a naturally existing flat/Euclidean space background, in the (idealised circumstances of an) absence of: matter, momentum, and energy.

²⁰⁵It is assumed that an analogy to circumstances involving the bending of a beam in "mechanics of solids" can achieve this, with an adjustment for the local time rate cf. the fastest clock in the universe (recall subsection 4.4.7).

²⁰⁶Later we shall more fully discuss how the mechanism involves the self-interference of atoms/molecules.

supplementary space curvature/perturbation, gravitation (overall) cannot be (solely) a “gauge” theory, as the three microscopic forces are. The ramifications of this non-particle reduction (and non-solely-particle unification) approach are now explored in relation to Noether’s theorem. Gravitation is preferably seen as a “geometric” theory.

4.6.1 Preliminary Remarks: macroscopic-microscopic ‘separation’

A total macro-micro separation is not being espoused. Rather, we are examining how the very different characteristics exhibited by the quantum mechanical (QM) and non-QM realms, may coexist. Particularly, when a physical phenomena spans both domains to comprise an integrated system. We shall argue that energy is common to both realms, and its conservation therefore spans both domains; much in the manner that the emission and absorption of EM radiation/photons involves both a quantum system and the external world. This *electromagnetic* micro-macro situation is somewhat of a precedent for our *inertial mass*-based model.

Restricting ourselves solely to the macroscopic level, the instantiation of (gravito-quantum) rotating space-warps to explain the Pioneer anomaly appears to violate energy conservation; but (we shall see that) for the combined micro- *and* macroscopic system proposed herein, energy’s conservation is maintained. In fact, energy conservation (and geometric constraints) are necessary in order to quantify the acceleration/gravitational amplitude and mass “carrying capacity” of a rotating space-warp — which in turn is indicative of a (virtual) non-inertial (mass in ‘motion’) circumstance at the QM level of matter (see subsection 4.7.5 for a fuller account of the model).

4.6.2 Preliminary Remarks: invariance, energy and acceleration

Concerning the benefits of homogeneity and/or invariance, two remarks need to be made. Firstly, the model is able to cope with the fact that energy is a frame dependent concept, whereas GR’s energy-momentum tensor is not; because (cycle-based) specific energy ($\Delta\epsilon$) is homogeneous throughout (the whole of) space. Note that the (global) frame is the whole universe. Secondly, (amplitude) homogeneity makes coordinate variations non-existent. Thus, by default we cannot deny that: *coordinate* acceleration and (systemic) physical acceleration are the same (cf. GR); with these also equivalent to, and indicative of, a ‘gravitational’ phenomenon (in its broadest sense/usage).

4.6.3 Background to Noether’s Theorem

Noether’s theorem is considered to precisely illustrate a deep connection between conservation laws and symmetries in nature. Noether’s theorem is restricted to systems which obey the principle of least action *and* have a Lagrangian and a Hamiltonian, which then determine the field equations. It is an important result, especially for symmetries in elementary

particle physics. The symmetries of Noether’s theorem relate to the invariance of the *laws* of physics. The homogeneity and isotropy of space, and the homogeneity of time, then imply conservation of momentum (linear and angular) and energy, respectively. Indeed: “...any symmetry of the Lagrangian corresponds to a conserved quantity (and *vice versa*).”²⁰⁷ Noether’s theorem is *not* seen to apply to the model being proposed herein, because the dual-level ‘system’ being considered cannot be described by a single Lagrangian and/or Hamiltonian; but aspects associated with Noether’s theorem *are* conceptually valuable to the new model’s formulation and discussion.

4.6.4 What remains in need of clarification

Let us recall our hypothesised (supplementary) ontological stance regarding noumenal/*background* space and (hidden background) sequential temporal evolution — mainly developed in section 4.4. In the idealised circumstances of an absence of mass, momentum and (mass- and radiation-based) energy, (the) background space *is* inherently homogeneous and isotropic. Further, the phenomenal-noumenal (pivot-pause-pivot-pause...) sequential arrangement of all-inclusive (i.e. cosmologically-‘wide’) ‘moments of reality’ — occurring all together²⁰⁸ — allows us to say that: (non-observational) universal ‘time’ *is* homogeneous (i.e. simultaneous)²⁰⁹. Thus, conservation of energy, and conservation of linear and angular momentum apply; and they are possibly valid assumptions/principles for the model’s real world application. We now investigate the veracity of this possibility.

Two subtleties shall need to be addressed. Firstly, in the unique circumstances of the model and a supplemented ontology, does the presence of: mass, momentum, energy, and (hence) a non-uniform (observational) time (rate), deny all of these conservation laws/principles? Secondly, (the ‘quantity’) energy, by way of its ‘dimensional’ make-up, also involves a mass ‘dimension’, in addition to the (spatial) length and time dimensions. Thus, if a rotating space-warp has a total energy, how (then) is the mass aspect of this energy to be understood?

4.6.5 On the failure of local conservation of energy in general relativity

Gravitational energy in GR cannot be conserved *locally* as it is in classical theories; although, in the presence of asymptotically flat spacetime (S/T), global energy conservation applies.

Somewhat in response to section 4.2, it appears that it is the assumed existence of a *local* nature of gravitational potential that necessarily thwarts local

²⁰⁷<http://www.mathpages.com/home/kmath564/kmath564.htm>

²⁰⁸With a beyond observation process-interval between these coordinated and cosmologically all inclusive (staccato-like) ‘moments’ (recall subsections 4.2.10 and 4.4.8).

²⁰⁹Notwithstanding the fact that *observationally* clocks throughout the universe run at different rates — depending on their local circumstances.

gravitational energy conservation. This is because a gravitational ‘potential’ is largely based upon (the antiquated notion of) gravitational ‘force’ applied over a distance, leading to an energy in the sense of physical work. Logically, a local failure of (this type of) energy conservation in GR is *not* a sufficient reason (as yet) to deny the applicability of energy conservation to/in a *global/cosmological* (curved space) physical system.

In GR, the non-homogeneity (or non-simultaneity) of time gives sufficient reason for the denial of conservation of energy. The inappropriateness of this denial, in the case of our model, shall (now) be discussed in subsection 4.6.6.

4.6.6 Homogeneity, isotropy, and the model being pursued

Accepting the dual-ontology outlined in section 4.4, relied upon a distinction between formalism and ontology. The former concerns: “what we can say about reality”, and the latter concerns: “what reality is” — which possibly also includes aspects behind the scenes, i.e. beyond (direct) observations. In the *idealised* situation of an absence of matter, momentum and energy, and by way of embracing a dual-ontology, we may embrace the (classical-like) symmetries of hidden/background space — (meaning/implying) that it is the same at every location (homogeneous) and in every direction (isotropic).

By way of reality’s underlying homogeneity of ‘time’ (i.e. a simultaneous digital cosmological temporal evolution), energy conservation is implied — in both real and idealised conditions. By way of (noumenal) *time’s* (priority and) homogeneity, energy (cf. momentum) conservation may be discussed — *both* micro- and macroscopically. For the purposes of the model, energy needs to span (and interweave) a physical phenomenon involving both the microscopic and macroscopic realms *together*. In other words, energy conservation is seen to ‘operate’ across both/all systems through (a common systemic) time. In the model, we shall see that energy and its linkage between the two realms, is the major (guiding) principle embraced; with the magnitude of this energy (per atom/molecule) ultimately lying within limits imposed by (Δt and \hbar in) Heisenberg’s uncertainty principle (HUP).

Time isotropy does not exist at the level of the new model’s *formalism*, because a total *virtual* microscopic (QM) energy is *asymmetrically/irreversibly* re-expressed externally as the (*real*) energy of a rotating space-warp. Further, to allow the space-warp’s (real) existence at the macroscopic level, i.e. the level of curved space, the model forgoes the isotropy and homogeneity of idealised (background) space. The correct implication (from this) is not that the model violates conservation of linear and angular momentum; but rather that a traditional reductive particle-based approach to conservation of momentum is inappropriate (and inapplicable) to the model’s multi-realm (system) — and its HUP-based circumstances.

The model cannot be based upon an action principle in one big/general system, as a reductive (and relativistic) approach envisages, because Noether’s theo-

rem (only) applies to single level ‘conservative’ systems. Thus, Noether’s theorem is not applicable in a dual-realm (i.e. microscopic plus macroscopic) physical system, that also employs a phenomena-to-noumena-like ontological distinction. Only the conservation of (a non-vector, scalar) energy, can ‘survive’ this complexity. Indeed, conservation of energy²¹⁰ is the linchpin of the model, with energy’s ‘universality’ and applicability to a field theory (see Section 6) providing strong support.

4.6.7 Briefly on dual-level, and non-local physical behaviour

In the preface to “Science and Ultimate Reality” (Barrow, Davies, & Harper 2004, xvii-xviii), Freeman J. Dyson discusses an example of (macroscopic) classical mechanics and quantum mechanics being both required in an explanation. A description of Uranium 235 fission by a slow neutron sits at the border of microscopic and macroscopic physics, with a classical liquid drop conceptualisation being very useful. Thus, (once again) the model’s approach is not unprecedented.

How is the mass dimension of energy, which has dimensions [$\frac{ML^2}{T^2}$], pertinent to the model? We shall see that: at the QM (virtual energy) level, it arises (partly) by way of Planck’s constant, i.e. a minimum angular momentum quantity; and at the macroscopic level, a (new) ‘non-local mass’ (m^*) quantity is (necessarily) introduced in addition to the ‘invariant’ specific energy (Δe) of rotating space-warps. This new type of (non-local) mass relates to the fact that: “... QMs is ‘non-local’ because the state of a quantum system is spread throughout a region of space (Davies 2004, p.20).” Similarly, by way of the virtual nature of the QM energy involved, this non-local mass is always spread throughout a volume (V) of space. In other words, it is never localised; but it can/does act upon a localised mass, e.g. the Pioneer spacecraft. Indeed, we shall see that: $m^*V = \text{constant}$ (in subsection 6.6.5).

4.6.8 Asymmetry in the model and two asymmetries in physical reality

Unlike Noether’s theorem (recall the beginning of subsection 4.6.3), the model herein is considered to illustrate a deep connection between a conservation law (i.e. energy) and an *asymmetry* in nature.

In Section 5 we shall see that the virtual quantum mechanical (QM) energy arises from the inertial mass (and spin) within an atom or molecule. By way of relative angular momentum, a (virtual) non-inertial configuration is attained within (a great many) *atoms/molecules*. This arises by way of: their macroscopic (celestial) motion (orbital path) in curved space-time; a small change/offset in *geometric* phase; and the dominant strength of electromagnetic forces within an atom/molecule. This virtual QM energy is ultimately an expression of a (different or newly recognised type of) asymmetry in nature.

²¹⁰Specifically (and additionally) involving a cyclic rate of angular momentum (offset), which is yet to be elucidated.

What comprises the *broad* basis for the existence of this (virtual) QM (angular momentum rate) asymmetry? Gravitation (in practice) involves only positive mass (i.e. an absence of antimatter), and hence locally there is only positive curvature. Further, the model exploits the unidirectionality (i.e. arrow) of time. These two physical asymmetries pertain to two primary aspects of reality. Indeed, physical dimensionality (standardly) employs mass and time as two of its *fundamental* dimensions. Thus, asymmetry is deeply imbedded in reality, and it is conceivable that in (positively) curved spacetime, an irreversible inertial mass-based process — that is asymmetrical in time — is indirectly behind the Pioneer anomaly.

4.6.9 The model's unique circumstances

Regarding the model's rotating space-warps²¹¹, homogeneity of space and the isotropy of space and time are clearly absent. We shall now discuss the very restricted, and minor, sense in which this occurs.

In the model, the energy of the asymmetry associated with the inertial to non-inertial frame offset — and a virtual to actual physical (circumstance) offset — equals the energy of a supplementary (external) real gravitational/accelerational field effect (a non-isotropic rotating space-warp²¹²). Note that over the time of a single rotation (this external field effect) is equally distributed around an equilibrium field strength — in that it imparts (on average) no 'overall' additional/extra curvature to that already provided by general relativity. The constant amplitude of the space-warp means that, over extended time spans and *on the whole*, an idealised 'background space' in one sense retains its inherent homogeneity and isotropy.

The existence of this rotating space-warp is ultimately based upon the uncertainty principle of QMs, and quantum *discreteness* encountering the effects of (non-discrete or) *continuous* curved spacetime. Thus, the warp merely represents an uncertainty principle based "wobble room" (perturbation) around an equilibrium set of circumstances established by GR (i.e. standard gravitation). Importantly, such a mechanism ensures the ongoing stable existence of the *mass* aspects of QM systems in curved spacetime circumstances, especially angular momentum (see Section 5 for a more detailed discussion).

Energy is the (common) 'currency' of the model's micro- to macroscopic mediation (or re-expression). The physical energy being discussed is a virtual QM intrinsic angular momentum per cycle time (i.e. a virtual spin energy). We shall see that because the underlying quantum mechanical asymmetry is cyclic, and process based, it reasserts itself after every completed spin-orbital cycle; thus maintaining the continuous rotation of the (effectively rigid²¹³) space-warp. Note, that for the QM energy to be virtual, it needs to be below the

minimum energy level of any atomic/molecular QM interaction; and that the not insignificant magnitude of this virtual energy arises from it being (very nearly the same magnitude and) shared by every atom/molecule throughout a (suitable) spin-orbit coupled moon.

4.6.10 Concluding remarks

The previous discussion has shown that: Noether's theorem, which is a part of the theoretical physicist's "principle" based approach to reality, has effectively over-reached regarding that which *cannot* exist in reality. To model a real Pioneer anomaly, asymmetry and energy conservation (shall) play a crucial role in linking together the microscopic (i.e. QM) and macroscopic (i.e. gravitational) realms/domains. Without this approach, it is hard to conceive how any meaningful physical linkage would otherwise be attained. In the language of philosophers of science, (herein) we are pursuing a (mainly) constructive theory (and model), rather than a principle theory approach.

4.7 Summary and concluding remarks regarding SR and GR

This section provides an overview of conclusions reached regarding our new stance concerning special and general relativity, as well as an encapsulation of the model to be presented (and further developed).

4.7.1 Incompleteness cf. incorrectness

Let it be said that in no way is either SR and/or GR's approach to 'spacetime' and physical phenomena seen to be *incorrect*, as regards relevant observational evidence. Our concern is whether they provide a conceptually *complete* theory of "gravitation", in the widest sense of the word. By way of issues surrounding QM non-locality, and the assumed existence of a real Pioneer anomaly²¹⁴, there is sufficient justification for hypothesising an ontological extension to our (singular) understanding of what space and time 'are'; and how they relate to, and coexist with, each other.

4.7.2 Aspects of our new approach to Special Relativity and Gravitation

In the spirit of (Sir) Arthur Stanley Eddington we saw (he saw) (QMs and) GR "...as fundamentally 'epistemological' in character, meaning that they provided insight into *how* we see the world, rather than *what* the world is (Stanley 2005, p.37)." Thus, in line with certain philosophers of physics, general covariance (GCoV) is regarded as simply a new mathematical technique, not an expression of physical content.

In the spirit of Vladimir A. Fock, who sought to develop a new, non-local, point of view of Einstein's theory (Fock 1959, Preface), we recognise the need

²¹¹Later we see that these are actually (in three spatial dimensions) rotating axisymmetric warped space-'cylinders', of global size/scale or 'proportions' — rather than a thin (two-dimensional) disk (also) of global/universal extent.

²¹²With its associated non-local mass magnitude (m^*).

²¹³In the sense of not deforming from its initial 'shape'.

²¹⁴Not to mention that: "no one understands why empty space should have [dark] energy (Krauss 2004, p.83)." Note that theory based upon vacuum energy (seeking to explain dark energy) differs from measurements by about 120 orders of magnitude, and thus this cannot be a valid hypothesis.

(in some cases) for a global perspective upon gravitation. Further, herein we share Fock’s preference for viewing SR as a “Theory of Invariance” and GR as a “Relativistic Theory of Gravitation” — in order to guide our supplementation of gravitation — but his wish to have privileged systems of coordinates (and ‘recast’ Einstein’s theory) is not embraced.

4.7.3 Benefits of a new supplementary conception of space and time

To appease a (real) Pioneer anomaly, we are required to accept the “dual-ontology” outlined in this Section. This dual-ontology involves appreciating the ‘coexistence’ of *both* a relative (observational), and a global (hidden background), conception of space and time. The former is constrained to ‘working with’ spacetime, whereas the latter allows space and time to be treated as physically separate notions. The latter conception, by way of a new appreciation of universal sequential evolution (that cannot be directly appreciated by way of observations), is consistent with QM non-locality.

This ‘other’ global (hidden background) approach/perspective, posits a type of hidden cosmological substratum; i.e. a space continuum that is somewhat analogous to a fluid mechanical molecular mass continuum. The nature of global (sequential) evolution is such that an effective (hidden background) time simultaneity exists; but observationally, clocks will run at different rates depending on their local circumstances²¹⁵.

Further, we need to allow the standard notions of emptiness and zero specific energy to coexist with their ‘opposites’, i.e. fullness and maximum specific energy (set at c^2). The latter is an (everywhere) uniform and (‘everywhen’) unchanging background potential (specific) energy field. We then need to hypothesise that the magnitude of this ‘field’ is ‘complex’, in it has both real and imaginary components. A ramification of this supplementary conception, is that SR’s Lorentz transformations can be seen as arising from relative motion introducing an imaginary component of specific energy, so as to reduce the availability of space and time to observations. Observationally, this (*loss* of availability) *appears* as time dilation, length contraction, and mass dilation. We may then think of this as involving a “drawing-down” upon the uniform (hidden background) specific energy field (or potential hill).

The ‘subtlety’ of GR, is seen to arise from multiple draw-downs coexisting (e.g. via mass and the motion of mass); hence the (somewhat restrictive) requirement of general covariance in its formalism. We conjectured (subsection 4.4.1) that: GR’s need for general covariance (GCoV) is a direct consequence of (further) adding gravitational effects to the (already additional) conformal structure demanded by SR’s Lorentz symmetry. From our new (or complementary) perspective we may alternatively recognise an intrinsic (cf. extrinsic) imaginary (number based) space ‘dimension’ into

²¹⁵Note that a hidden variable background time (itself) is *not* being proposed; only a hidden cosmological (sequential existence) go-pause-go-pause-... *simultaneity* is being proposed (recall subsection 4.2.10).

which space ‘curves’ or deforms. Although this new conceptualisation assists our understanding, where observations are concerned, we must almost always necessarily defer to standard GR. The Pioneer anomaly and its explanation is the exception to this rule.

The consequence of this supplementary (noumenal cf. phenomenal) conceptualisation is that a different type of ‘gravitational’ field, as compared to any describable within GR, can exist. Subsection 4.7.5 briefly outlines how this field arises from a different ‘level’ of matter; i.e. mass ‘within’ (a great many) atoms/molecules — subjected to celestial closed loop/path orbital (and spin) motion — cf. macroscopic mass/matter (itself), its motion, and energy.

4.7.4 Summary remarks and distinguishing the new approach to ‘gravitation’

A reduction and unification agenda, Noether’s theorem, GR’s principle of GCoV, SR’s non-simultaneity, and the lack of local energy conservation in GR, are seen to *not* stand in the way of the model being presented; if the reader is prepared to relax their standard “concept package” of reality — (so as) to embrace a conceivable alternative. If one’s standard “concept package” is not relaxed, then one can only have a high degree of scepticism regarding the real (‘gravitational’) existence of the Pioneer anomaly. The model requires the ‘universality’ of energy conservation to be applicable/restored (in its specific circumstances), and the importance of an exclusively ‘principle’-based approach in *macroscopic* physics to be strongly downplayed.

The new approach being pursued, via the awkward Pioneer observational evidence, involves/includes:

1. a macroscopic constructive theory, that arises in response to a microscopic asymmetry,
2. an energy basis, and a circulatory field effect,
3. continuum mechanics (involving space & non-local quantum mechanical mass — separately),
4. a richer ontology, and acceptance of hidden variables — albeit at the level of the full system, and
5. recognition of two quite distinct, yet coexistent, physical realms; i.e. the micro- and macroscopic.

This stands in stark contrast to the general thrust (standard approach) of contemporary physics which utilises/employs (respectively):

1. a principle based approach to theory (including an emphasis upon symmetry),
2. particle exchange as being responsible for force,
3. Lagrangian and Hamiltonian based formalisms,
4. an “operationalist” approach to the world, and
5. a four force unification agenda (3 micro+GR).

This write-up doesn’t seek to discredit Special or General Relativity, nor disparage the three force “standard model”; rather, it seeks to supplement and complement this approach — which has failed, for more than 25 years, to make *substantial* ‘scientific’ (cf. mathematical) progress towards its objective of unifying the three force quantum mechanical microscopic world and General Relativity.

4.7.5 Previewing the mechanism that indirectly explains the Pioneer anomaly

The ramifications of our (noumenal plus phenomenal²¹⁶) dual-ontological stance upon energy conservation was examined in section 4.6. We saw that the global approach, of the supplementary/complementary ontology, also allows conservation of energy to regain its ascendancy — at least with regard to the unique circumstances of the (new) mechanism/model.

The model involves geodesic motion in curved spacetime influencing the (spin-based) geometric phase of (all elementary fermion/matter particles within) an atomic/molecular quantum mechanical system (see Section 5). This geometric phase influence is (effectively) common to (a great many) QM *systems* i.e. atoms and molecules within a ‘suitable’ celestial body. This suitability requires the celestial body to be a (predominantly solid-body) moon in spin-orbit coupled motion around its host planet, which in turn orbits the Sun — i.e. three celestial bodies are involved in the model. Also involved are inertial effects pertaining to QM intrinsic spin. The relative (spin) geometric phase change ‘results’ in a not insignificant (non-inertial) QM intrinsic angular momentum (rate), or (spin) energy. It is ‘non-inertial’ because atoms/molecules are dominated by the electromagnetic force, and because of the ‘internal’ spin-orbit coupling of atoms/molecules the geometric phase change/offset is overridden/denied. A ‘rate of angular momentum’ is involved, because self-interference of the atoms/molecules per completed orbital/cyclic loop is a feature of the mechanism to be proposed.

Only when this (‘internal’, relative, and non-inertial) energy offset is virtual, i.e. existing below a decoherence ‘trigger’ at the first or a minimum QM energy level, does the situation result in a non-local *external* re-expression of the (same) energy magnitude. The re-expression of the total virtual internal energy (ΔE) as a real external energy is a one-way process. It relies on energy being a scalar quantity, and having a multi-faceted ‘nature’. Scalar energy is the common ‘currency’ in the ‘mediation’ of this rather complicated physical phenomenon; with ‘universal’ energy conservation *the* ‘governing’ principle.

The external re-expression of (virtual QM) energy expresses itself as a neo-classical-like rotating and asymmetric perturbation of space, that ‘piggy-backs’ upon and distorts the pre-existing gravitational field (as determined by GR). This perturbation takes the form of a universal (or cosmological) size/scale rotating space-warp (RSW) (introduced/discussed in Section 3), whose energy — upon completion of one full rotation of the RSW — is also ΔE . Associated with this (now non-virtual) energy is both: a non-local mass distribution, and a specific energy distribution (Δe). The latter, and an associated acceleration amplitude (Δa), is the same everywhere (i.e. cosmologically) throughout the field. It is the (distributed) non-local mass, at a point in the field, that reduces as we move

away from the RSW’s source; and as such ΔE is ‘dissipated’ away from its source (atoms and molecules).

The representation and conceptualisation of this effect relies on the dual-ontology previously mentioned. The existence of a hidden background temporal coordination (or simultaneity), and a global background space continuum, together with energy as drawing-down from a maximum (reservoir) value, permits the representation of the mechanism to be expressed in a ‘neo-classical’ mathematical formalism (that is separate from GR’s formalism). Subsequently, use of a metric shall *not* be required.

The model is comprehensively quantified in Section 6. Several of these RSWs (coexist and) combine/superposition to produce the Pioneer anomaly. Preempting this, (for each RSW) the (virtual) QM energy (ΔE) is proportional to a rotational rate $[\Delta t^{-1}]$, multiplied by the reduced Planck constant (\hbar) and the number of atoms/molecules (N_m). The rotating space-warp’s energy (also ΔE) is proportional to (half) the warp/wave amplitude squared, i.e. acceleration squared. Later we shall see that $\Delta E = \frac{1}{2}m_1^*\Delta a^2\Delta t^2$ where Δt is the duration of a single rotation, and m_1^* is a non-local mass encompassing a given (initial) volume.

In Section 5 it shall become clear that only a minimal geometric phase shift can lead to the *virtual* QM energy associated with the mechanism, and this is dependent upon a number of things; one of which is the existence of the *weak* gravitational fields typical of our solar system — and almost certainly other similar solar systems. Subsequently, the neo-classical approach employed herein is not compromised by circumstances pertaining to very strong gravitational fields.

5 Outlining quantum & general aspects of the model

In this Section the quantum mechanical (QM) aspects of the model are primarily outlined. Each section steps up the level of detail in which the model is presented. Due to the length of this Section, the economical reader may prefer to merely read the summary at the end of each section — 5.1 to 5.7. The main aim of this Section is discussed in section 5.1.1.

5.1 Revising and extending the relationships between the microscopic and macroscopic ‘realms’

Drawing upon the alternative ontology presented in Section 4, this section (5.1) seeks to elaborate upon the differences between the microscopic and macroscopic realms previously discussed in subsection 4.2.6. It then highlights various ramifications ensuing from this. The discussion is at times general and at other times model specific. This is an important step in our attempt to establish a second ‘means’ (or manner) by which non-Euclidean geometry is achieved. Preceding this we declare a major aim of this paper/treatise.

²¹⁶In a scientific observational sense.

5.1.1 Main aim, and non-locality and locality at non-extreme temperatures

In very general terms, the main aim of this Section and the paper is to (indirectly) relate curved space-time (rather than GR *per se*) to a quantum mechanical system's 'state' at non-extreme temperatures/energies. Of great influence in this regard has been Raymond Chiao (Chiao 2004). His, and our, primary concern is the apparent tension between QMs' non-locality and GR's locality. He addresses this issue in conjunction with Berry's geometric phase (see section 5.2). Note that 'locality' implies both: a particle basis, *and* communication being mitigated at (or below) the speed of light. The ramifications of questioning a particle basis are investigated in this section, i.e. section 5.1.

When examining an internal-QM to external-GR physical relationship, Raymond Chiao and others examine either sub-atomic particles or electromagnetic radiation's photons. In contrast, our interest shall involve (wave-like) angular momentum aspects of the internal 'motion' within *atoms and molecules*. Thus, the Dirac equation in 'flat' or curved spacetime shall not concern us. Of interest shall be: maximum QM energy uncertainty, a minimum difference in QM energy levels; and (quantitatively) Dirac's constant (i.e. Planck's constant $\div 2\pi = \hbar/2\pi = \hbar$) shall be important.

5.1.2 Altering the perceived relationship between gravitation and EM

Some close parallels are *assumed* to exist between gravitation and electromagnetism (EM), e.g. the inverse square law. The dual (wave and particle) nature of electromagnetic radiation/photons spans the micro-*and* macroscopic realms. Note that mechanistically, EM exhibits quite distinct behaviour in the two realms: for example, the role of EM in an electric motor is very distinct from its role in an atom. To compliment the massless photon, the existence of a (massless) graviton particle has been confidently postulated, but there is a subtle difference between EM and gravitation.

In EM the force between two charges doesn't depend upon a third charge, whereas with gravitational influences this is not the case. If we deny the existence of a graviton particle and hence a *gauge theory* approach to gravitation, then a purely particle based (gravitational) momentum to energy link flounders, because graviton momentum can no longer be defined. Unconventionally, we shall allow energy's manner of existence to not be solely restricted to a *local* particle (and force) basis. As such, the *energy* of a rotating space-warp can be associated with both: the *specific energy* of a rotating space-warp/space-deformation, together with a (new) *non-local* distribution of *mass* — with the latter elaborated upon in section 6.6.

This is consistent with our previous denial of "mass in SR" always being secondary to energy, at the *macroscopic* level; which ensured the subsequent revival of the notion of relativistic mass (see subsections 4.4.5 and 4.4.9). In contrast, *microscopically* the relationship between the energy and the momentum of a photon is simply mitigated by the speed of light. Deny-

ing a (physical) reduction to the graviton (particle), in turn denies both: that mass is *always* secondary to energy, and (the validity of) a gauge theory approach to gravitation. In place of a particle emphasis, gravitation — in the sense of a gravitational acceleration (that can act upon a body) — is given a geometric emphasis; and subsequently, it is quite different to, and distinct from, the other (three microscopic) 'forces'.

5.1.3 The size of 'microscopic systems', and mass & energy by number

Our criteria for a macro-micro (theoretical) dividing line is taken to be: whether the size of a micro/QM object is determined by its QM wavelike nature — a 'boundary' condition for the 'microscopic' QM realm one might say. For electrons and atoms, i.e. an elementary particle and a QM bound system (respectively), this *is* the case²¹⁷. Subsequently, the largest microscopic systems are necessarily atoms and molecules²¹⁸.

If in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence passed on to the next generation of creatures, what statement would contain the most information in the fewest words? I believe it is ... that *all [macroscopic] things are made of atoms* ... Richard P. Feynman (Rohlf 1994, p.1)

We emphasise the somewhat neglected idea that bulk mass (or large mass) is effectively a sum of its constituent atoms/molecules. Loosely speaking, we have "mass by number". Microscopically, mass is secondary to energy. These two points together exhibit the possibility that an excess "virtual energy by number" may arise. This notion was introduced in subsection 4.2.2, and herein it shall be applied with regard to an energy uncertainty upper limit associated with the atoms/molecules 'constituting' a celestial three-body spin and orbital system.

Additionally, when discussing the macroscopic in terms of the microscopic, we shall confine ourselves to the largest microscopic systems: i.e. atoms/molecules. They represent the total angular momentum of all smaller constituent particles (or lesser elementary fermion wave aspects) — as discussed in the following quote.

Formally [i.e. the equations involved], however, there is no difference between the spin of a single particle and the total angular momentum of any system regarded as a whole, neglecting its internal structure. It is therefore evident that the transformation properties of spinors apply equally to the behaviour, with respect to rotations in space, of the wave functions ψ_{jm} of any particle or system of particles

²¹⁷The wavelength of an electron 'inside' an atom is equal to the *size* of an atom. The wavelength of the neutron gives the nucleus its size. The wavelength of a quark inside a proton is the same order of magnitude as the size of a proton (Rohlf 1994, p.138).

²¹⁸The molecules may indeed be very very large.

with total angular momentum j , independent of whether orbital or spin angular momentum is concerned (Landau & Lifshitz 1965, p.200).

5.1.4 A mass emphasis as compared to an electric and magnetic emphasis

It is the mass, inertial, and mechanical aspects of an atom/molecule's angular momentum that shall be our primary concern herein, even though atomic/molecular angular momentum is dominated by electromagnetic fields and the electrical charge of its constituent particles.

Importantly (re: the model being pursued), it shall be the (indirect) effect of an *external field*, gravitational cf. electric or magnetic, that shall concern us; in particular, its (non-negligible) affect upon spin-orbital angular momentum coupling within an atom/molecule — in certain 'third' (celestial) body circumstances.

Note that the (internal) position of the centre of mass of atoms/molecules is effectively fixed cf. the distribution of electrical charge; and (recalling subsection 4.1.6) if the correspondence principle holds, then this centre of mass moves (according to Ehrenfest's theorem) along a classical trajectory. Further, as a property of atoms/molecules, the mass interaction of neighbouring atoms/molecules is negligible cf. the electrical interactions of (outer) electrons in atoms/molecules with their neighbours.

Regarding QMs and the behaviour of atoms/molecules in macroscopic bodies (e.g. metals, and chemistry), it is just the outer electron effects that are predominantly significant; whereas, the inertial effects related to curved spacetime (proposed herein) involve *all* the mass of each atom/molecule — in an 'appropriate' macroscopic (celestial 'third') body.

Unlike the case with electromagnetic aspects of atoms/molecules, each atom's/molecule's mass, and (as we shall see) virtual spin, is treated as *distinct* from its neighbouring atoms/molecules. Thus, if the (energy) influence upon a collection of neighbouring atom/molecules is the same — e.g. via motion in a gravitational field — the total energy may then be described by way of a simple *addition* of the individual atomic/molecular contributions. Loosely speaking, this is an "energy by number" situation; i.e. energy is proportional to the number of atoms/molecules involved.

5.1.5 A point of order re: QMs, classical physics and macroscopic systems

Elaborating upon comments made in a footnote of subsection 3.2.15, we note — by way of paraphrasing Vedral (2011, p.40) — that the criteria for treating a macroscopic system as purely classical depends upon the way quantum systems interact with one another, rather than merely depending upon the *size* of the system itself. Furthermore:

Larger things tend to be more susceptible to decoherence than smaller ones, which justifies why physicists can usually get

away with regarding quantum mechanics as a theory of the microworld. But in many cases, the information leakage can be slowed or stopped, ... [In summary, the] division between the quantum and classical worlds appears not to be fundamental ... [and as such] quantum mechanics applies on all scales (Vedral 2011, p.41,43).

Two comments ensue: firstly, information leakage in the model is 'thwarted' by way of the influence (we have proposed) being so small that it cannot register its presence within the digital/quantum system itself; effectively, it is a (proper) fraction of any minimal quantum change. Secondly, there *is* a clear division between solely quantum-based and solely classical-based *descriptions*/models of the world; the challenge herein is incorporating features of both descriptions (and worlds/domains) into a single model.

Finally, experiments now leave very little room for the following proposals to operate: (1) [Roger Penrose in the 1980s suggested] "that gravity might cause QMs to give way to classical physics for objects more massive than 20 micrograms, ... [or (2)] ... that large numbers of particles spontaneously behave classically (Vedral 2011, p.43)."

5.1.6 Quantum probability density and acceleration/gravitational field undulations — single particle case

Celestial bodies in stable orbits move along geodesics, and are *not* acted upon by gravitational *forces* (*per se*). It is worth mentioning that (via the time independent Schrödinger equation) for a (solitary) *free* 'particle' travelling at a constant speed²¹⁹, as atoms/molecules 'slaved' to a moon roughly/effectively do; (then) regarding the wavefunction, the probability density has a *constant amplitude* (for all x and t)²²⁰. In our case the "free particle(s)" we consider are the constituent atoms/molecules of a bulk mass in celestial motion.

This subsection's (hypothetical and idealised) examination of a single free particle essentially disregards all but one atom/molecule in a celestial body; and thus it is inappropriate when/where *real* quantum mechanical interactions upon or between atoms/molecules occur. For a *virtual* effect, common to all atoms/molecules in a celestial body, the following discussion could (conceivably) be valid.

Using a large number of wavefunctions to generate a group of travelling waves, the wave 'packet' *spreads out* from its (original) localised position. Similarly, and in addition to the rotating space-warp's acceleration amplitude remaining constant *out* to 'infinity', the space-warp's (non-local) mass at a point in space reduces with the spherical volume enclosed around a

²¹⁹Note that we are dealing with the *zero potential*; and 'particle' also refers to atoms and molecules.

²²⁰The unobserved (single) particle is equally likely to be found anywhere on the geodesic (i.e. a straight line in curved spacetime) that forms a closed orbit. If this were the case, the momentum (and energy) are known with complete precision.

moon — going to zero at (mathematical) ‘infinity’ (see section 6.6). Thus, the nature of the hypothesised rotating space-warp’s associated (non-local) mass distribution is not totally dissimilar to the quantum mechanical (probability) wave function of a *single* free particle. We also note that interference effects are possible, as are self-interference effects — with the latter being especially relevant to our model.

5.1.7 De Broglie matter waves, decoherence, and reduction

By advocating a standard two realm non-reductive approach, we also necessarily question the nature of the assumed existence of de Broglie matter waves for *whole macroscopic* objects, e.g. a pitched baseball. We cite a Paul Davies thought experiment, concerning a perceived quantum violation of the equivalence principle, involving two perfectly elastic balls; one made of rubber and one made of steel, dropped from the same height.

...quantum mechanically, there will be the phenomenon of tunneling, in which the two balls can penetrate into the classically forbidden region *above* their turning points. The extra time spent by the balls in the classically forbidden region due to tunneling will depend on their mass (and thus on their composition) (Chiao 2004, p.268).

A reductive assumption underlies the discussion of this concern. Essentially by way of decoherence, we might alternatively conclude that a discussion of quantum tunneling effects regarding macroscopic objects (at room temperature) is invalid. This alternative stance, and Paul Davies’ perceived concern, supports the contention that the microscopic and the macroscopic are quite distinct realms/‘worlds’, with distinct behaviour²²¹. This is *not* to say they cannot both be involved in a phenomenon; rather, they may (herein) coexist together but not in a “unified” and reductive manner — at least as regards the ‘mass era’ of the universe (cf. the hot ‘radiation era’).

Decoherence, as an approach to reality is generally loyal to a perceived reductive agenda; in that “behind the scenes” a macroscopic body is considered to still possess a matter wave field associated with the *whole* object cf. a macroscopic body being essentially representative of the sum of its (QM) parts. This unverified (whole body) conjecture is denied by our non-reductive micro-macro phenomenal stance.

The alternative is that, there is a (dual) coexistence, with the domain of independent matter waves only extending to the largest QM systems, with these being atoms/molecules. Thus, in a macroscopic body, and by way of decoherence we *may* still retain latent QM features; but there is now a huge multitude of ‘individual’ matter waves superpositioned “behind the scenes”. Thus, the whole object’s matter waves exist

simply by way of the sum/superposition of its parts — which may in turn involve further (and unknown) processes at the level of the whole object (i.e. emergent processes).

Finally, regarding decoherence, we shall not need to employ concepts such as the density matrix, or mixed states; nor shall Schrödinger’s equation concern us — the latter because we are not concerned with *finding* the allowed energy levels of QM systems (especially atoms/molecules), nor determining the positional probabilities of particles. Our emphasis lies solely with the intrinsic angular momentum (spin vs. orbital) aspects of microscopic (i.e. QM) matter, subjected to a common *external* effect; that then (under very special circumstances) expresses itself (also) externally to the QM system (or systems) concerned.

5.1.8 A system comprising four levels of matter and the system as a whole

Due to the different physical: phenomena, laws, and principles involved, a ‘complete’ system relevant to our model involves five different levels of matter and size.

Note that the word “macroscopic” is reserved for use in the sense of “not microscopic” — thus applying at levels three, four and five. These matter/size levels are:

1. Elementary fermion particles — the smallest microscopic level;
2. atoms/molecules — conceived as a composite of (all) their constituent elementary fermion (spin-1/2) particles — the largest microscopic level;
3. the (composite) sum of all atoms/molecules, i.e. bulk (lunar) mass — the ‘mesoscopic’ level²²²;
4. three-body celestial motion involving a moon in spin-orbit tidal lock orbiting a planet, with this moon-planet pair orbiting a central star/sun — the celestial (and bulk mass in motion) level.
5. Finally, there is the full system to consider which incorporates *all* four (lower) levels as well as the background space-time²²³ ‘stage’ — (effectively) the universal (and cosmological) level.

5.1.9 The transition between the microscopic and macroscopic realms

The transition between the microscopic and macroscopic realms remains a contentious (and open) issue (recall subsection 4.1.8). By altering the conventional/standard relationship of the microscopic to the

²²²Although not the conventional use of the word, we are avoiding the use of the word “macroscopic” (here). Additionally, mesoscopic is used to indicate that certain statistical properties have meaning at this level.

²²³Generally, we shall use the word “spacetime” when referring to ‘everyday’ or general relativistic usage, but we shall use “space-time” when the model’s non-local basis and (supplementary) noumenal/hidden background ontology needs to be recognized/appreciated. Note that some quotations (presented in this paper) use “space-time” rather than “spacetime”.

²²¹N. Bohr, W. Heisenberg, and J. von Neumann all argued for a strong distinction between the quantum realm and the classical world.

macroscopic world we attain a different understanding of the demarkation and transition between the two realms.

Bohr’s correspondence principle (CP) is seen to apply to atoms/molecules, but its application is restricted to QMs that obeys Schrödinger’s equation — i.e. quantum (orbital) mechanics, with the QM spin exempt/excluded. Regarding ‘orbital’ angular momentum, as in the case of the Rydberg atom where (the orbital angular momentum’s quantum number) $l \rightarrow \infty$; the behaviour of the quantum mechanical system ‘reduces’ to classical physics in the limit of large quantum numbers. In contrast, the spin of a quantum system never ‘goes to’ infinity. Subsequently, for QM spin the correspondence principle does *not* apply. This is one reason we are told to *never* conceptualise QM spin in a classical manner. While this is true for QM systems on their own, it may not always be heuristically useful — especially for a QM system existing in curved spacetime; (because) we shall argue that (in a rare type of third-body system) *externally* imposed effects of spacetime curvature can *attempt* to offset QM spin phase relative to the phase demanded by EM spin-orbit coupling. This occurs in a non-discrete (i.e. continuous/analog) manner — albeit only in the form of a tiny (sub-quanta) virtual effect.

A conceptual visualisation in the form of a relative over-spin, is beneficial for our explanation — albeit not physically realistic. Also note that a “virtual” effect is an effect that is not actually occurring nor is it directly observable, but may still be physically relevant — (in our case) at the (external) universal physical level. Our virtual effect is below the first energy level — and/or a minimum change in the (practical) energy levels — of a quantum atomic/molecular system, and thus it is associated with (proper) *fractions* of Planck’s constant (\hbar), where $h = 2\pi\hbar$.

The use of $\hbar \rightarrow 0$ to indicate the macroscopic limit, so that $\hbar l$ is finite as $l \rightarrow \infty$, is *not* embraced unequivocally. Its use assumes that reduction is always justified. It appeals to the relative smallness of \hbar . Indeed if a (fermion *spin*) offset effect is common to a great number of atoms/molecules, the use of the $\hbar \rightarrow 0$ macroscopic limit is inappropriate. Thus, this use of $\hbar \rightarrow 0$ (i.e. very very small) is only appropriate for a single atom/molecule. A hypothetical intrinsic angular momentum discrepancy/offset does not always have a classical/macroscopic limit analogous to (‘interior’) orbital angular momentum. Thus, any blanket denial of spin affecting a macroscopic system, in the form of a (virtual) angular momentum offset at the universal/systemic level, appears to be unjustified — albeit only in the special (vast number) additive and externally ‘driven’ circumstances of the model.

5.1.10 Summary remarks for section 5.1

This section has outlined how the model is able to restrict itself to the study of (only) atoms and molecules: as a composite of all their sub-system fermion elementary particle/wave constituents, and independent of interactions with neighbouring atoms/molecules. The model’s emphasis (with regard to quantum mechanics)

lies with atomic/molecular inertial spin effects — via atomic/molecular geodesic motion in curved spacetime — rather than the usually dominant electromagnetic based effects. Thus, standard QM approaches cannot be applied, e.g. involving Schrödinger’s equation.

By way of (further) questioning both the physical/ontological and theoretical aspects of a scientific reduction agenda, and emphasising the differences in the micro- and macroscopic realms/domains, it was argued that these realms coexist primarily in an independent manner — at least as far as our formal/representative understanding of them is concerned. This standpoint influences issues such as: the transition between the two realms; the superposition of atomic/molecular effects; and highlights a subtle difference between the classical limits of QM spin (i.e. intrinsic angular momentum) vs./cf. orbital angular momentum — in that QM spin has no classical limit (nor a classical analogue). Thus, (in exceptional circumstances) QM spin may be (additively) relevant to a macroscopic body compromising a great many atoms and molecules; the model herein shall exploit this case.

5.2 Berry’s geometric phase in celestial systems

5.2.1 Curved space, time and geometric phase

In Section 4 we examined what may be termed an “observational ontological oversight”, arrived at by way of our questioning that “observations are reality” (without remainder). Subsections 4.5.14 and 4.5.15 utilised a ‘dual-ontology’ to reach an alternative/supplementary interpretation of curved ‘space-time’. By placing all the adjustment for the presence of: mass, momentum, and energy, relative to an (idealised) empty universe Euclidean space and time, into an imaginary (number based) space dimension(s) we may effectively speak of curved space, whilst employing time simultaneity — so as to (more generally) appease QM non-locality. Note that time rates *are* variable (in the presence of mass, momentum, and energy).

This ‘meta’-physical approach, involving “hidden physicality” (and hence hidden variables), facilitates this section’s (i.e. 5.2’s) conceptualisation and quantification of a relative and virtual (spin) geometric phase (offset) — for the motion of atoms/molecules *along a geodesic* in curved spacetime, cf. motion in ‘flat’ space circumstances. Indeed the use of the standard mathematical formalisms of both GR and QMs implies that *no* geometric phase offset, real or virtual, should exist for motion upon a geodesic — whatever the circumstances. The phase offset described herein necessarily applies only within the third-body of a three-body celestial system — relative to spin phases in the rest of the system. Later we shall see that QM self-interference is involved (see subsection 5.4.3).

Note that *observationally*, QMs, SR, and GR all hold as before; but regarding the (virtual) geometric phase (GP) offset, the additive/supplementary use of a curved space and ‘staccato’ noumenal/background time simultaneity idealisation is our only option — if

we are to provide an explanation for a ‘real’ Pioneer anomaly incorporating non-local considerations.

5.2.2 Background to Berry’s geometric phase and anholonomy

A quantum system’s wavefunction may not return to its original phase after its parameters cycle ‘slowly’ around a circuit — a circumstance referred to as ‘anholonomy’, or preferably ‘holonomy’ by some authors. If a stationary state for a QM system is maintained, as it travels around a circuit, it may acquire a phase shift that is of a geometric or topological origin — which is quite different to the usual dynamical phase changes associated with Schrödinger’s equation²²⁴. The QM system is said to:

“...record its (path) history in a deeply geometrical way, whose natural formulation ... involves phase functions hidden in parameter-space regions which the system has not visited (Berry 1984).”

Note that geometric phase is always *relative*. This is elaborated upon in subsections 5.4.5 and 5.4.6.

There are two types of anholonomies: topological phases (e.g. the Möbius strip) and geometric phases — “geometric” because phase shift depends only on the geometry of the circuit in the parameter space [which is sometimes real physical space] (Berry 1988, p.28).

In Cai, Papini, & Wood (1990) the classical nature of Berry’s phase for photons, specifically the rotation of a linearly polarised beam travelling along a single helically wound optical fibre (or, in AmE, fiber), is shown to arise from the intrinsic topological structure of Maxwell’s theory — if the Minkowski (flat) spacetime is considered as a background. The phase is (theoretically) developed in the context of fibre-bundle theory.

By way of contrast, we are examining the geometric phase of QM *intrinsic* angular momentum (i.e. spin) in the context of: motion in (observational or phenomenal) curved spacetime — or equivalently and alternatively, motion in (noumenal time and) curved space. Note that orbital angular momentum phase is not affected. Of particular interest shall be:

- the path of atoms/molecules when ‘attached’ to (i.e. are a part of) celestial bodies,
- the inertial ‘commitment’ of the QM system,
- and the projection of QM spin by way of a particular direction in space; with this direction being externally established by way of the *lunar spin axis* or the plane of lunar orbital motion.

For large moons this orbital plane usually has very low inclination relative to the planet’s equatorial plane²²⁵, and the lunar spin axis is very nearly perpendicular

²²⁴Note that dynamical phase shift is proportional to time, whereas geometric phase is path-based and never (directly) dependent upon time.

²²⁵A planet’s equatorial plane and a large moon’s orbital plane are usually very nearly parallel — with Neptune’s captured moon Triton being an exception to this (low relative inclination) rule.

to the moon’s orbital plane²²⁶, i.e. the obliquities are very small — and effectively zero in some cases.

5.2.3 Finer detail upon the use of Berry’s geometric phase in the model

We now turn to the use of (Berry’s) geometric phase in the model being hypothesised/formulated.

Firstly, we recognise the *global* and cyclic nature of (relative) geometric phase change (or offset); which implies the curvature (i.e. non-Euclidean geometry), and possibly topology, of background space and time is important — especially its relationship to the ‘spin’ of QM systems.

...only for a cyclic evolution the change in geometric phase β can be defined invariantly. This makes it appear that β is global in the sense that it can be unambiguously defined only for cyclic evolution (Anandan & Aharonov 1988, p.1868).

Three comments ensue:

- a) The global (rather than local) origin of β implies a global phase ‘datum’, relative to which β can be referenced²²⁷.
- b) A cyclic process conceivably establishes a (virtual) spin *energy* — in the sense of a rate of angular momentum. With regard to the new mechanism (under development), the external expression of energy associated with an internal energy (of equal/common magnitude) indicates the effect is non-cumulative; i.e. the phase offset and virtual energy are not carried over to the next cycle.
- c) In a two-body gravitational system the “reduced mass” greatly simplifies the orbital description, so that the two-body problem can be solved as if it were a one-body problem. In any solely two-body situation the (new) geometric phase change is seen to be absent.

Only with a three-body (gravitationally bound) celestial system²²⁸, with its multiple types/sources of space-time curvature, can (a virtual) relative phase offset occur; with this offset being for a moon’s constituent atoms/molecules — and (equally for) all their constituent elementary fermion/matter particles/waves — relative to the rest of the system. Note that the model *also* requires celestial spin-orbit resonance (i.e. phase-lock or synchronous motion) for a moon in relation to its host planet. The importance of this is discussed in subsection 5.4.5. The host planet (and Sun) remain unaffected, thus effectively forming a ‘datum’ of no atomic/molecular geometric (spin) phase change. Later we see (subsection 6.3.5) that the

²²⁶One way in which discrepancies can exist is via a (spin) rotation “axial precession” effect e.g. Earth’s moon.

²²⁷Certainly, for EM effects in a laboratory curved spacetime is usually neglected.

²²⁸Asteroids occasionally have satellites, e.g. Ida and its moon Dactyl; but a naturally occurring stable four-body (gravitationally) bound system remains unobserved. An artificial satellite in the form of a spacecraft, orbiting our moon, effectively creates a four-body bound system.

(particular case of the) Sun-Earth-Moon system fails to be geometrically suitable, and thus does not produce/‘coexist’ with a rotating space-warp.

Secondly, the external ‘impetus’ to an internal (virtual) change in geometric phase (i.e. β) requires a relationship between celestial kinematics and QM geometric phase be pursued.

The essential difference between our [Anandan & Aharonov] approach and Berry’s approach is that we regard β as a geometric phase associated with the motion [i.e. kinematics] of the state of the quantum system and not with the motion of the Hamiltonian as Berry did (Anandan & Aharonov 1988, p.1864).

In our case the motion of atoms/molecules within (and ‘slaved’ to) a celestial moon is examined. With the model requiring celestial spin-orbit resonance, the lunar-constituent atoms/molecules undergo both (classical) spin rotational motion *and* ‘linear’/geodesic motion (in curved spacetime) — at the same ‘time’.

It shall be seen that (pure/Euclidean) geometry also plays an important role in determining geometric phase in the model²²⁹; in conjunction with the ‘geometry’ associated with spacetime curvature. Finally, we note that the celestial motion of atoms/molecules (within a large/bulk mass) is both: adiabatic, and “force-free” by way of being motion ‘upon’ a geodesic. Thus, within the model, dynamical phase concerns may be (and are) neglected.

Thirdly, in certain EM two-body circumstances:

...geometric phase depends only on C
[the closed curve] and is independent of
how the evolution around C takes place
(Anandan & Aharonov 1988, p.1866).

In our three-body moon-planet-sun system, two distinct “closed curves” can be recognised: i.e. a planet around its sun, and a moon around its host planet²³⁰. In both cases the orbital paths *are* dictated by gravity (and kinematics). In the model we consider one full (sidereal) orbital cycle of a moon as the primary closed curve. Later (section 5.6.7) we propose that β (the closed curve geometric phase change) is dependent upon *both* the moon’s closed path and the planet’s angular rotation/progression around the Sun in this time — which is substantially less than a (2π rad) planetary year.

Note that the value of β is only *indirectly* based upon the mass and speed of the bodies concerned. Thus, once the (celestial) orbital features are in place, it is only the paths that are important; and the basis for our empirical *determination* of (quantum mechanical) geometric phase change/offset can be considered solely “geometric” — i.e. closed curves, arcs, lengths and angles. Loosely speaking, celestial kinematics and dynamics establish a ‘path’ geometry,

²²⁹We shall use the expression “geometric phase”, rather than “Berry’s phase”, to distinguish it from the latter which is often associated with the motion of the Hamiltonian (in parameter space).

²³⁰Since Jupiter and Saturn’s moons dominate the model, the larger body’s core is essentially the centre of rotation.

which then establishes the (spin) phase offset of lunar atoms/molecules at the end of a closed (orbital) circuit relative to the spin phase at the beginning of the (moon’s) closed loop/circuit. One again we note that *orbital angular momentum phase is unaffected*. That a geometric/topological *quantum mechanical* fermion wave phase effect should have a purely geometric *celestial* basis is initially surprising but not unreasonable — especially since this (classical/Euclidean) path-based geometry is ‘coexistent’ with celestial (‘phenomenal’) curved spacetime²³¹.

Outstanding, at this stage is that: it appears we may need to accredit the background curved space continuum with some additional (topological) fine detail (see subsection 5.3.3).

5.2.4 A different role for geometric phase in a ‘mechanistic’ system

Within atoms/molecules, i.e. (microscopic) QM systems, the electromagnetic force dominates any gravitational ‘forces’ by a very large amount; the ratio of the electrical to the gravitational forces between a proton and an electron is about 10^{40} . Subsequently, any physical effects, that are solely based upon mass cf. mass *and* electric charge, are always considered negligible. We are considering an exception to this ‘rule’. Regarding geometric phase, the presence of curved spacetime is (currently) seen to merely alter relationships that are dominated by electromagnetic (EM) concerns.

Herein, we examine a curved spacetime effect upon a *non-electromagnetic* aspect of an atom or molecule — hence this is referred to as a “mechanistic” approach. We shall take geometric phase into a domain where its effects are considered to be non-existent, or at best negligible; with this being: the non-inertial status of the intrinsic angular momentum of an autonomous (moon-based) bound QM system translating/propagating along a geodesic in curved spacetime.

5.2.5 Geometric phase & celestial motion

The effects of spacetime curvature upon the geometric phase of different, although neighbouring, quantum mechanical (QM) systems are generally considered to be *all* the same; but on larger scales, i.e. involving different celestial bodies, this local idealisation is completely inappropriate. For example, a moon orbits its host planet at a distance, and hence its path in curved spacetime is profoundly different to the (curved spacetime) path of the planet.

At laboratory scales curvature effects differing between QM systems are imperceptible, and up to moon or planet size are very nearly identical — because the paths of neighbouring atoms/molecules are almost *parallel* in spacetime. In other words, throughout a given celestial body (particularly moons), the geometric (spin) phase offset (at the ‘level’ of individual atoms/molecules) is effectively the same — essentially

²³¹Or “coexistent with curved *space*”, from our alternative noumenal/background perspective, i.e. the second of our ‘dual’ perspectives.

because any given moon’s radius is much smaller than the moon-planet semi-major axis.

5.2.6 The model encapsulated

The following brief overview of the new mechanism encapsulates much of what shall be discussed in the rest of Section 5.

For an atom/molecule (within an appropriately configured third celestial body) moving in curved spacetime, a (relative) virtual geometric phase offset (per loop duration/time) effectively ‘equates’ to a (virtual) over-spin of intrinsic angular momentum per unit loop duration/time; with this being a spin *energy*. Further, this internal energy offset — by way of being below the first/lowest energy level²³² — is (effectively) shared by every atom/molecule in an appropriate moon; and thus, upon summation (throughout a moon) the virtual energy offset ‘produced’ is significant — by way of an extremely small number ($< \frac{1}{2}\hbar\Delta t^{-1}$) being multiplied by an extremely large number of atoms/molecules.

The multifarious orientations of the elementary fermion particles within atoms/molecules all ‘receive’ the same *externally orientated* phase change effect. Thus, the *virtual* phase effect is effectively coherent. In other words, the magnitude of the phase offset and the (common/shared) spin axes’ orientations are extrinsically based²³³ — with the latter only ‘being’ possible in the case of a virtual effect/offset. Note that only minor differences/variations in phase change occur over the extended body, because the body’s radius is very small compared to the scale of the spacetime motion and curvature/geometry concerned. We shall assume that all (third-body) geometric phase changes are equal. Only at and above the decoherence threshold, i.e. greater than a (fermion) half-wavelength (2π) geometric phase offset, could the actual (i.e. real) multifarious spin orientations be exposed. Below this threshold, the virtual effect’s common application (i.e. same direction and magnitude for all atoms/molecules) facilitates the existence of a new type of (externalised/environmental) ‘condensate’ behaviour — (that is) not necessarily physically associated with a constructive superposition of (virtual) geometric phase offsets in the QM wavefunctions (see subsection 5.2.8).

The appeasement of this virtual energy imbalance, by way of a *systemic* (i.e. global) conservation of energy, necessitates a (real) physical expression of energy *external* to the many²³⁴ QM systems. This external energy expression imposes itself as a field upon the pre-existing gravitational field. This unrecognised ability of nature acts to always maintain the (internal) stability of atoms/molecules in the presence of curved spacetime. Importantly, QMs and GR’s curved spacetime are *both* involved in the generation of this new ‘gravitational’ field effect (i.e. rotating space-warps with an associated non-local mass distribution), and yet QMs

and GR maintain their *distinct* ‘character’ — rebuffing most physicists’ expectation that the next step in their relationship is to be one of unification and reduction.

5.2.7 Commenting upon two assumptions associated with Berry’s phase

Berry (1988, p.31) discusses two assumptions implicit in his derivation of the geometric phase.

1. The environmental parameters *governing a system* can be determined (at least in principle) to arbitrarily high precision, and that
2. the environment remains unaffected by any phase changes it induces in the system.

Regarding the first assumption, the environmental parameters herein involve the geometric and kinematic characteristics of lunar and planetary motion; and these *are* known to high precision.

Regarding the second assumption, in contrast to EM instantiations of Berry’s phase, the environment — in a different sense of the word — *is* necessarily affected in the model’s case; by an amount equal to the *virtual* energy offset it induces ‘within’ a bound QM system (or systems). It is the QM atomic/molecular system that is unaffected — as far as a physically *real* change is concerned. Interestingly, if a physically real change is induced in the QM system, then decoherence ensures the environment remains unaffected — this is further discussed in section 5.3.

5.2.8 Towards coherence and condensate behaviour, in the graviton’s absence

Internal (virtual) QM *coherent* behaviour and a (condensate) external acceleration/gravitational field resultant effect is being pursued herein; (but) with the photon particle mediating the EM *force*, and the graviton particle denied, things are necessarily quite different.

The term “condensate” is used here in the sense of ‘singular’ behaviour, but also in the sense of an effect related and common to (i.e. shared by) a great many atoms/molecules (N_m). Thus, an *additive* approach to the quantification of virtual spin energy is possible. Note that our phenomenon of interest lies *below* the lowest (i.e. first) energy level of a bound QM system, whereas a Bose-Einstein condensate occurs *at* the first energy level; and as such the singular behaviour discussed is *not* that of a real physical (state of matter) condensate. Nevertheless, in the model, quantum effects become apparent on a macroscopic scale.

In QMs, geometric phase (and gauge theory) are usually discussed in terms of such things as fibre bundles. Importantly, relative electromagnetic based geometric phase effects are seen to extend “all the way up” to Maxwell’s equations. Thus, geometric phase effects appear to be related to the *long-range* effects of classical electromagnetic fields, and hence they are also conceivably steeped in curved spacetime’s large-scale (geometric) fields/influences. Subsequently, a back-reaction involving a long-range field response, external to the QM systems concerned, is not unreasonable.

²³²Or below a minimum change in discrete energy levels.

²³³Extrinsic (definition): not part of the essential nature of something, and coming or operating from outside.

²³⁴Of the order of $\sim 10^{50}$ atoms/molecules in a large solar system moon.

This new long-range geometric phase-based effect requires us to not disregard subtle properties of background space and time *in themselves*. Coherence and condensate-like behaviour are further discussed in section 5.7.

5.2.9 Summary remarks for section 5.2

Building upon the supplementary ontology discussed in Section 4, this subsection introduced the manner in which (Berry’s) geometric phase is used in the model. After a brief introduction, the very restricted conditions of its instantiation were outlined, representing a departure from standard applications of Berry’s phase — which typically involve neutrons and electromagnetic phenomena.

These restrictions involve: (1) application to the inertial mass aspects of atoms and molecules within the (lunar) third-body of a three-body celestial system, *moving* ‘along’ a geodesic in curved spacetime; (2) a relative phase shift that is restricted to intrinsic angular momentum (i.e. spin) phases, with orbital angular momentum-based geometric phase remaining unaffected; and (3) a dependence upon closed orbital loop motion (and self-interference).

For relative geometric phase shifts less than half a fermion wavelength (i.e. $< 2\pi$ rad), the phase ‘shift’ represents a virtual effect within an atom/molecule. Lunar motion, relative to its host planet, determines the orientation of this (virtual) phase shift for all (lunar) atoms/molecules; and since a moon’s diameter is small relative to its semi-major axis, the magnitude of the phase shift is (also) effectively the same throughout an appropriate moon. This permits an additive approach to spin ‘energy’ — i.e. change of intrinsic angular momentum per closed loop duration/time (Δt).

The general acceptance of a linkage between (standard) geometric phase effects and long-range classical EM effects is seen to support our hypothesis²³⁵ that: curved spacetime based geometric phase effects can externally ‘express’ (a *virtual* QM energy) into a (new type of) long-range acceleration/gravitational field — albeit only for circumstances where a minimum (internal) QM energy ‘level’ is *not* attained and (such that) geometric phase-based decoherence is not ‘triggered’.

5.3 Spin, spinors, decoherence and Hannay angle

5.3.1 The model’s use of QM spin and spinors

Note that herein the QM mathematical device of a “spinor” plays no quantitative role in the model, but spinors are necessarily a part of the discussion. With the physical energy involved in QM systems of primary importance, the concept of “spin” (i.e. intrinsic angular momentum) is crucial to the discussion — particularly in terms of the physicality of a phase change. Note that a 4π spin phase change of a fermion particle results in a return to its initial state — all other things being equal (*ceteris paribus*).

²³⁵By way of similarity, and hence analogy.

5.3.2 Fermions, spin, spinors, Hannay angle, and decoherence

By utilising both microscopic *and* macroscopic phenomena (i.e. quantum mechanical and non-QM gravitational/accelerational field phenomena) in the model, a *solely* QM approach is clearly not possible. In QMs, physical quantities are quadratic in their wave functions. A 2π change in the wave phase (of fermion particles) was historically (pre-1967) seen to be merely associated with a sign change of the wave function — with no associated experimental consequences; whereas nowadays experimental consequences are well-appreciated. Generally, only interference experiments are considered an appropriate vehicle for the investigation of geometric phase effects. Our investigation goes beyond this restriction.

If we restrict discussion of the phase change to 2π ‘jumps’, this suits the reductive/dicrete particle approach of EM, but not the continuous (i.e. analog) geometric phase changes proposed herein — that lie below a minimum energy level²³⁶, and are seen to arise from an external analog curved spacetime effect. Subtleties of spinor sign change, related to the case of a 2π rotation of one system *relative* to another system are briefly discussed in subsection 5.3.3.

In (non-relativistic) QMs, spin states may be simply represented by a two-component spinor (spin up and down); where “spinors” are wave functions of the intrinsic angular momentum of (usually) elementary particles. Note that spinors are also associated with a dilation effect, in addition to the rotational aspect. Additionally, we note that a phase, for the purposes of QMs, is not a state of matter but a complex number of unit modulus, an element of group $U(1)$. Herein, we cannot restrict ourselves to a solely QM basis — with its mathematical group theory emphasis²³⁷ — but we *can* restrict the quantification (and conceptualisation) of the model to a solely *classical* basis — e.g. energy, momentum, mass, and (something akin to) classical spin. How this is achieved is outlined in the following paragraph.

The *external* basis/origin of the QM (geometric) phase change/offset proposed herein acts similarly upon both the up and down spin components — of all elementary fermion particles, and (in equal measure) an atom/molecule as a whole. This simplicity, regarding how the external effect acts, allows the spin (geometric) phase offset to be (loosely) described and conceptualised as a (classical) Hannay angle — with this being the mechanical/classical analogue of Berry’s geometric phase. Indeed the intrinsic spin of ‘quantum’ particles should not be considered classically, but the external origin of the internal effect lends itself to this simple and useful *visualisation*. An example of a Hannay angle is the angular rotation associated with Fou-

²³⁶That is, below a level where the QM atomic/molecular system’s state is altered, so as to exhibit (local) particle-based consequences — that can be observed by experiments. Below such a level, mathematical consequences, and wave-based interference consequences cannot be ruled out; nor can our proposed externalisation of ‘unresolved’ (fractional quantum) energy be ruled out.

²³⁷Not to mention: Clifford algebra, Lie geometry, etc.

cault's pendulum. We can imagine the model's virtual (phase offset) effect as behaving like a simple classical "over-spin" (per loop/cycle). Note that the *attractive* (only) nature of mass-to-mass gravitational interactions results in a phase asymmetry that is restricted to an over-spin circumstance; whereas an under-spin circumstance (its *repulsive* opposite) cannot exist in a three-body (bound) *gravitational* system. This over-spin is clarified in subsection 5.5.8.

With our approach using 'classical' energy as a bridging quantity between the two realms, our classical rotation conceptualisation/visualisation is a doubly useful explanatory device. Thus, from an external perspective we are effectively *treating* the atoms/molecules "mechanistically" — not that this is how they are described mathematically within a (microscopic) QM *system*, e.g. an atom or molecule.

Fermion wave functions have a 4π [radian] periodicity with respect to either: a physical *rotation* (about an axis, and within a coordinate system), or a rotation of the coordinate system about the fermionic 'system'; and unlike bosons it takes two full rotations to return them to their original 'orientation'. Fermions that have undergone a 2π [radian] rotation around an axis experience a sign change in their wave function. Note that this is a *forced* rotation whereas celestial orbital motion is an *unforced* rotation. Thus, the relationship between forced rotation and phase does *not* simply 'transpose' into the model (see subsection 5.6.7 for the actual three-body relationship). In fact, ('unforced') planetary and lunar orbital motion is generally considered to be free of any interior QM phase changes; but for (third-body) lunar motion an unforeseen spin phase change/offset is seen to be indirectly implied by the (inconclusively explained) Pioneer observations.

This paper proposes that (relative) geometric phase changes can arise within three-body 'unforced' rotational systems. Planetary and lunar orbits involve (straight line) geodesic motion in curved spacetime; thus, this situation is quite unlike a rotation in 'flat' space. Nevertheless, it is proposed that the intrinsic angular momentum of atoms/molecules constituting (i.e. comprising) a moon are imposed upon by something analogous to a classical rotation in flat space — in that a geometric phase change/offset can arise. Note that the orientation of the spin axis associated with the *virtual* geometric phase offset is actually extrinsically determined. We further note that the quantitative behaviour of the phase shift is not necessarily analogous to that of a simple fermion rotation in flat spacetime. Decoherence is seen to occur when $\beta \geq 2\pi$, i.e. when a sign change in *any* fermion wave function occurs²³⁸. Note that this decoherence is *not* temperature related; it is (geometric) phase based. In short, an externally 'driven' phase effect is only "found out" by the QM system when it 'alters' the QM system — even if no 'observables' have changed.

²³⁸Note that with atoms/molecules in a moon having varied spin orientations, the first atoms/molecules to 'trigger' the macroscopic decoherence will be those with their spin axis most susceptible to the external effect — with this (conceivably) being (either) a spin axis perpendicular to a moon's orbital plane or parallel to the moon's spin axis.

Previously, we have indicated the model is based upon a situation *below* a maximum energy uncertainty or below a minimum (discrete) change in energy levels (which quantitatively are a function of the reduced Planck constant). We now make this physically specific by using the relative 2π phase advance as the threshold/dividing line between decoherence and non-decoherence of the system. Only for $0 < \beta < 2\pi$ can a rotating space-warp coexist; (and) it is this scenario that we are specifically examining²³⁹. For any *single* intra-atomic/molecular $\beta \geq 2\pi$ occurrence the 'bulk' moon is considered to lose (entirely) its ability to 'behave' in an (additive) QM condensate-like manner.

5.3.3 Physicalising the spinor sign change arising from a 2π relative rotation

In flat spacetime, a 2π (6π , 10π , etc.) [radian] rotation of either a physical fermion particle/system about *any* axis (within a coordinate system), or a rotation of the *coordinate system* (or an observer) itself around a stationary (spin-1/2) fermion particle or fermionic system, leads to the sign change of a spinor wave function, but not a change in state — with a 4π (8π , 12π , etc.) rotation returning the spinor to its initial sign. Beyond the quite abstract mathematics associated with spinors and operators, the physical interpretation of spinor sign change, arising from a physical rotation, deserves examination.

Silverman (1980) points out that for the spinor sign change to be physically relevant we need to discuss a *relative* situation²⁴⁰.

Only a part of the [whole] system can be rotated; part must remain fixed to provide a reference against which the rotation is measured. [Additionally] ... experiments revealing the 2π phase change of a spinor wave function resemble closely well-known classical optical phenomena (Silverman 1980, p.116).

To develop the model we need to make a distinction (and linkage) between a classical (2π) rotation/revolution and a quantum mechanical (2π) phase change. Our model involves: an inertial mass-based emphasis, curved spacetime, a three-body celestial system and a (fermionic system) atomic/molecular geometric phase change; whereas, as a rule, QM fermion particle/system 'rotation' is dominated by external *electromagnetic* effects²⁴¹. A 2π 'rotation' in an electromagnetic force-dominated QM system is seen to be *physically* generated; thus, it is effectively a 'forced' rotation — whereas celestial (geodesic) motion in *curved* spacetime is actually an 'unforced' (orbital) 'rotation',

²³⁹Possibly, for a *single* atom or molecule things may be different, especially if EM radiation absorption or emission is involved. See the discussion on change in the fine structure constant in subsection 6.2.6.

²⁴⁰If the *whole* (i.e. entire) system is rotated there can be no experimental consequences.

²⁴¹With a (rotating) external magnetic field being the most common means to induce a change, often involving neutron particles in a beam, e.g. a Larmor precession.

if we embrace a purely geometric (i.e. curved space-time) approach to gravitation.

In both cases the change in a QM sub-system's spinor is considered: physical²⁴², externally induced, and relative. To simply examine a mathematical (2π) particle or coordinate rotation upon a (mathematical) spinor overlooks all this. Indeed, Maxwell's equations of electromagnetism and GR's gravitation cover the two classes of external macroscopic physical effects that can conceivably influence the geometric phase of a QM particle, atom or molecule — with an atom/molecule having both electrical (plus magnetic) aspects and (inertial) mass aspects (in its own right).

It should *not* be readily thought that a spinor sign change (arising from a 2π relative rotation) requires background spacetime to have an additional and mysterious structure that interacts with (i.e. topologically couples to) QM spin/spinors²⁴³. Background space is seen to merely support (or act as) a (physically) relative 'stationary' phase or 'rotation' reference datum, but it doesn't need to have a spin structure intrinsically within itself.

Thus, we have reason to reject (principally by way of Ockham's razor) the assertion of Gough (1992), who cites Edmonds (1973, 1976), that a direct linkage between a 2π rotation of *coordinates*, in flat spacetime, and a QM 2π phase advance is possible — by way of a new 'hidden variable'/quantum number called "tumble"²⁴⁴. Although it should be noted that: "... conventional quantum mechanics requires lengthy arguments to justify the sign change, involving the obscure [yet, highly regarded] notion of fibre bundles (Gough 1992, p.169)."

We may downplay, by way of an internal to external distinction, the concern that Silverman (1980, p.122) raises, by way of Byrne, regarding how to interpret the interference of spatially separated and rotated²⁴⁵ spin- $\frac{1}{2}$ particles in a beam; because recently proposed interference free measurements may indeed be possible.

...for fermions, simultaneous observations of the relative rotations of the spins in the two beams and of the phase shift are incompatible [since observation of the relative rotation of neutrons destroys the interference pattern]. [Thus, argues Byrne:] ... the notion of relative rotation ceases to have meaning as it corresponds to nothing that is measurable (Byrne 1978).

Further, Silverman (1980, p.122) notes that with regard to the interference pattern: we cannot say that

²⁴²Note that the model makes real, by way of an (external) rotating space-warp, the 'condensate' QM *virtual* energy asymmetry.

²⁴³Even though in Section 4 of this paper, in order to appease non-locality, a noumenal/background/hidden 'universal substratum' and a different interpretation of SR have been propounded.

²⁴⁴The idea of a new quantum number "tumble", with its apparent similarity to spin, arose out of expressing the Dirac equation in (complex) quaternion form (Edmonds 1973). Edmonds explores ways to lift its degeneracy.

²⁴⁵By way of some electromagnetic process.

we can observe *directly* the sign reversal of a spinor wave function subjected to a 2π rotation.

To simply, but not exhaustively, understand the experiment (that Silverman discusses) we may allow both: an externally generated physical relative-rotation to act upon QM spin/spinors, and an QM/internal phase change, to coexist (at least conceptually and prior to measurement); in order to explain the interference effects when the two beams are recombined. Clearly, the classical conceptualisation of QM spin as a rotation remains forbidden, and it is preferable to avoid implementing the 'hidden' background associated with the quantum number "tumble".

Although the notion of tumble has its origin in Dirac's equation (and hence electrical charge) (Edmonds 1973), the conceptualisation of a phase change (by way of relative motion) is quite similar to the model's proposed geometric phase offset; with both necessarily recognising (in different ways) the existence of a 'global' (or systemic) reference frame — e.g. a barycentric solar system reference frame. Hence, the additional notion of tumble retains some appeal, especially in light of the fact that the hypothesised non-local existence of the rotating space-warps proposed herein lacks a truly 'deep' physical explanation²⁴⁶.

In summary, it can generally be said that quite elaborate mathematics is needed to represent the 'weirdness' of QM physical 'behaviour', and yet the simple notion of an externally induced fermion 2π phase advance, leading to spinor sign change, is easily appreciated. Externally induced (or forced) rotation and internal phase change are well suited (if not 'natural') partners. The model's quite different conditions, i.e. exclusively examining *mass*-based aspects of atoms/molecules undergoing unforced (closed loop) orbital motion, retains and exploits the simplicity of this conceptual linkage; now (i.e. in the model) between lunar celestial (orbital) '*rotation*' — within a systemic/barycentric reference frame — *and* relative geometric *phase*. Finally, we recall that in the mechanism/model macroscopic lunar spin-orbit resonance, also known as 'phase-lock', is additionally required.

5.3.4 Closing and summary remarks for section 5.3

The two major issues addressed in this section were: the 'physicality' of a particular type of (fermion) phase change, and the understanding of what that phase change might also involve/implicate.

In the model, a classical quantity — in the form of energy — shall be used to link a QM energy within atoms and molecules to a new type of macroscopic (gravitational) field effect. This QM (spin) energy is linked to intrinsic angular momentum, which in turn is linked to a relative spin-based geometric phase offset (β). Since this phase offset arises by way of geodesic motion and curved spacetime conditions *external* to the atoms/molecules (in question), a major simplification in the treatment of β is possible.

²⁴⁶The author confesses to limitations regarding his understanding of the subtleties of quantum mechanical 'reality'.

Our model is greatly simplified if we *conceptualise* β as an externally ‘driven’ classical (Hannay) angular rotation — of both the spin up and spin down components. This is a completely inappropriate representation of what is physically happening at the QM level, and a rigorous mathematical representation would preferably employ spinors, but the virtual nature of β permits this simplification.

Instead of a rotation in an Earth-based laboratory leading to a relative (QM) fermion phase change, our concern involves a celestial (orbital) ‘rotation’ — acting in unison with a celestial spin rotation. By establishing a (non-flat spacetime) barycentric-based reference frame, it can be said that we are examining the lunar (and planetary) rotations (i.e. orbits) of an atom/molecule — i.e. the quantum mechanical subsystems of bulk celestial matter. Only the *lunar* (third body) motion is of significance, in that its atoms and molecules experience a fermion phase change/offset (β) relative to the rest of the system — i.e. Sun (first body) and the moon’s host planet (second body).

A sign change in any affected wavefunction is seen to trigger full systemic decoherence, and thus a system characterised by a spinor would seem necessary. Nevertheless, because our concern is merely the magnitude of phase change, relative to a decoherence onset value of $\beta = 2\pi$, and because β shall be determined (later) solely by external (celestial) geometric and kinematic conditions, a rigorous internal representation of the atomic/molecular circumstances is *not* required — *nor* could it (alone) represent the more elaborate internal-external (and micro-macro) situation being studied.

Our approach is quite distinct from (standard) interference based experiments (used) as a means of investigating a relative geometric phase effect/offset; especially because the third body in a celestial gravitational system is involved, and a *virtual* phase offset ($\beta < 2\pi$) (as well as self-interference) are central to the model. Finally, we note that the virtual nature of the spin phase offset permits it to be associated with a quantum mechanical spin axis orientation that is externally (i.e. extrinsically) determined — with this orientation being perpendicular to the plane of lunar orbital motion, or parallel to the moon’s spin axis.

5.4 A new type of precession — a QM relative spin phase shift

5.4.1 Relating the model to other geometric phase based phenomena

The non-Euclidean ‘geometry’ associated with spacetime (S/T) curvature is a vital qualitative ingredient behind the geometric phase offset. By way of a comparison, geometric phase effects in EM situations (e.g. the Aharonov-Bohm and the Aharonov-Casher effects) are associated with “...a ‘vector-potential’ coupling [that] gives rise to a topological phase (Aharonov & Reznik 2000).” We have effectively cut out the ‘go-between’ coupling, and allow the path of an atom/molecule in curved spacetime to establish the geometric phase offset — albeit in quite special circumstances.

It is useful at this point to distinguish locally applicable geometrical properties of *the universe*, e.g. curved spacetime, from (exclusively) global characteristics of the universe, e.g. topological characteristics (such as shape).

It should also be noted that a gravitational analogue to the EM Aharonov-Bohm effect is discussed in the literature. This analogue to the EM situation is *not* what is being pursued herein. The situation being proposed is quite unique and different (recall subsection 5.1.2). The EM Aharonov-Bohm and the Aharonov-Casher effects have gravitational analogues that are associated with a classical time delay or lag (Reznik 1995). The model’s new situation does *not* have an EM analogue; it is thoroughly independent of electromagnetism. In other words, it is unique to “geometric” curved spacetime.

5.4.2 On the omission of global topological aspects from the model

Although the model accredits the geometric phase offset (β) solely to geodesic motion in curved spacetime (recall subsection 5.2.2 on two types of anholonomies), and we shall (later) quantify β merely by way of celestial kinematics and (Euclidean plane) geometry; (but) a topological aspect associated with quantum spin (existing throughout the universe) cannot be ruled out. Indeed, this topological aspect may (and probably will) form part of a richer explanation/model than the one given herein — especially considering the rotating space-warps (to be further discussed) are cosmological in their size/extent, and universal conservation of energy and conservation of angular momentum principles/laws are required by the model.

5.4.3 Comparison to the Aharonov-Bohm effect

The EM based Aharonov-Bohm (A-B) effect is considered to be the *only* type of non-locality associated with fermion waves. Herein, a second non-locality associated with fermion waves is being proposed, involving the mass/inertial aspect of atoms/molecules. The comparison is given in point form.

1. Importantly, we are (also) assuming the self-interference of fermions, i.e. the constructive interference of each elementary fermion wave/particle (within an atomic/molecular fermion system) with itself after a full loop/cycle²⁴⁷. Clearly, non-locality is implicated/involved; additionally so, because no field lines are crossed — via the geodesic motion of lunar atoms/molecules.
2. Our interest lies with the (spin) inertia of fermions (which have mass). Although Planck’s (or rather the reduced Planck) constant is vital to the (maximum) magnitude of the new effect

²⁴⁷This is somewhat analogous to the physical interpretation of self-interference given by Chiao (2006), who refers to the closed path Feynman-Onsager quantisation rule/condition (Donnelly 1967).

($\frac{1}{2}\hbar\Delta t^{-1}$), this energy magnitude acts independently of any electromagnetic field effects.

3. Whereas the Aharonov-Bohm effect is observed for neutrons and electrons (i.e. particles), we examine atoms/molecules. In particular, ramifications of their ('internal') spin-orbit coupling.
4. As for the A-B effect, the parameter space remains the ordinary space of the atom's motion, cf. the motion of the Hamiltonian; but this space is now *curved* space cf. Minkowski space.
5. The phase shift of interest in the model is virtual, cf. (in the A-B effect) the real phase shift of particles acted upon by topological macroscopic EM effects. The virtual phase shift arises from the motion of a celestial spin-orbit 'coupled' third-body in curved space — cf. an EM vector coupling (topological) effect in Minkowski space — but (herein) it is not determined by way of the parallel transport of a vector.
6. EM-based phase changes associated with particle or radiation emission/absorption are quantum-like (and gauge-based), whereas this motion and curvature based phase change allows arbitrary analog (i.e. non-digital) phase changes — both virtual and 'real', i.e. (respectively) below and above (or rather 'up to') a decoherence threshold.

5.4.4 A new type of 'precession'

The (electromagnetic) Aharonov-Bohm effect is considered to be analogous to the general relativistic frame-dragging effect²⁴⁸ of a body orbiting a rotating mass. Loosely speaking, we can say frame-dragging leads to a precession in the plane of (orbital) rotation.

General relativity also involves an (orbital) geodesic effect upon spinning celestial bodies. A spinning body's axis, when parallel transported around a massive object's curved spacetime, (in one complete revolution for argument's sake) does not end up pointing in exactly the same direction as before. There is a 'precession' of the polar axis. Loosely speaking, one can say: "... the gyroscope 'leans over' into the slope of the space-time curvature (Wikipedia: *Gravity Probe B*, 2011)."

The new "precession" being proposed herein is a function of curved spacetime but it is quite different:

- Involving, a (relative) *phase* of intrinsic angular momentum — and requiring atoms/molecules 'slaved' to a celestial body.
- It is a *virtual* effect, that is self-interference based and thus determined only after a complete orbital loop/cycle; in a suitably configured three-body spin-orbit coupled celestial system.
- This QM "spin phase precession", relative to initial spin phase and an unchanged orbital phase, does not arise from a (local) change in a celestial body's *orientation*. See section 5.5 for further discussion.

Loosely speaking, we may also visualise (i.e. imagine) the (virtual) spin phase precession/offset as a QM "over-spin", although it is the relative virtual spin *phase offset* that is our primary focus.

5.4.5 The three faces of one and the same (magnitude) geometric phase offset

It helps to recognise three different (conceptual) types of *relative* phase change (per loop/cycle), that are physically equivalent *if* moon-planet spin-orbit resonance/coupling is 'active', and a cosmological substratum is recognised (recall section 4.4). Firstly, internal to an atom/molecule we have: the QM phase associated with intrinsic angular momentum (i.e. spin) relative to (the unchanging, non-topological) orbital angular momentum phase. Secondly, we have an internal (lunar-based) spin phase offset relative to a background global reference 'datum'; with this datum including the sun and host planet's atoms/molecules — i.e. everything *other* than the spin aspect of atomic/molecular constituents within the third body of suitably configured celestial systems. Thirdly, we have the spin phase offset, after the closed loop motion, relative to its (own) pre-loop/initial 'value' — by way of self-interference.

5.4.6 The need for two types of spin-orbit 'coupling': macro- and microscopic

Lunar orbital resonance (i.e. macroscopic moon-planet spin-orbit resonance/phase-lock) ensures an exact externally orientated (1:1) spin-orbit relationship for all lunar atoms/molecules per "closed path loop" (i.e. one orbit). This externally imposed exact phase-matching establishes a (resonance) 'datum', against which any unforeseen externally driven atomic/molecular geometric phase drift may be referenced. Subsequently, two different broad types of relative phase offset are conceivable. The first is exclusively internal/micro: i.e. spin vs. orbital, in conjunction with spin (virtual) vs. spin (actual); and the second is lunar (atoms/molecules) vs. an external reference 'datum' — which includes the unaffected spin and orbital geometric phases of planetary and solar atoms/molecules. In both cases the same conceivable offset — occurring upon the completion of a closed path loop — is being described; i.e. a lunar (atomic) spin phase offset relative to either: an internal unchanging QM spin-orbital phase configuration; and/or an external background space phase 'datum'.

In the model, any QM system (particularly atoms and molecules) whose spin phase is offset by less than half a wavelength, must 'maintain' (and not lose) its (discrete) quantum spin-orbit 'configuration'; (and) thus, the energy associated with the virtual geometric phase offset must be externalized into the space-time 'environment' — if a systemic (i.e. universal) conservation of energy principle applies. Actually, this is a conservation of angular momentum over the lunar spin-orbital duration/time Δt . Thus, the *virtual* spin phase offset 'exists' relative to its own unchanging (actual) lunar atomic/molecular spin phase datum (see

²⁴⁸Also known as the Lense-Thirring effect.

subsection 5.5.8); with this being a *local* datum cf. a *background* space phase datum.

5.4.7 Global reference frames and lunar geodetic precession rate

In relation to the Sun's tidal acceleration Kenneth Nordtvedt recognises that:

...de Sitter's 'geodetically' rotating inertial frames accompanying the Earth in its motion around the Sun induce a precession of the lunar orbit which is *slightly greater* than the geodetic precession rate²⁴⁹, ...[because] the Moon is not a locally isolated dynamical system, ...[Thus, the] Moon's dynamics in the geodetically rotating inertial frame is intrinsically different from its dynamics in a globally fixed frame,²⁵⁰ ... (Nordtvedt 1996, abstract).

Somewhat similarly, in our model there is a tiny (virtual) discrepancy in the form of a QM spin phase 'precession'²⁵¹; arising from the planet-moon system's orbital motion in the 'barycentered' solar system frame — that would (always) be absent if only the local (two-body orbital) dynamics was considered.

It is prudent to (once again) note that a GR-based understanding of the motion of larger bodies in the solar system, especially the Earth-Moon system, is *not* (at all) being questioned. Indeed the "closed cycle/loop" referred to, regarding QM geometric phase, is an *orbital* loop that includes (non-Newtonian mechanical) general relativistic-based effects.

5.4.8 Summary remarks for section 5.4

The section's main objective has been a discussion of the proposed geometric phase (GP) offset, and how it is different to GP shifts in (primarily) electromagnetic-based situations. Specifically, we make a comparison with the Aharonov-Bohm (A-B) effect, because the A-B effect (until now) has been considered to be the only phenomenon that involves a non-locality associated with fermion waves.

We distinguish this (unique) GP offset from gravitational analogues of the (EM) Aharonov-Bohm and Aharonov-Casher effects, where the A-B effect is considered to be analogous to general relativity's frame-dragging. The new GP offset has been portrayed as (or likened to) a new type of (spin) precession, in that it is (also) a function of motion in curved spacetime. Obviously, it is quite distinct from geodetic precession in that it involves a relative and *virtual phase* offset

²⁴⁹The Lense-Thirring effect is neglected in the article [see p.1318].

²⁵⁰That is, a frame fixed with respect to distant inertial frames. This is the barycentric frame (comprising the Sun, planets, Earth, Moon, asteroids, etc.) employed in the generation of ephemerides for the Moon and other solar system bodies.

²⁵¹In the form of an intrinsic angular momentum phase shift/offset per (closed loop) cycle time.

that is figuratively likened to a classical Hannay angle offset.

The special circumstances required for this new effect have been discussed. Firstly, by way of applying to all (third-body) elementary fermion particles, this GP offset is also seen as applying — in equal measure as that applied to each (and every) 'constituent' fermion particle — to whole *quantum mechanical* systems (i.e. atoms or molecules). We note that atoms/molecules have 'internal' spin-orbit coupling. Secondly, we require the parent body of the atoms/molecules (i.e. a moon) to be in celestial spin-orbit resonance relative to its host planet. These conditions ensure that a minimal phase offset can occur, i.e. less than half a fermion wavelength (2π), with this circumstance being physically and quantitatively not insignificant.

Finally, the relative nature of the geometric phase offset (β) was further discussed. It is seen as both: an internal spin vs. orbital effect involving angular momentum *phase*; and secondly, as a local vs. global/systemic effect involving spin phase cf. a background (spin phase) datum. Additionally, we introduced the idea that β is also a virtual spin phase offset vs. the actual spin phase as fixed by the EM dominated 'internal' spin-orbit coupling — which acts to suppress (and make virtual) the 'would be' phase advance.

5.5 Spin phase, orientation, and spin-turning

This section brings us to a core aspect of Section 5.

5.5.1 Spin phase and probability changes

Measurements yield only two discrete values of (fermion) spin, i.e. $-\hbar/2$ or $+\hbar/2$. In our case, the projection plane of a QM system's spin is fixed by the plane of lunar motion. Relative phase changes²⁵² [i.e. relative to a systemic (phase) reference frame] are seen to alter the probability of finding a lunar-based (and atomic/molecular-based) elementary fermion particle in an spin-up or spin-down state. Loosely speaking, a 2π fermion phase shift, for example, would alter a 100% spin-up probability to a 100% spin-down probability — with this alteration also associated with the sign change of a spinor. A π (radian) phase shift/offset, i.e. effectively 1/4 of a fermion wavelength, alters a 100% spin-up probability to a 50% spin-up and 50% spin-down probability²⁵³. Note that this discussion, and that of subsection 5.5.2, draws upon the discussion in a Bernstein & Phillips (1981, pp.97-99) Scientific American article; but we shall *not* employ the notions of: base space, total space and fibre bundles (preferring the model-specific stance adopted in subsection 5.4.4).

²⁵²Either a lunar (atomic/molecular) vs. planet/global spin phase offset, or equivalently, a lunar atom/molecule's initial spin phase vs. its (virtual) final phase.

²⁵³Where probabilities are calculated from the mathematical description of each state of a particle as a wave function.

5.5.2 ‘Turning’ as compared to ‘rotation’

Usually (although not in our case), any change in spin probability, that is conceived (in an idealised manner) as arising from an externally driven (physical) rotation/precession of a QM system’s spin ‘vector’, also implies a change in the spin orientation. Some authors prefer to speak of a ‘turning’ rather than a rotation to reinforce the non-classicality of QM spin. In our case, this turning is *not* an axis orientation turning; rather, it is a phase turning associated with a fixed/given spin vector’s orientation — (classically visualised) much in the manner of the Earth’s (daily) turning on its axis. In the case of our model, we shall see that maximum virtual energy occurs at a relative phase offset occurs of π radians, with the 0 and 2π radian offsets being *quantitatively* equivalent. Note that coherent (virtual) phase offsets greater than 2π radians are not considered because they are physically impossible; by way of (actual) decoherence at 2π radians, which will ‘reveal’, and would also be incompatible with, the *real*/actual multifarious orientations of spin in the numerous atoms/molecules of a bulk moon.

5.5.3 Spin orientation and virtual spin phase offset orientation

The *external* planar (orbital) rotation of a moon about its host planet fixes both the projection angle of (internal) atomic/molecular spin, and also the spin axis orientation (the z axis) for the *virtual* phase ‘turning’ associated with motion in curved spacetime. This circumstance applies to *every* atom/molecule ‘in’ a moon so long as it remains below the decoherence limit.

In the solar system, planets and their moons orbit close to the ecliptic plane; (and) the orbital orientations of large moons, with the exception of Neptune’s Triton, are closely aligned with a planet’s equatorial plane. Fortunately, we shall see later (subsections 6.2.2 and 6.5.8) that the model’s motion retardation effect for (the Pioneer) spacecraft is the same irrespective of the S/C’s inclination to lunar orbital planes; although for Earth-based measurements a correction for the angle between spacecraft path and line-of-sight observation direction is required (see Table 6 of Section 6).

5.5.4 Standard approaches to the interaction of spin with inertia and gravity

Two names that loom large in the field of gravity’s interaction with quantum mechanical systems are Bahram Mashhoon and Giorgio Papini. Their emphasis is upon applications to particles (cf. atoms/molecules) and:

- utilises a *localised* approach to the formalism (cf. a non-local, closed loop, Aharonov-Bohm like effect);
- involves the (local) interaction of spin with: inertia, rotation, and gravity; and
- concentrates upon motion that *crosses* gravitational field lines, i.e. non-geodesic motion.

In broad terms, the interest for Papini, Mashhoon, and others lies in the fact that: “External gravitational fields induce phase factors [and only phase factors] in the wave functions of particles (Papini 2008, abstract).” It was Cai & Papini (1989) who obtained a covariant generalization of Berry’s phase. That general covariance of GR can be incorporated in the theory is a significant feat.

We can finally conclude that the validity of covariant wave equations in an inertial-gravitational context finds support in experimental verifications of some of the effects they predict [concerning neutrons and either: the Earth’s rotation, or the Earth’s gravitational field], in tests of the general relativistic deflection of light rays and also in the phase wrap-up in global position system measurements (Papini 2008, p.15).

Our main concern is to comment upon their proposed spin-rotation coupling²⁵⁴ and its extension to spin-rotation-gravity coupling (Mashhoon 2000, section 3).

In light of the fact that the model’s non-local effect is restricted to (‘overall’ atomic/molecular) intrinsic angular momentum (spin), while leaving orbital angular momentum unaffected, it is *crucial* to note that *locally*:

In derivations based on the covariant Dirac equation, the coupling of inertia and gravitation to spin is identical to that for orbital angular momentum (Papini 2008, p.16).

Thus, a local approach to establishing the model’s new (virtual) spin geometric phase offset, involving GR and QMs, is denied. Consequently, the offset is (necessarily) systemic and non-local (as per subsection 5.4.3).

Clearly, the new mechanism must be different, and not included in the broad scope of situations examined by this field. Were it not for the Pioneer anomaly, the conceivable presence of this new (non-local) curved spacetime/gravitational field effect upon certain quantum systems, which is *completely* ‘back-reacted’ into a gravitational field (in a different/new format), would have remained far from consideration.

5.5.5 Further reasons for denying a local approach to the phase shift/offset

There are only two theoretical options regarding the basis of the new phase offset. Either it is solely related to an atom/molecule’s local physical motion, or an additional systemic and non-local effect upon fermion spin phase is also present — as is the case with the Aharonov-Bohm effect. The former (i.e. local) scenario is preferable, but for motion along a geodesic this subsection shall present further reasons why it cannot be justified. Thus, by default we need to embrace

²⁵⁴For fermions, there exists at present only indirect evidence for the existence of spin-rotation coupling (Mashhoon & Kaiser 2006).

the latter scenario i.e. a non-local/global effect arising from (closed loop) motion in curved spacetime.

From a purely *local* perspective the (shared by every atom/molecule) QM virtual spin phase offset may be conceived as a (virtual) angular offset/excess (or “over-turning”); with this being a measure of the curvature enclosed by the path of the moon in curved spacetime. Note that in this (offset) case, parallel transport of a vector *is* (in some sense) necessarily implied. We may then equate the common angular excess of all the atoms/molecules in a moon to a (virtual) lunar bulk-mass “over-spin” — relative to a scenario involving uncurved (flat) space. It was tempting (for the author) to then assume this virtual “over-spin” is actually an unrealised/thwarted (macro) spin geodesic precession effect — thwarted by way of tidal forces that lead to celestial spin-orbit resonance. Thus, the *angular* offset could then be derived from a moon’s path in relativistic curved spacetime, so as to make unnecessary any non-local basis (ultimately involving self-interference) ‘behind’ the *phase* offset. But, there are problems with such an expectation/approach.

If we idealise the moon-planet-sun three-body motion as: (all bodies) lying in the same (ecliptic) plane (with the lunar and planet spin axes orthogonal to this plane), then it is clear that the over-spin does/could not arise from a precession of the orientation of a moon’s spin-axis. Thus, the notion of a geodesic spin-axis precession is not relevant to the model. Furthermore, the macroscopic (Hannay angle) “over-spin” concept fails to be analogous to the geodesic precession of a spinning gyroscope — as is examined in the Gravity Probe B experiment²⁵⁵. In other words, utilising GR’s representation of spacetime curvature and the concept of parallel transport of a vector, there is no way to link curved spacetime with a *spin* over-rotation angle. It is just not in the ‘machinery’ of general relativity, (even) incorporating the parallel transport of a vector, to do this. Furthermore, the heuristic and conceptual use of a classical Hannay angle, regarding (an extrinsic/external imposition upon) QM spin, would be inappropriately physicalised.

The failure of a solely local approach is further supported by the inability of the QM spin (at a sub-system level) to be physically understood as a classical rotation — of any kind. Only a virtual effect *thwarted* by the dominant electromagnetic intra-atomic ‘forces’ is conceivable, and thus the thwarting of a physically *real* rotation (on any level) is inappropriate and not possible. General relativity demands that the local frame can always be configured as an inertial frame; in section 5.6 we see why this, in special celestial ‘third-body’ circumstances, is not (always) the case — at least virtually — at the QM (atomic/molecular) systemic level.

5.5.6 Ramifications of denying a local and reductive approach (re: phase shift)

Subsection 5.5.5 showed how a solely reductive and local account of the model has fatal flaws. Thus, regarding the model’s physical associations, we are restricted to only discussing a relative phase-shift (albeit virtual). Any reference to spin *rotation* angles lacks physical validity, concerning *both*: a moon (as a whole), and at the quantum mechanical level of its constituent atoms/molecules — as well as the latter’s constituent elementary (fermion) particles.

The representation of a *virtual* relative geometric phase offset by way of either: a Hannay angular offset, an over-turning angle, or over-spin angle, nevertheless remains a useful heuristic/visual tool in the model. For atoms/molecules we are restricted to speaking only of spinor or spin phase changes²⁵⁶, but systemically/globally we allow ourselves to envisage, by way of the conceptual tool of visualisation, a spin-rotation or spin-turning based offset (as per subsection 5.5.4).

5.5.7 Aspects of a (third-body) non-local virtual geometric phase offset

It remains true that the virtual angular offset, or rather virtual geometric phase offset, is (still) derivable from a moon’s path in relativistic curved spacetime; but herein it shall simply be the (configurational) *geometry* of the lunar and planetary *closed paths* that shall quantify the relative phase shift (see subsection 6.3.5).

Non-locality in the explanation cannot be avoided. Our (global/non-local) fermion self-interference scenario involves an unappreciated and subtle three-body curved spacetime effect upon the phase of a (given) spinor’s wavefunction. We embrace a phase-offset approach, rather than an orientation based approach. For explanatory simplicity we have chosen to (somewhat inappropriately) visualise this (externally ‘imposed’ effect) as a (relative) over-spin.

This spin phase offset exists in conjunction with what Michael Berry termed: “... phase functions hidden in parameter-space regions which the system has not visited (Berry 1984).” Regarding the model, this quoted phrase is best altered to: “phase effects hidden in background space-time that the system cannot recognise within itself, but systemically (i.e. globally) are physically relevant.”

The non-local basis of the spin phase offset is seen to extend the geometric (two-body) ‘curious problem of spinor sign change’ (and 4π fermion wavelength) to a new scenario; this being an appropriate third-body of a three-body celestial system. Note that in both (the two- and three-body) cases the simple/pure geometric relationship of (macroscopic physical) celestial (planetary *and* lunar) orbital rotation angles to (microscopic) phase shift is determined within a plane, i.e. it is a relationship describable in two dimensions.

²⁵⁵An alternative name for the Pioneer anomaly could be (the) Gravity Probe C experiment — even though two spacecraft were involved.

²⁵⁶Mathematically we would need to deal with ‘spinors’, whereas for physical understanding the term ‘spin’ is preferred.

5.5.8 What lies ahead: the spin offset via a non-inertial path cf. an inertial path

In section 5.6 the physical significance (and ramifications) of the spin phase offset's existence is examined. Up to this point we have simply examined the basis and motivation for a phase offset. It shall be argued that a virtual (intrinsic angular momentum) phase offset represents a phase offset between a background global *inertial* frame and a local frame that has remained inertial (and unforced) in all ways *except* for the unique spin circumstances outlined herein.

Since the phase offset — upon the completion of a geodesic closed loop (orbital) motion and one complete lunar-celestial spin cycle/loop — is virtual, there is an absence of any actual (i.e. real) (fermion) wave interference. Thus, at all times we retain (cf. the implications of Berry's phase), a circumstance where (local) quantum eigenstates are not made 'multi-valued' — i.e. they (themselves) remain unaffected by the (externally induced) virtual phase offset.

In other words, we shall argue that the system's spin phase lags behind that which curved space (eventually) requires after the lunar closed loop (and spin) motion(s). Thus, the atomic/molecular system itself 'under-turns' relative to a spin inertial frame — with this being the phase excess or over-turning we have been discussing. It is the internal/QM spin-orbit coupling of an atom/molecule that both: leads to a non-inertial circumstance; and thwarts any internal (sub- 2π) spin phase change, because orbital phase is unaffected. Thus, a *virtual* spin phase offset has *real* physical implications — presuming that it is recognised/appreciated by the global system it is a part of. In QMs this situation can lead to an *equal and opposite* compensating *phase* shift in the environment (Berry 1988, p.31). *In our model the 'reaction' in the environment is based upon energy rather than phase.*

Importantly, in our case — i.e. the model's hypothesised case — the virtual phase shift of intrinsic angular momentum also allows an *energy* value to be determined. The spin energy deficiency needs to be "made up" somewhere else in the larger system (i.e. the universe). With no available means of internal expression this energy, totalled up for all the atoms/molecules in a (suitable) moon (N_m), gets expressed *externally* as a (single) rotating space-warp of amplitude Δa — rotating in the *same* direction and at the same angular velocity as a moon's 1:1 spin-rotational (i.e. spin-orbit resonant) motion²⁵⁷. Note that decoherence negates (or invalidates) any external influence arising from a greater than 2π rad geometric phase offset/change (i.e. $\beta \geq 2\pi$ rad).

Finally, we note that this space-warp 'rotation' — introduced and discussed in Section 3 — is quite different to the *internal/interior* virtual/non-inertial phase shift or phase-'turning' per lunar spin-orbital

cycle discussed throughout (this) Section 5. Initially, the mechanism is treated as a *planar* phenomena, but the space-warp's extension into three (physical) dimensional space is required (see subsection 6.2.2).

5.5.9 Summary remarks for section 5.5

This subsection has discussed further aspects of the model's virtual and non-discrete geometric phase offset (β), and finishes with a preview of what this offset is physically representative of; i.e. a distinction between (or difference involving) an inertial and a non-inertial situation.

Whilst the standard relationship of a change in phase to a change in the probability of measuring a 'system' as spin-up or spin-down is retained, the external means by which the spin phase shift is 'generated' is quite *different*; in that it does not arise (solely) from an externally induced rotation of a spinor (or particle's) spin vector orientation. This difference is best understood as arising from the non-standard 'influence' of a gravitational (i.e. curved spacetime) field upon the (wavefunction-based, and intrinsic angular momentum-based) phase of (certain third-body) quantum mechanical sub-systems — i.e. the atoms and molecules of a moon. This interaction differs from standard approaches in three ways: (1) atoms/molecules as composite elementary fermion particle systems are examined cf. merely particles; (2) our concern lies with geodesic motion cf. effects arising from the crossing of gravitational field lines; and (3) most importantly, the basis for the geometric phase change/offset is (necessarily) global/systemic and non-local cf. a localised approach that involves the parallel transport of a spin vector's orientation. The unavoidability of this final distinction has been discussed in some detail and reinforces the fact that: the geometric phase change cannot be solely attributed to a local motion and/or rotation, nor some form of local spin-rotation-gravity coupling.

We also noted that: for this *virtual* geometric phase shift, the projection plane of each (and every) lunar (intra-atomic/molecular) elementary fermion particle's spin axis is fixed (externally) by either: the bulk mass moon's spin axis, or the plane of lunar orbital motion around its host planet. Finally, it was mentioned that a $\beta = \pi$ phase offset (i.e. one quarter of a fermion wavelength) is associated with the maximum (internally unresolved) *virtual* spin energy — of every atom/molecule in a (solid) bulk lunar 'mass'. This total internally unresolved energy is expressed (externally) by way of a 'real' rotating space-warp, and a spherical non-local mass distribution — for the latter, see/await subsection 6.4.2 for an introduction, and section 6.6 for a comprehensive discussion.

5.6 Further discussion upon the non-local geometrical (and topological) spin phase offset

5.6.1 The inertia of intrinsic spin

²⁵⁷ By way of energy conservation demands, the 'universal' system effectively 'preempts' the offset dilemma and 'externalises' the energy involved — over the course of the cycle time Δt . Additional background/hidden temporal effects are implied — see Price (1996) for an account of how concerns arising from this situation may/might be appeased.

Mashhoon & Kaiser (2006, introduction) note that: “... in quantum theory the inertial properties of a particle are determined by its inertial mass as well as spin.” We extend (the use of) ‘particle’ (in this quote) to the bound *systems* of atoms and molecules. Our interest involves both spin *and* indirectly inertial mass — the latter by way of the *number* of atoms/molecules being ‘equivalently’ influenced at the same time, *and* (more importantly) the role of mass in intrinsic angular *momentum* (i.e. spin).

The interpretation of our model/mechanism is that the spin phase offset indicates the local spin phase’s lag (at the end of a loop in curved space-time) relative to the spin phase required to maintain an inertial spin frame — with respect to a global or systemic (inertial) spin frame. This systemic (spin) reference frame implies there is a topological aspect to the geometric phase (recall subsection 5.4.2). As regards geometric phase changes, the atoms/molecules effectively remain in a stationary state throughout the cycle/loop — with only phase and probabilities experiencing a (virtual) alteration.

The *orbital* characteristics of atoms/molecules are *not* (additionally) affected by third-body cyclic motion in curved spacetime, i.e. there is no latent or hidden virtual effect upon the phase of (lunar-based) atomic/molecular orbital²⁵⁸ angular momentum.

Local atomic/molecular spin-orbit coupling dictates that the spin’s phase is necessarily ‘slaved’ to the orbital characteristics of an atom/molecule — which is dominated by: electromagnetic forces, Schrödinger’s equation, Planck’s constant, discreteness, etc.; as compared to minuscule (internal) gravitational (attractive) ‘forces’ which are $\sim 10^{-40}$ times weaker *in the atom*. Subsequently, (within atoms/molecules) any inertial ‘forces/effects’ acting upon the elementary fermion/matter particle’s spin (alone), ‘driven’ by (external) gravitational curvature effects, have hitherto been assumed negligible, or (alternatively) have simply not been appreciated/anticipated.

Locally, the (relative) spin phase offset — common to every elementary fermion wave/particle and hence (equally) to an atom/molecule ‘overall’ — is a background/hidden effect. Only a system ‘greater’ than just the atom/molecule can ‘register’ its presence — and only then if it is below the (2π phase shift) decoherence limit. The under-turning of an atom’s spin (i.e. its non-inertial spin behaviour) fixes a (per loop) rate of angular momentum offset — i.e. a non-inertial spin *energy* difference (per atom/molecule).

Implicit in this discussion is the need to assume the existence of a global (but *not* absolute) reference frame where both gravitational and quantum mechanical systems (and effects) harmoniously coexist in nature. A systemic conservation of (spin) energy is also embraced (recall section 4.6). Also implicit is the model’s use of noumenal/background/hidden time simultaneity in conjunction with quantum mechanical ‘non-locality’ (as discussed in Section 4).

²⁵⁸The word “orbital” has a somewhat loose applicability to QM circumstances.

5.6.2 Ramification of no graviton particle: a virtual fermion condensate

Firstly, with a presumption of the non-existence of graviton particles — recall subsections 4.2.9, 5.1.2 and 5.2.8 — there is no gravitational momentum transfer (of information) by particles; thus, only a sign change of a spinor ‘activates’ decoherence. In contrast to a charged particle-to-photon coupling, the absence of a mass to graviton coupling means an unlimited quantum coherence cannot be ruled out — i.e. a large number of quantum ‘particles’ cooperating in a single quantum ‘state’. This ‘state’ is actually the energy associated with the shared virtual phase difference — with a common (externally determined) projection angle of atomic/molecular spin and a common/shared spin-axis orientation facilitating this “large number” effect (recall subsection 5.5.3).

The physical absence of charge interference, because charges are decohered by their own electrical field, leads to a charge superselection rule. Similarly, although by default, there is no graviton interference. Thus, below the level of decoherence, something akin to a (virtual) fermion ‘condensate’ is possible. Without a graviton particle a QM system, influenced by curved spacetime effects, can remain isolated from its environment — but only so far as ‘real’ (i.e. particle-based) physical communication is concerned.

To simply believe a large mass is necessarily classical fails to fully appreciate the *qualitative* manner by which decoherence *is* achieved (recall subsection 5.1.5), and overlooks the exceptional circumstances, based upon a *quantifiable* phase offset, discussed herein.

The EM gauge and phase transformations that satisfy Schrödinger’s equation and gauge invariance are only relevant in so far as stipulating a minimum quanta of intrinsic angular momentum (i.e. spin) (for fermions) of $\frac{1}{2}\hbar$. By way of quantum discreteness and uncertainty the *minimum* (actual/real) energy associated with: a particle’s, atom’s, or molecule’s QM spin is $\frac{1}{2}\hbar\Delta t^{-1}$ (recall subsection 4.2.2); which is actually a *maximum* possible virtual energy level (per atom/molecule) in the new mechanism’s spin-based inertial offset situation (see subsection 6.3.2). A prerequisite of this energy (in the model/mechanism) is the coexistence of atomic spin *and* orbital behaviour, and their (interaction or) coupling.

5.6.3 Discussing relationships between spin and orbital angular momentum

The additional QM/microscopic spin phase advance needed to achieve inertial status, is thwarted by the much stronger local EM effects. Note that internal QM orbital angular momentum is *only* a function of EM effects, and it is (not surprisingly) completely *distinct* from macroscopic celestial orbital angular momentum — including GR’s additional geodetic (orbital) precession, and any “frame-dragging” effects. With regard to macroscopic spin angular momentum and QM spin this distinction is even more pronounced.

Recalling subsection 5.5.5, it is wrong to consider

the shared (sub-half-wavelength) microscopic spin-phase offset, as representing a virtual macroscopic “over-spin” that is thwarted by tidal effects. We also noted that even if a macroscopic geodetic precession-like effect did exist, it could not be ‘thwarted’ — so as to ‘establish’ an unresolved non-inertial energy. Additionally, with microscopic over-spins always below a minimum (actual change) energy level, and decohered (i.e. lost) at or above it, tidal effects can *never* be said to physically impede (quantised) lunar over-spin. All that remains physically relevant is that, at a microscopic level, the many (virtual) ‘over-turnings’ remain active in the ‘awareness’ of the global system as a whole — i.e. in the ‘environment’/surroundings of the quantum mechanical (atomic/molecular) systems.

It is the unique ability of a global/systemic reference frame to access how the spin probabilities of QM atoms/molecules are counterfactually²⁵⁹ influenced, and the inability of any microscopic state change or macroscopic motion to appease this change, that necessitates the existence of the external rotating space-warp. The total QM over-turning *energy* (imbalance) cannot be denied — *if* it is below the (2π phase offset) decoherence limit. A crucial feature of the model is the fact that the counterfactual/virtual offset scenario *is* the *inertial* frame circumstance. At, or above, the decoherence limit this subtle situation, involving both macroscopic GR’s curved spacetime and microscopic QMs, is absent; and effectively only macroscopic tidal effects²⁶⁰ act upon the celestial third-body’s spin — as is currently envisaged by theorists.

Importantly, without sufficiently strong tidal effects, the absence of planet-moon spin-orbit resonance would mean that the finely tuned self-interference arising from:

- celestial (moon to planet) spin-orbit resonance,
- motion of celestial mass ‘upon’ a geodesic,
- the vast majority of atoms/molecules in a moon being in a fixed location relative to the moon’s geocentre²⁶¹, and
- QM atomic/molecular spin-orbit coupling,

is not applicable, and thus the decoherence limit is (considered) easily breached. Decoherence ensures that GR plus tidal effects are sufficient to successfully account for most three-body gravitational phenomena. We are examining the exception to the rule, with celestial and QM spin-orbit resonance/coupling (respectively) crucial to the new mechanism’s existence.

With regard to the QMs of atoms/molecules, spin and orbital angular momentum are linked in an *inter-dependent* manner. Classically, spin and orbital angular momentum are (usually) quite *independent* — with the spin-orbit synchronisation of a moon-planet system being an exceptional case. Thus, a *distinction*

²⁵⁹‘Counterfactual’ (adjective) is defined as: relating to or expressing what (is conceivable but) has not happened or is not the case; or running contrary to the facts (i.e. reality).

²⁶⁰Tidal effects arise from celestial bodies being non-rigid; notwithstanding the fact that moons are (primarily) solid bodies.

²⁶¹In other words, large moons are essentially (i.e. dominantly) “solid” bodies.

between how (external) curved space (cf. flat space) influences an atom/molecule’s intrinsic spin and orbital momentum is not unduly surprising — with our (moon-based) case’s specific ‘closed curve’ (recall subsection 5.2.3) involving (orbital) celestial motion along a geodesic, as well as celestial spin.

As a final encapsulating remark, it may be said that the environment’s ‘awareness’ of each atom/molecule’s non-inertial (microscopic) spin status, throughout a (generally) solid moon, does not entail non-geodesic (macroscopic) motion, but it does entail a new (macroscopic) gravitational/accelerational phenomenon in the form of a rotating space-warp²⁶².

5.6.4 Uncertainty, angular momentum and energy in quantum mechanics

Heisenberg’s uncertainty principle tells us we may not know simultaneously the exact values of two components of the angular momentum; in our case orbital and spin angular momentum. Even though the position of electrons in an atom (for example) may only be known by way of probability, the value of its angular momentum is an exact (or sharp) value.

[In an atom] the angular momentum operator commutes with the Hamiltonian of the system. By Heisenberg’s uncertainty relation this means that the angular momentum can assume a sharp value simultaneously with the energy (eigenvalue of the Hamiltonian)²⁶³.

This is supportive of the existence of an exact value of virtual (intrinsic) angular momentum in the model, and its change over cycle loop time Δt (i.e. virtual energy).

5.6.5 A distinction between macroscopic spin and orbital angular momentum facilitates the existence of a microscopic/QM non-inertial situation

Although macroscopic geodetic precession and the model’s proposed microscopic spin phase offset (or spin-turning) arise from the *same* motion in curved spacetime, these effects are otherwise totally distinct. With external curved spacetime acting upon the QM nature of atoms/molecules, any distinction between (and independence of) spin and orbital angular momentum at the classical level is easily carried through/‘down’ to the microscopic level. Thus, there is no compulsion to have something (local and reductive) like a geodetic spin-orbit coupling effect, with an unknown extra celestial/macroscopic spin precession (cf. NMs and GR) accompanying the extra macroscopic orbital precession (of GR cf. NMs) for example. Macroscopically/celestially, spin and orbital angular momentum are quite distinct, and subsequently, the effects of geodesic celestial motion/momentum upon the (geometric) phase associated with QM spin and orbital

²⁶²With an associated non-local mass distribution.

²⁶³Sourced from Wikipedia (modified 3 January 2012 at 21:31): *Angular momentum coupling*.

momentum in atoms/molecules can conceivably retain this independence, i.e. the effect can be ‘uncoupled’.

What remains true is that *together*: the virtual microscopic spin phase offset(s), which may be figuratively and collectively conceived as a (virtual) lunar-*spin* angular excess, and general relativistic subtleties such as the geodetic effect(s) etc.; act to physically establish what is inertial motion — in the sense of “least/no action” applying. Subsequently, for “mass in motion”, at the QM level, there is no good reason to deny an inertial-to-non-inertial ‘frame’ offset — albeit virtual — concerning the spin phase and (thus) the *mass* ‘within’ atoms/molecules. This offset (over the course of a cycle/loop) is effectively a non-inertial intrinsic angular momentum (rate); it arises from atomic/molecular spin-orbit coupling being quantised and dominated by electromagnetic concerns (including electrical *charge*), rather than the (inertial) *mass* of its constituent sub-atomic (fermion) particles — such that the actual/real spin phase is necessarily coupled/slaved to (the unaffected) orbital phase.

The confidence of Papini (subsection 5.5.4) in assigning *all* orbital and spin angular momentum the same coupling to inertia and gravitation is (thus) cast into doubt, at least below the level of any change in QM state and/or a sign change of a spinor. This concern is not with regard to the (local) *crossing* of gravitational field lines, but rather with regard to the geometric phase offset effect proposed herein — arising from the closed loop *geodesic* motion/trajectory of atoms/molecules within a ‘third’ celestial body.

5.6.6 Can dynamic curved spacetime considerations determine the model’s virtual spin phase offset?

In response to subsection 5.6.5 it could be argued that: because general relativity itself is seen to encompass “all that is inertial”, it follows (somewhat trivially) that (even) a quantitative fibre bundle analysis of (lunar atomic/molecular elementary fermion particle) QM spin in curved spacetime, with its basis in GR’s *local* curvature, cannot lead to an inertial vs. non-inertial offset at the QM level. Clearly, this is not a ‘watertight’ argument.

In subsection 5.2.2 it was mentioned that a relative geometric/Berry phase shift for photons has been shown to arise from the intrinsic topological structure of Maxwell’s theory. Conceivably, the means by which the model’s geometric phase offsets *could* be established is by way of the (dynamic) geometry (and topology) of curved spacetime, involving a metric tensor — albeit in a *three* body system. One stumbling block for such an approach is its complexity. Another possibility would be to use the (approximate) classical/Newtonian gravitational potentials. Such top-down reductive approaches cannot be ruled out, even though the closed loop (local) path followed is a *geodesic*, and regardless of the fact that curvature is an inherently ‘non-local’ phenomenon — in the sense of being distributed over a region of space.

For the sake of simplicity, and essentially by default, in Section 6 and initially in subsection 5.6.7, we

shall employ an *quasi-empirical* purely-geometric (and kinematical) basis to quantify the (third-body) relative QM spin phase offsets; as compared to a dynamic approach involving either: a metric tensor, gravitational potential, or a Lagrangian and/or Hamiltonian. This approach may or may not be the ‘best’ quantitative approach, but its use (and further development) in this paper achieves consistency with (i.e. matches) all the various observational evidence pertaining to the Pioneer anomaly. In agreement with the third quotation in subsection 5.2.3 we assume that: each geometric phase (offset) depends on a closed (orbital) curve (C) and (its quantitative determination) is independent of how the evolution around C takes place.

Notwithstanding our quasi-empirical approach, we nevertheless achieve a relative superiority and comprehensiveness of explanation (cf. other models arguing for a real non-systematic based Pioneer anomaly), that *quantitatively* is quite conceivably only of provisional (or stepping-stone) status. On the other hand, *qualitatively*/conceptually there is little room for the model to manoeuvre, and the approach is either: fundamentally misguided, or has merit and can lead to further progress. The latter is elucidated later in this paper (particularly subsections 6.2.4 to 6.2.6).

What does stand (somewhat) in the way of a *dynamic approach*, to the geometric phase offset, is the model’s embrace of non-local physical concerns/considerations, with this (embrace) exhibited by both: the manner with which we argued that a supplementary (to GR) type of acceleration/gravitational field can exist, and the need to introduce the concept of “non-local mass” (at the macroscopic level). Further, it remains unclear as to whether dynamic curved spacetime considerations could (ever) determine the model’s virtual spin phase offset (per orbital/cycle-loop). For our purposes a solely *geometric-kinematical approach* (involving three celestial bodies) is utilised, thus removing *all* possible conflict with non-locality (particularly with regard to QM spin entanglement).

5.6.7 Quantifying the geometric phase offset in the third-body of a three-body gravitational system

The model’s ‘*third-body*’ (virtual and mechanistic) spin phase offset²⁶⁴ can be *conceptually* appreciated as a virtual (Hannay angle) over-spin; and physically as a distinction between an inertial and a non-inertial frame. This unorthodox phase offset requires the presence of curved spacetime, but it goes beyond purely local and ‘force’-based considerations.

We seek to precisely determine the geometric phase offset for specific three-body celestial motion circumstances (e.g. Sun–Jupiter–Ganymede²⁶⁵). In the model, the phase change’s magnitude is a continuous (i.e. non-discrete) quantity, cf. subsection 5.3.3 where the QMs based discussion was (implicitly) restricted to discrete 2π (and $4\pi, 6\pi, 8\pi, \dots$) phase changes.

²⁶⁴For atoms/molecules upon the completion of a lunar orbit (closed circuit).

²⁶⁵Or more precisely: the (solar system’s) Barycentre–Jupiter–Ganymede.

For phenomena solely involving quantum mechanics ‘plus’ electromagnetism (QMs+EM) (with special relativistic effects included), the conventional approach to spin phase change (and spinor sign change) cannot be applied to the virtual spin phase/Hannay angle (offset) conceptualised within our (QM+curved S/T) model. With the Pioneer spacecraft observations dictating the model’s structure, a purely *geometric approach* is the simplest and most elegant option²⁶⁶; especially in the presumed absence of a graviton particle. Fortunately, this (semi-empirical) closed-curve based approach ‘bares fruit’²⁶⁷.

In the model, the relative phase advance (of the inertial spin frame) is restricted to atoms/molecules ‘experiencing’ (lunar) ‘third-body’ spin-orbit resonant (or phase-locked) motion; with the atoms/molecules making up the rest of the system (i.e. planets and the sun) *unaffected*. Distinctively, the (2π rad magnitude) ‘decoherence’ spin phase advance — of lunar atoms/molecules over one spin-orbit cycle (Δt) — needs to play a further crucial role, as regards its relationship to the geometry (and kinematics) of the (second-body) host planet’s orbital motion (over Δt).

By way of fitting the model to the observations: for *one* lunar spin-orbit cycle; *and* a planetary orbital progression angle (around the Sun), designated θ , of $\tan^{-1}(8\pi)^{-1} \approx 2.2785^\circ$; the lunar phase advance (applicable to intrinsic angular momentum) is seen to be 2π radians. This specific and unique geometric configuration, although highly unlikely in real moon-planet-sun circumstances, becomes our reference scenario. It is used to determine (i.e. quantify) the (virtual) lunar atomic/molecular (spin) geometric phase offsets (over one spin-orbit cycle) — of physically real three-body celestial systems (e.g. Sun–Jupiter–Ganymede). This is more fully examined in section 6.3.

The geometry of actual lunar and planetary orbits, arcs, angles, etc., ‘supported’ by the presence of spacetime’s curvature/geometry, is all that is needed to establish any active third-body phase offset β (as long as it is $< 2\pi$ radians). The Earth’s moon (Luna), whose origin is collision based, is not a space-warp ‘generator’ because its geometric phase advance far exceeds the decoherence limit of 2π radians — where a spinor sign change occurs. Effectively, the spinor sign change (associated with a 2π phase offset) in flat space has been extended to a curved space scenario — involving *three* bodies — and hence the extra (planetary progression) angle is required. Thus, our three-part reference scenario is:

- a 2π (geometric) phase offset ($\beta_{\text{ref}} = 2\pi$ rad),
- for one (closed loop) lunar orbit and one lunar spin cycle [i.e. (in both cases) a 2π rad rotation],
- where the moon’s host planet advances $\approx 2.28^\circ$ [i.e. $\tan^{-1}(8\pi)^{-1}$] around the sun in this time.

²⁶⁶These words may give the sceptical reader some solace. “...for geometry, you know, is the gate of science, and the gate is so low and small that one can only enter it as a little child (William K. Clifford).”

²⁶⁷Recall that two conceivable (but unlikely) alternative methods for establishing the geometric phase offsets were discussed in subsection 5.6.6.

This ‘yardstick’ (three-part) relationship allows us to quantify the ‘gravitational’ geometric phase offset β of a third-body’s atomic/molecular constituents. For cases where $\theta \neq \tan^{-1}(8\pi)^{-1}$ over one lunar orbit then $\beta \neq 2\pi$. This is an important step in the eventual determination of the total energy of each rotating space-warp, and subsequently the Pioneer anomaly. In section 6.1.1 we outline the steps involved.

5.6.8 Concluding and summary remarks for section 5.6

Of vital importance to the model is the fact that: in quantum theory the inertial properties of a particle, atom, or molecule are determined by its inertial mass *as well as* spin.

A core issue of this subsection is the physical relevance of the virtual spin (geometric) phase offset upon the completion of a closed loop/orbit; which then quantifies a rate of intrinsic angular momentum, i.e. a virtual (spin) energy offset — within (and only within) an appropriate ‘third’ (lunar) celestial body. Fortuitously, the (lunar-based) atoms/molecules (effectively) all experience the same virtual geometric phase offset relative to the (pre-loop) initial phase or an unchanging/unaffected phase. In the “special circumstances” of our model, a frame offset is present because each atom/molecule’s *actual* (local) spin phase configuration (at the completion of a closed orbit) ‘has become’ non-inertial; with the virtual offset being indicative of the departure from the inertial (spin) circumstance. This inertial vs. non-inertial offset circumstance is dependent upon the local existence of a *spin status* that itself is influenced (non-locally) by: the curved-spacetime-geodesic closed-path/orbital *motion* of (lunar-based) atoms and molecules, and all their constituent elementary fermion particles.

This physical phenomenon has both local *and* systemic/non-local aspects. There is a systemic (or global) awareness of the (relative) local asymmetry that is (then) collectively appeased externally (in the environment) by way of both: a rotating space-warp, and an accompanying non-local mass distribution. *Individually*, neither quantum mechanics nor general relativity suggest that this offset could exist, with the proposed phenomenon (necessarily) involving features relevant to *both* of these distinctly different theories — in particular QM spin, QM spin-orbit coupling, non-local geometric phase effects, and curved spacetime.

The (aforementioned) ‘special circumstances’ required include: (1) a topological and geometric spin phase ‘precession’ — albeit virtual — that does not affect orbital (angular momentum) phase in any way; and (2) a lack of decoherence triggered by the presence of gravitons, which is distinct from the case with electromagnetism — where electrical charges are decohered by their own electrical field. This is because a solely geometric approach to gravitation is seen to deny the (very) existence of the graviton particle (recall Section 4). Thus, a significant quantum coherence of the non-inertial offset effect and virtual fermion (and atomic/molecular) condensate behaviour — so vital to the model — cannot be ruled out (see section 5.7).

Further, the ‘fineness’ of the spin phase offset (i.e. $< 2\pi$ rad), requires near exact self-interference of atoms/molecules after closed loop motion. This is attained by way of: third-body celestial (macroscopic) spin-orbit resonance²⁶⁸, and QM (EM dominated) spin-orbit coupling; lunar geodesic motion; and an absence of atomic/molecular motion relative to a lunar geocentre (i.e. the solid body content of a moon).

The preliminary (empirical) quantification of the geometric phase offset was also outlined. The determination of relative geometric phase needs to go beyond a standard electromagnetic and QMs based approach. Celestial orbital *geometry*²⁶⁹ (alone) is sufficient to allow the quantification of the spin phase offset in all actual (*three*-body) moon-planet-sun configurations. This is achieved by way of recognising, and adjusting from, an idealised (three-pronged) reference scenario that gives: a 2π phase offset, over the course of one lunar spin-orbit cycle, for a specific planetary progression angle (θ) around its (host) Sun²⁷⁰. Later (subsection 6.3), we see that the Earth’s moon exceeds the 2π decoherence limit, and thus it doesn’t have/generate an associated rotating space-warp.

5.7 Energy, coherence and condensate behaviour in the model

5.7.1 Constructive superposition of virtual phase

The geometric phase effects upon neighbouring atoms/molecules in a celestial body, arising indirectly from spacetime curvature, may be considered equivalent because of their (effectively) parallel celestial (orbital) motion. Certainly, the motion/path of lunar atoms is very different to those of its ‘host’ planet — within a systemic (barycentric) reference frame.

That the spin phase offset is common to all lunar atoms/molecules means that effectively there is no geometric phase shift difference between different lunar atoms/molecules²⁷¹. Thus, their respective phase shifts act in concert, i.e. coherently yielding a type of constructive superposition — albeit virtual. Additionally, by denying the existence of gravitons, the coupling between mass and gravitons is non-existent, and hence there exists no limitation upon the coherence and superposition of (qualities and quantities involving) spin and ‘mass’²⁷². In our case, this is indicative

²⁶⁸That is, lunar synchronisation or ‘phase-lock’ around its host planet (the second-body).

²⁶⁹This is in its traditional (non-general relativistic) sense, involving: closed curves, arcs, angles, etc.; i.e. plane Euclidean geometry as compared to non-Euclidean geometry.

²⁷⁰Or more precisely, the solar system’s barycentre or centre of mass, which is (effectively) the three-body system’s ‘first-body’ or (alternatively its) ‘central-body’.

²⁷¹In other words, the variation throughout a moon is negligible. A “solid” body is also implied, i.e. effectively negligible contributions from molten material and water — see subsection 6.5.11 for further discussion.

²⁷²In analogy to: “If the electromagnetic coupling between charge and photons is turned off, then there exists no limitation on the coherence of superpositions of charge (Aharonov & Susskind 1967, p.1430).”

of a proportion of the (finite) minimum intrinsic angular momentum of each and every elementary fermion particle within each and every atom/molecule²⁷³; that in themselves are now appreciated to be not immune from an unforeseen (global/systemic) inertial commitment/frame — and from which they ‘stand apart’.

Now, with the virtual phase shift representing a virtual (spin) energy, constructive superposition allows a simple additive summation of these individual energies to give the total energy — and this energy is then expressed externally, i.e. in the environment, as a rotating space-warp. Note that the rotation of the space-warp is in the same sense/direction as the moon, so as to make up for the system’s ‘under-spin’ relative to inertial (spin) conditions. This scenario is seen to obey a global (i.e. universal or systemic) conservation of energy principle, that encompasses both the micro- and macroscopic realms (at the same ‘time’ and in ‘unison’) — i.e. QM and gravitational energies (respectively).

5.7.2 A space-warp so as to disallow a global/non-local phase change

An external rotating space-warp, with both an associated constant acceleration/gravitational undulation amplitude (Δa) and non-local ‘mass’ distribution, effectively quantifies the energy it takes to maintain/regain an ‘all-round’ conserved energy scenario. Additionally, the non-local mass is seen to exhibit (something analogous to) a spreading of a quantum wave packet over time (recall subsection 5.1.5). See section 6.6 for how this (new concept of) non-local mass, acting upon a given condensed/compact physical mass in space, changes with radius away from a moon — i.e. the mass involved is a field based quantity, as is Δa .

For any QM system exhibiting ‘interior/internal’ spin-orbit coupling, ‘slaved’ to a macroscopic third-body moving along a geodesic, the departure of atomic/molecular-based spin from inertial conditions (upon completion of a closed loop), due to external gravitational and configuration conditions/circumstances, must remain an *external* gravitational condition. Rephrasing this, a systemic or global curved space and motion *input* effect produces a systemic rotating curved space *output* — albeit a totally different phenomenon. Remember, that with the ‘internal’/QM effects of the celestial orbital motion essentially uniform throughout the bulk matter of the lunar body, quantum mechanical coherence behaviour exists. Thus, a new type of external ‘coordinated’ behaviour is conceivable — in what is termed the “environment” of the (collective) QM sub-systems.

Non-locality is necessary (so as) to allow the internal and external behaviours to coexist at (and over) the same (duration of) time (Δt).

5.7.3 The benefit of a rotating space-warp

It can be argued that the external physical effect/behaviour ensuing from the lunar ‘constituent’

²⁷³This is quantified in subsection 6.3.2.

atoms/molecules, allows these internal QM sub-systems to be effectively (and continuously) in a stable configuration — that coexists with ‘unforced’ movement along a geodesic. The rotating space-warp acts to screen the state of each QM sub-system (i.e. an atom or molecule) from the model’s new non-local (virtual) spin phase offset. Thus, in the presence of analog curved space-time, ‘nature’ *always* ensures the on-going internal stability of these QM systems — particularly in the rare circumstance of when a coherent effect exists *below* both: a minimum change in energy levels, and a phase-based decoherence threshold.

Although not physically *measurable*, the total (background/hidden) virtual QM energy *quantifies* the energy of the real (macroscopic) rotating space-warp — by way of a global principle of energy conservation.

5.7.4 Inertial energy cf. fictitious force

The following discussion draws upon the notion of inertial (or fictitious) forces²⁷⁴, e.g. centrifugal force. We shall (somewhat loosely) extend this notion to the energy discussed herein, by way of there being a rate of ‘change’ of (QM) momentum involved (albeit a non-instantaneous change/process). In what follows, we shall restrict the discussion of an inertial frame to the spin status of quantum mechanical systems.

The term “virtual energy” (as used previously) is now seen to be indicative of the inability of a quantum mechanical system to either: express or ‘match’ the (global) inertial requirements of spin energy. From a global/systemic perspective the energy is real: representing an *inertial energy*²⁷⁵; i.e. an ‘apparent’ energy resulting from the intrinsic angular *momentum* of atoms/molecules in a non-inertial frame of reference — albeit only upon the *completion* of a closed circuit or cycle (spanning a duration Δt). In the model, by way of a QM (cyclic) rate of intrinsic angular momentum change, we establish a relative (inertial to non-inertial frame) spin *energy* offset (i.e. discrepancy); with this energy magnitude now being alternatively understood, and referred to, as an ‘inertial’ energy.

Unlike the case with “inertial force” we are not primarily dealing with macroscopic/classical “mass in motion”²⁷⁶, nor ‘felt’²⁷⁷ fictitious/inertial forces, even though the energy is based upon an inertial to non-inertial frame distinction. At the microscopic/QM level, the non-inertial spin energy is not *locally* ‘registered’; (firstly) because it is a non-local systemic effect (below a ‘discrete’ minimum energy level difference), and (secondly) because there is no such thing as QM spin (rotational) *motion* in our understanding of: an elementary particle, a subatomic particle, and

an atom/molecule²⁷⁸. Nevertheless, total virtual QM *spin energy* is made (externally) physically real, and this new term “inertial energy” nicely encapsulates a core conceptual feature of the new mechanism.

One could possibly say the new mechanism exhibits “spinertia”; defined as the ability of a QM system, or a collection of QM sub-systems, to export its/their non-inertial (rate of intrinsic angular momentum) energy. This phenomenon requires: QM self-interference, 3-body orbital (celestial) motion in curved spacetime, a non-local (relative) geometric phase change, as well as other special requirements outlined in subsections 5.6.3, 5.6.7 and elsewhere.

5.7.5 Decoherence, and the statistically additive nature of total energy

The internal to external, and virtual to real, bifurcations of the spin (inertial) energy previously discussed is supported by and/or related to one central aspect of decoherence theory.

Standard QM decoherence theory speaks of the separation of a quantum system into a subsystem (the relevant/distinguished part) and its environment (the irrelevant/ignored part) (Joos et al. 2003, p.227). The (external space-time) environment plays a special role in our new mechanism. Further, we note that:

Decoherence follows from the irreversible coupling of the observed system to the outside world reservoir. In this process, the quantum superposition is turned into a statistical mixture, for which all the information on the system can be described in classical terms, so our usual perception of the world is recovered (Davidovich et al. 1996, introduction).

The classical concept of energy — so central to QMs — is fundamental to the model. To this we add that within “statistical physics” *energy* plays a primary and unique role, and total energy is attained *additively*.

Post-decoherence, QM effects are considered to be ‘delocalised’ rather than destroyed. Our proposed mechanism takes delocalisation (of mass especially) to a conceptual limit, by allowing both the constant acceleration/gravitational amplitude of the space-warp and the (de-localised or) non-local mass component to extend to the ‘end’ of the universe — albeit with the (non-local) mass²⁷⁹ at a ‘point’ in the field decreasing with the volume of space enclosed around a (third-body) moon (i.e. as r^{-3}).

Our denial of the graviton particle’s existence removes the standard particle-exchange means by which

²⁷⁴Fictitious forces arise from the *acceleration* of a non-inertial reference frame itself. They are proportional to mass, and do not arise from a (local) physical interaction.

²⁷⁵In the sense of inertial (or fictitious) *force*.

²⁷⁶Although we note that *macroscopically*, the motion of the atoms and molecules (slaved to a moon) is ‘along’ a (force-free) geodesic.

²⁷⁷Unless we entertain the notion that when this energy is expressed externally in an atom/molecule’s environment, it is ‘felt’ there — i.e. ‘experienced’ in the environment itself.

²⁷⁸Quoting Werner Heisenberg: “The atom of modern physics can be symbolized only through a partial differential equation in an abstract space of many dimensions. All its properties are inferential; no material properties can be directly attributed to it. That is to say, any picture of the atom that our imagination is able to invent is for that very reason defective. An understanding of the atomic world in that primary sensuous fashion ... is impossible”.

²⁷⁹As compared to gravitational mass (active or passive) or inertial mass.

QM decoherence could be ‘activated’. We shall consider the (neo-classical) space-warp as representative of QM *decoherence* only in the sense that it involves an internal to external (physical) ‘information’ exchange, and/or an internal to external (physical) “exchange” of energy ‘information’ — albeit an exchange that occurs ‘instantaneously’/non-locally²⁸⁰, is ongoing, and is irreversible (i.e. one-way only).

[Post-decoherence] (almost) all information about quantum phases has migrated into correlations with the environment and is thus no longer accessible in observations of the subsystem alone (Joos et al. 2003, p.227).

Indeed, for the model’s mechanism, its virtual phase information is never directly (i.e. locally) accessible²⁸¹. This is further examined in subsection 5.7.6 and it is not so much phase that ‘migrates’ into the environment, rather it is (some proportion of the minimum ‘internally expressible’) atomic/molecular spin energy ($\frac{1}{2}\hbar/\Delta t$) that is *exported* (*en masse*) into the environment (or surroundings).

Note that all elementary fermion particles within an atom/molecule share the same geometric phase offset; and it is this shared/common phase offset that applies to the (composite) atom/molecule (i.e. as a whole). This *atomic/molecular* spin phase offset, over a cyclic loop duration Δt , is indicative of a *non-inertial* rate of intrinsic angular momentum (i.e. spin energy).

5.7.6 Quantum coherence, environmental energy, and quantum entanglement

We shall now further consider a rotating space-warp as representative of *quantum coherence* in some way; thus implying that the effects of the internal (virtual) superposition are somehow ‘retained’. In subsection 5.7.5 we spoke of QM effects being delocalised rather than destroyed.

The effective equivalence of the (virtual spin) phase offset for all elementary fermion particles within an atom/molecule, and for all atoms/molecules in a moon, means that the phase offset (relative to that of an inertial frame) may be termed a ‘pure’ phase offset rather than a ‘mixed’ phase offset. Further, the total *energy* associated with this phase offset — which is proportional to the number of atoms/molecules in a moon — may be thought of as somewhat like a pure quantum ‘state’, albeit incapable of expression internally (i.e. within a bound atomic/molecular QM system). Such a pure coherent quantum state is well suited to a ‘singular’ form of expression, e.g. as an external QM ‘condensate’.

²⁸⁰ “Non-local” as in a process that acts at the noumenal/hidden background ‘level’, rather than at the phenomenal particle-exchange level (recall subsection 4.2.10); and involving an unforeseen and new/‘foreign’ (non-local) process that is yet to be fully understood — see section 6.6 for further discussion.

²⁸¹ Further, virtual phase is not there (at the QM level) to be measured, because (via non-local instantaneous ‘causation’) the external compensating rotating space-warp is present.

Consequently, we may think of the energy of a rotating space-warp, with its associated non-local mass distribution, (loosely speaking) as a single classical energy ‘state’ in the (QM systems’) environment — albeit a *process* that is expressed over a finite cycle time (Δt). Further, this (effectively) exact coherence ensures that the total energy of the rotating space-warp is equal to the simple additive *sum* of the individual (atomic/molecular) QM energies — with these being (effectively) all equal.

One can go on to say there is a type of quantum *entanglement* between the (perfectly coherent) pure (offset) ‘state’ of the (atomic/molecular) QM sub-systems together, and the (singular) energy process external to (i.e. in the environment of) this conglomerate (lunar) QM “super-system”. It is the internal *and* external domains/realms²⁸² of the full/whole (conglomerate) quantum system (i.e. a moon) that have to be described (non-locally) with reference to one another, in the sense that these domains are spatially separated²⁸³, and ‘instantaneously’ coordinated in time — as far as observational physics is concerned.

Significantly, although the two domains/realms are physically entwined, their formalisms remain distinct. Thus, we require both quantum mechanical quantities (especially \hbar) and classical quantities to express their (equal) energy magnitudes (see section 6.3). Indeed, the total internal/virtual energy is our *only* means for establishing the environment’s real energy magnitude — involving both the space-warp’s acceleration magnitude, and the amount of non-local mass (at a given radius) enclosed by a given volume.

Finally, for the new mechanism, let us additionally designate a (new) quantum “*phase transition*”, in the sense of a major qualitative change in system behaviour. Transition is dependent upon the amount of spin phase offset; with a solitary $> 2\pi$ phase offset (‘interior’ to any atom/molecule²⁸⁴) being enough to induce a comprehensive (full lunar body) *decoherence* ‘event’. The different actual spin axis orientations within different atoms/molecules (comprising a bulk lunar body) become apparent to the global system, and the (externally orientated *geometric*) phase offsets are exposed as mixed or non-pure. Thus, ‘post’-decoherence, i.e. $\beta > 2\pi$ over (a spin and) an orbital duration Δt , a common virtual phase offset becomes meaningless and the lunar-based rotating space-warps cannot occur. This is because the wiggle room allowed by (the minimum quantum angular momentum) $\frac{1}{2}\hbar$ in Heisenberg’s uncertainty principle has been breached, and subsequently a moon reverts back to non-QM (i.e. exclusively classical/macrosopic) behaviour — as is currently envisaged. At the celestial/macrosopic level

²⁸² Quantum entanglement is usually discussed in relation to two ‘objects’ as compared to a distinction between micro- and macrosopic realms/domains.

²⁸³ Thus, a dividing radius may exist to demarcate where the external far-field begins. This is seen to be loosely analogous to the EM wave situation where, at a particular distance (from an aperture or slit), Fresnel diffraction (also known as near-field diffraction) changes to Fraunhofer (far-field) diffraction.

²⁸⁴ Recalling that these are the largest bound quantum mechanical systems.

this is typical behaviour, as compared to the “exception to the rule” situation modelled herein.

5.7.7 Concluding and summary remarks for section 5.7

This subsection has built upon earlier work that involved: (1) recognising a hidden (or noumenal) global/system background process that observationally implies a form of time simultaneity, thus facilitating non-locality (subsection 4.2.10); (2) accepting global conservation of angular momentum and global conservation of energy (in particular), (with the latter) involving spin energy in particular; and (3) appreciating that a fractional analog geometric offset can not be expressed by a ‘digital’/quantum ($\frac{1}{2}\hbar$) spin effect.

For explanatory depth we have sought to extend certain QM concepts such as: coherence, condensate, decoherence, and entanglement into the model. With the geometric phase offset being (effectively) equal throughout a (suitable) non-decohered moon, as is (subsequently) the intrinsic angular momentum offset rate (per loop), the total virtual/proper-fractional (quantum) energy involved can (simply) be determined additively — by way of multiplying the single atomic/molecular energy value by the number of atoms/molecules in a moon (N_m).

This (total) energy is effectively a non-inertial energy, somewhat analogous to inertial/fictitious force effects. With a single atomic/molecular 2π phase offset triggering decoherence throughout a moon, this (2π) level of phase offset can be associated with a new type of (quantum mechanical) ‘phase’ transition. Further, the same rate of spin phase (and rate of intrinsic angular momentum) offset can be considered a coherent effect throughout the lunar system — albeit a virtual effect ‘internal’ to the moon, involving every single elementary fermion particle nested within *every single* (composite) atom and molecule²⁸⁵.

The (singular) resolution of this additive energy offset can be understood by way of an analogy to quantum decoherence, in that we make the universe a two-part system with the ‘relevant’ part being the lunar atoms/molecules, and the environment being all the (cosmological) space that is external to these ‘sub-systems’. The model suggests that: observations of these (atomic/molecular) sub-systems alone will yield no sign of their individual latent/virtual energies.

The external expression of the total lunar virtual energy as a real energy, in the form of a rotating space-warp (RSW) and non-local mass distribution, can be associated with a number of things. Firstly, it appeases conservation of (universal spin) energy; secondly, it illuminates the presence of non-local (and instantaneous) spin entanglement; thirdly, we may argue that the local (fermion-based) quantum coherence behaviour entails a (wave-like) condensate behaviour (i.e. the RSW); and fourthly, internal ‘information’ is *transferred* into the environment in an *irreversible* manner. This latter characteristic is further supported by the need for a (classical) *energy* basis as a means to (or

‘currency’ by which we can) globally/systemically relate the internal atomic/molecular QMs to an external neo-classical (new kind of gravitational/accelerational field) effect.

The mechanism/effect being proposed involves both: a constant acceleration/gravitational field amplitude (Δa), albeit sinusoidally varying at a given ‘fixed’ location in the field in response to the space-warp’s rotation around its source region/‘point’; and a variable non-local mass quantity [$m^*(r)$], that accounts for dispersion of the energy at increasing distances from its (lunar) source region — see Section 6 for further discussion and quantification.

We note that an *external* curved spacetime dependent effect upon (internal) atoms/molecules in (third-body) geodesic celestial motion has resulted in a (curved space-time) effect also *external* to these QM sub-systems of a lunar bulk-mass. The benefit of the presence of this “insourcing–outsourcing” scenario is that analog *geometric* phase changes and digital QM energy levels are never locally in conflict. In short, the environment ‘deals with’ (‘fall-out’ from) the non-flat spacetime based effect/circumstance, and thus ‘nature’ (*in toto*) is inherently stable in the presence of this potentially conflicting (analog vs. digital) situation.

6 General features and the quantification of the model

Having qualitatively justified the model, in Sections 2 to 5, we now turn to the model’s quantification. Firstly, we briefly discuss errors and encapsulate the model’s features; then (beginning at section 6.3) the ‘flesh’ of quantification is applied to this framework.

6.1 Discussion of errors and major features of the model

6.1.1 Looking ahead: steps to explaining the Pioneer anomaly

We have worked ‘backwards’ from the Pioneer observational evidence/constraints, and assumed a ‘real’ (non-heat based) anomaly. The model is able to empirically *match*, with the assistance of pure/classical geometry:

- the physical characteristics of moon-planet-Sun motions [e.g. lunar orbital period (Δt), lunar mass (or rather number of atoms/molecules), semi-major axis (length), planetary angular progression angle (per lunar orbit), etc.];
- *to the* geometric phase offsets (β) experienced collectively (in a given moon) by all of its atoms and molecules (N_m), relative to the optimum²⁸⁶ (π rad) phase offset. This offset is expressed as an energy efficiency factor η (where $0 \leq \eta \leq 1$).
- This efficiency factor then ‘quantifies’ the total (virtual) asymmetric/non-inertial internal energy (ΔE_w), which is related to a time/process-based (maximum) energy uncertainty within

²⁸⁵That is, assuming the (non-rigid) moon is completely comprised of ‘solid’ material (cf. ‘fluid’ material).

²⁸⁶As in having the maximum possible physical effect.

atoms/molecules; (and) via conservation of energy, ΔE_w (the exact externalised energy) is expressed as an external rotating space-warp and an (initial/inception) non-local mass (m_1^*).

- The external energy of each suitable moon is dependent upon: a (*squared* value of) acceleration/gravitational undulation *amplitude* (Δa), a process time (Δt), and a non-local²⁸⁷ mass ‘continuity’/distribution quantification — again by way of employing (classical) plane geometry.
- Together, by way of superposition, several of these rotating space-warps, of different period (Δt) and amplitude (Δa), result in moving ‘low mass’ (celestial) bodies experiencing oscillatory/unsteady speed perturbations — around an equilibrium speed value governed by general relativistic gravitation (recall section 3.2).
- This superposition of individual unsteady motion (perturbation) effects, by way of the coexistence of multiple rotating space-warps, ‘causes’ the overall shortfall (cf. predictions) in motion (over time) of the Pioneer spacecraft — i.e. the Pioneer (acceleration) anomaly (a_p)²⁸⁸.

Although the approach used is semi-classical, conceptually ‘vivid’ and empirical, it fits all the (awkward) observational evidence (see section 2.4). Additionally, it has predictive capability (see section 6.2), and a significant further (dark energy related) ramification (developed in Section 7). Curved space-time, independent of special relativity, plays a vital role. While such a method is possibly anathema to mathematical physicists, this approach is in all likelihood merely an intermediate step along a path to fully understanding how the microscopic and macroscopic domains, described by QMs and GR, interrelate in the physical world — to yield something new, unexpected, and progressive.

6.1.2 Interim summary: Core issues of the model encapsulated

Arguably, *the* core issue is that: in the presence of curved spacetime, a macroscopic object comprising innumerable quantum mechanical (atomic/molecular) sub-systems can engage with its external surroundings to maintain its on-going (‘internal’) stability. Thus, each sub-system is able to “carry on” (locally) as if the non-local effects of curved spacetime upon its geometric phase, acting in concert with background global space-time topological aspects, were absent.

The external expression of the (shared/common) internal (virtual) spin phase offset²⁸⁹ involves both:

²⁸⁷Non-local also in the sense of a wave’s energy distribution being distributed over a region of space.

²⁸⁸Larger objects, such as moons and planets, are not affected because their mass is above the cut-off masses of the dominant (RSW ‘generating’) moon-planet systems — i.e. Jupiter’s: Io, Europa, Ganymede, and Callisto; Saturn’s Titan; and to a much lesser extent Neptune’s Triton (see subsection 6.6.6 and Table 7).

²⁸⁹The (virtual) spin phase offset is ‘relative’ (in a broad sense of the word) to three things: i) actual spin-orbit (coupled) phases; ii) a background datum; and iii) itself, regarding: initial and (after Δt) ‘final’ cyclic-loop motion status.

an external (gravito-quantum) rotating space-warp, quantified by a *specific* energy; and an external mass dilatory effect (which is volume dependent). Together, these external effects ‘match’ (and substitute for) the total internal “*under-spin*” energy (per loop/cycle) — with the latter being a *virtual* energy. Note that the phase offset is associated with an angular momentum offset, and that the under-spin is relative to an inertial circumstance/configuration. Alternatively, we may say that an inertial frame *over-spins* relative to the actual QM spin phase — which is held in spin-orbit coupling by (the dominating) electromagnetic forces.

Ignoring special relativistic effects, the virtual internal and *real* external effects coexist in ‘universal’²⁹⁰ real time²⁹¹ (alternatively, a ‘systemic simultaneous pseudotime’²⁹²); with the space-warp rotating at the same rate, and in the same rotational direction/sense, as a moon’s spin. Note that additionally a (‘hidden background’) global/systemic frame is required to exist, and that the *measured* effects of special and general relativity must be upheld²⁹³. All in all, conservation of the *full* system’s energy is maintained by the field-based ‘reaction’ — to each (moon-planet-Sun) system’s (total) virtual energy offset/shortfall.

The presence of both: shallow gravitational wells (pertaining to moons and planets), and a spin-orbit coupled moon-planet system (orbiting around a sun) are required — with the moon being the *third* celestial body. Energy is seen as the linchpin that relates a mechanical aspect of the micro/QM world to the macro (‘gravitational’) world. The need to accept a (specific) “energy of a rotating space-warp” is unorthodox; further, the additional acceleration/gravitational field associated with this energy is supplementary to, and independent of, GR’s spacetime curvature. Thus, we are transcending the implicit restrictions of GR’s general principle of relativity, and the sole use of a metric theory to explain ‘gravitational’ influences — in their widest sense. This is further discussed in section 6.7.

It is *important* to note that locally the asymmetry-based or offset-based virtual QM energy, although conceivable, does *not* physically/actually “exist”. What does exist (in its place) is the external rotating space-warp phenomenon. Importantly, the internal virtual (and non-inertial) energy acts to quantify the external energy magnitude. The importance of a ‘universal’

²⁹⁰The systemic (or global) perspective for rotating space-warps ‘takes in’ the whole universe.

²⁹¹Recalling section 4.4, this is a (hidden) background time utilised by the Universe ‘in itself’, especially for QM non-local effects. It is not, and cannot be, indicated by direct observational measurements involving EM radiation.

²⁹²By way of the American physicist John Cramer, we may alternatively refer to this ‘other’ time as a *pseudotime*. In words appearing upon John Gribbin’s homepage (click on “Quantum mysteries”, then “Solving the mysteries”): “Cramer’s pseudotime is a semantic device allowing us to stand outside of (observational) time.” As far as clocks in the everyday world are concerned, any (pseudo-temporal) ‘gaps’ between digital sequential moments/frames (of observational/physical reality) are instantaneous — by way of them lying beyond observation-based ‘apprehension’.

²⁹³They *are* upheld; (in part) because, on average and ‘overall’, the general relativistic gravitational field’s strength is ‘unchanged’ by the space-warp’s presence.

conservation of energy principle, to cope with this situation, is paramount. Thus, at all times and in all circumstances, the *mass* based aspect (cf. electrical charge based aspect) of QM systems moving in curved spacetime is *stable*. ‘All up’, (discrete) QM systems obey SR, and can always cope with the additional effects pertaining to: (analog) GR and Heisenberg’s (energy and time based) uncertainty principle ‘demonstrated’ herein — at least in the universe’s ‘mass era’.

The restriction of the energy magnitude to that: spanning/within a finite time loop/cycle (Δt), and involving an angular momentum less than (half) Dirac’s constant ($\frac{1}{2}\hbar$), assists (systemic) stability — in that things can only be minimally perturbed from an equilibrium circumstance. By virtue of these (and other) circumstances, the Pioneer anomaly has an extremely small magnitude — notwithstanding the vast number of atoms/molecules in each (contributing) moon.

6.1.3 Error discussion and aspects of the model’s idealisation

This subsection highlights how the Pioneer anomaly’s error far exceeds the *numerical* error, and comfortably exceeds the total error, of the/our *idealised* model. Note that the Pioneer anomaly’s magnitude-to-error ratio is 8.74:1.33 (with both values $\times 10^{-8}$ cm s $^{-2}$). Known to many more significant figures are the physical quantities in the model’s quantification: Planck’s (and Dirac’s) constant, lunar masses, celestial body orbital periods and distances. Further, the model’s geometric approach uses an *exact* (celestial) angle: $\tan^{-1}(8\pi)^{-1}$, and is free of adjustable parameters, i.e. free of parametrization²⁹⁴. Subsequently, the uncertainty or ‘error’ in the model’s (eventual) quantification of total anomalous acceleration (a_p) is well below the ‘observational’ error attributable to the Pioneer spacecraft’s (measured) anomalous acceleration (a_P).

Obviously, if the model is flawed, then this small error is unjustified. Loosely speaking, the conceptual modelling ‘error’ (or rather inaccuracy) is (potentially) very much greater than the model’s numerical error.

The model, at this stage, is idealised and restricted; in point form these (‘simplifications’) are:

- i) Solar mass \gg planetary mass \gg lunar mass; noting that the Earth’s moon shall fail to be a (rotating) space-warp ‘generator’.
- ii) The relationship between lunar mass and its number of constituent atoms/molecules is based upon Carbon 12 (and any alteration in this relationship due to binding energy is considered negligible).
- iii) Large moons dominate the anomaly: Jupiter’s four Galilean moons and Saturn’s Titan almost completely quantify the anomaly.
- iv) Moons are considered to be (non-rigid) solid bodies, so as to not compromise the pure effect of a common geometric phase offset. The minor ramification of a (possible) variation from this idealisation is discussed in subsection 6.5.11.

²⁹⁴Should the observational value of the Pioneer anomaly be revised significantly, the model — with its a_p value — *could not* be adjusted to fit this altered a_P value.

- v) Orbits are considered to be circular. The influence of (small) planetary and (very small) lunar orbital eccentricities is negligible (and conceivably nonexistent) — see subsections 5.2.3 and 6.3.6.
- vi) In the global/systemic reference frame, over a single lunar cycle (which is always < 17 days), any change of the Pioneer spacecraft’s (barycentric) position, trajectory, and hence observational inclination angle, has a negligible influence upon the anomalous acceleration (i.e. *rate* of speed short-fall).

Assuming the idealised model is valid and (later) adjustments/corrections to it (see Table 6) are also valid, the computational/numerical errors are very small ($< 0.1 \times 10^{-8}$ cm s $^{-2}$). It is the veracity of the model that is primarily at stake here, and further discussion of the model’s numerical errors is not pursued.

A possible modelling error also lies in corrections to the idealised model. Later (in Table 6) we see that the adjustments/corrections to the idealised model for Pioneer 10 are up to 0.56×10^{-8} cm s $^{-2}$. Thus, we get a (conservative) *total error* magnitude of 0.66×10^{-8} cm s $^{-2}$, which is about half the observational and experimental error of 1.33×10^{-8} cm s $^{-2}$. If the idealised model’s corrections are valid, then a more refined (i.e. less idealised) model may well be accurate to better than 0.10×10^{-8} cm s $^{-2}$.

6.1.4 The relationship between: QMs, GR, intrinsic ‘spin’ and conservation

For our model, the concept of intrinsic spin has allowed an inter-linking between: (micro) QMs, (macro) celestial motion, and ‘gravitation’ (in its broadest sense); whereas gauge field theoretical approaches have (to date) not achieved *this* type of (quantum mechanical to gravitational) link, together with falsifiable predictions involving observable physical phenomena. This linkage is achieved by way of a deeply conceptual model involving: micro- and macroscopic spin-orbit coupling²⁹⁵, a relative (virtual) phase effect, (together with) a non-inertial vs. inertial frame offset, conservation of (systemic) spin energy, and a denial of the graviton particle’s existence — amongst other things.

A non-*special* relativistic formal representation of quantum mechanical spin, in an atomic/molecular system, is independent of (and additional to) Schrödinger’s equation²⁹⁶. Similarly, a virtual (lunar or) *third-body*-based (intrinsic) over-spin — which itself is dependent upon: a common (to all atoms/molecules) fermion spin phase offset, quantum entanglement, and self-interference — is independent of (and beyond) *general* relativity’s formalism, but not necessarily curved spacetime *per se*.

Note that the (tiny) Lense-Thirring effect arises from a *central* body’s spin, and similarly, the spin-down of a binary pulsar (or dual-star) by way of grav-

²⁹⁵The latter involving the second and third bodies of a (sun-planet-moon) three-body celestial system.

²⁹⁶Note that Dirac’s equation incorporates *special* relativistic effects, but not general relativistic effects, and it provides a description of elementary spin- $\frac{1}{2}$ particles.

itational wave ‘output’ is also a *two-body* general relativistic effect. The new mechanism features both: tidal effects of non-point-like (moon/planet) masses, and requires a *three-body* celestial system; both of these features lie outside the ‘umbrella’ of GR’s applicability.

It is as if (QM) intrinsic spin’s failure to be a part of either Schrödinger’s equation or GR’s formalism (re: a third-body) has paved the way for a micro-macro spin (conservation) relationship to mitigate any effects arising from a (digital) QM system existing in (analog) curved spacetime — all the while allowing Schrödinger’s (and Dirac’s) equation and GR’s formalism to dominate their essentially separate domains. In short, the general relationship between QMs and gravitation appears to be one of (almost completely) detached coexistence, rather than fully entwined (reductive) unification; and in order to establish our (admittedly minor) physical link between gravitation and QMs, a common ontological ground is required. The dual-ontology pursued in this paper achieves this (see section 4.5). Also required is a reassertion of the need for a global barycentric reference frame²⁹⁷, and the resurrection of conservation of angular momentum and energy principles (section 4.6) that span both the micro- and macroscopic realms (at the same time).

Interestingly, a global (classical-like) conservation of angular momentum holds impressively in a (macroscopic only) moon-planet system. The Earth’s slowing spin rate leads to the Moon’s orbital radius (and period) increasing — by way of celestial spin-orbit coupling in conjunction with tidal effects and tidal locking.

... empirical observations, many of which actually come from analysis of the LLR [Lunar laser ranging] data itself, suggest that these conservation laws [energy, momentum, and angular momentum] hold to high precision (Nordtvedt 1999, A104-5).

Note that this comment is made in spite of the fact that for GR there is not a general law of energy conservation, just a restricted version involving an $r \rightarrow \infty$ requirement (recall subsection 4.6.5).

6.2 Extensions to, and peripheral issues influenced by, the model

In this section a number of the model’s pragmatic features are briefly addressed, including: its extension into three spatial dimensions, predictions, future observations of interest, and ramifications of the model upon the current understanding of various issues.

Note that any *change* in a Sun-planet-moon’s geometrical (configuration) circumstances, that impacts upon the offset/imbalance energy magnitude (ΔE_w) and hence its associated rotating space-warp, is propagated at the speed of light. Over human and spacecraft lifetimes, the effect of changes in planetary and lunar orbital characteristics upon the model/mechanism (as presented herein) are negligible.

²⁹⁷In conjunction with a systemic Barycentric Dynamical Time (TDB), with this being how NASA treats the *observations* of spacecraft in the outer solar system — cf. a (theory of) general relativity based approach to S/C motion.

6.2.1 Extending the model into three spatial dimensions

We need to distinguish three different aspects of the model’s extension into three spatial dimensions:

- i) an internal quantum mechanical spin aspect;
- ii) an orthogonal extension to the (external) planar rotating space-warp (discussed in Section 3);
- iii) and motion of a spacecraft at an angle to the plane of space-warp rotation.

The orbital basis for a moon-planet offset in (overall²⁹⁸) atomic/molecular geometric phase (β), and subsequently virtual intrinsic angular momentum (S_z), only considers a (planar) projection of spin. The ensuing/associated rotating space-warp is initially idealised as a two dimensional planar phenomenon. Due to the virtual nature of the QM energy, corrections for the (internal) three dimensionality of atoms/molecules, and their total spin magnitude, are *not* required.

The extension of the two-dimensional space-warp, we have been considering (up to this point), into three spatial dimensions is addressed in subsection 6.2.2.

Fortuitously, for (spacecraft) motion at an angle to the lunar (and space-warp) plane of rotation, the motion shortfall along the spacecraft’s *path* is unaffected²⁹⁹; but a minor correction for the *geometric* inclination angle of the spacecraft path to the observational line-of-sight is required (see subsection 6.5.8 and Table 6). This correction employs the cosine of the angle(s) between spacecraft path and line-of-sight measurement. Two orthogonal angles are required: the first (longitudinal) within the ecliptic plane, and the second (latitudinal) involving inclination above or below the ecliptic plane.

6.2.2 Extending the planar rotating space-warp into three dimensions

If we assume some form of analogy with irrotational, non-viscous fluid flow (i.e. potential flow), then by way of Kelvin’s circulation theorem and Helmholtz’s theorems two things follow. Firstly, the strength of the rotating space-warp, as indicated by the acceleration amplitude, is constant along its length — i.e. orthogonal to the two-dimensional warp-plane. Secondly, because the space-warp does not start or end on a boundary, it either extends to infinity or forms a universal sized closed loop. Thus, the (infinitesimally thin) planar rotating space-warp, implied by observations and hypothesised in Section 3, is necessarily a thickly rotating space-warp that effectively extends to ‘infinity’ — in the warp plane and also orthogo-

²⁹⁸By way of the same geometric phase offset applying to all elementary fermion/matter particles within each atomic/molecular quantum mechanical system.

²⁹⁹The Pioneer spacecraft (S/C), whether moving parallel to the lunar orbital plane, or at an angle to it, will ‘experience’ the full strength of the space-warp’s cyclical acceleration amplitude. In subsection 6.2.2 we see that regardless of the spacecraft’s distance above or below the ecliptic plane, the retardation effect (upon the S/C) is also the full effect.

nal to this plane³⁰⁰ (see Figure 8). Energy dissipation/dispersion, involving the dispersion of (non-local) *mass* rather than (constant/uniform) specific energy, is discussed in section 6.6.

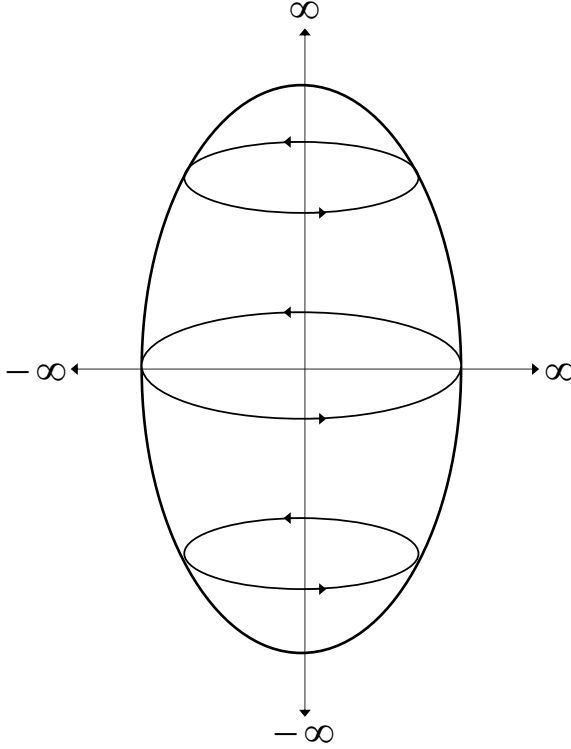


Figure 8: Schematic diagram illustrating the cosmological extent of the rotating space-warps in three spatial dimensions. Each and every horizontal sectional disk extends to ‘infinity’, with an orthogonal depth (upwards and downwards) that also extends to ‘infinity’ — i.e. the end of the universe.

The extension to a volume from a plane is not problematic because we are dealing with a constant specific energy (at all points throughout a plane) when quantifying the space-warp, i.e. $\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2$ (recall subsections 3.2.6 and 3.2.11). That the effect extends to ‘infinity’ above and below the lunar orbital plane was somewhat unexpected, but it appears necessary if a (space) continuum mechanics approach is employed. Compatibility with GR, via Δe ’s (universal) invariance, is maintained.

Interestingly, pursuing this conceptual analogy implies that compressibility effects in fluid mechanics might be analogous to special relativity’s effects in space continuum mechanics, which is compatible with our previous contention that ‘measured time’ is not necessarily all there is to ‘time’. The orthogonal extension of the space-warp to ‘infinity’ (or the end of the universe) is consistent with our stance upon background time simultaneity (recall section 4.4) — which

³⁰⁰This extension to infinity, or the end of the universe, might be mitigated by unforeseen/unknown effects.

was introduced so as to ‘deal with’ and more fully understand the notions of QM (spin) entanglement and non-locality, both in general and in the model.

6.2.3 Briefly on different observations to improve our understanding of a_p

There are a number of future space missions whose data may be beneficial to an analysis of the Pioneer anomaly.

- i) A dedicated mission to examine the Pioneer anomaly, e.g. Dittus, Turyshev, Lämmerzahl, & et al. (2005).
- ii) The New Horizons mission³⁰¹, especially an investigation of the “Saturn jump” discussed in subsection 3.5.4 and Nieto (2008). Is the change in anomalous ‘acceleration’ related to the Saturn encounter itself, or is it simply a distance from the Sun effect? Assuming the former, does the Pioneer anomaly begin at Saturn encounter or do we have a sudden large increase from a much lower (non-zero) value? Herein, the latter scenario is consistent with the model.
- iii) Further detailed analysis of past and future Earth flybys³⁰² by spacecraft³⁰³, e.g. the “Deep Impact” spacecraft in January 2008 and 2009. A clear confirmation of a real anomalous Earth flyby effect, in preferably only some flyby cases, would possibly lend support to a real (non-heat based) Pioneer anomaly. This view, and also the existence of anomalous *decreases* in K.E., is supported by the papers of Anderson et al. (2008) and Nieto & Anderson (2009, Section 3) — although Turyshev & Toth (2009, p.174) are sceptical. See subsection 6.2.4 for further discussion.
- iv) Performance of the LISA Pathfinder mission [a precursor to the Laser Interferometer Space Antenna (LISA) mission]. The accurate positioning of the spacecraft (at the Lagrange 1 point), to a few millionths of a millimetre in space³⁰⁴, should be affected by the anomalous acceleration (a_p).

Finally, for a comprehensive overview of space missions (past, present, and future) the Lämmerzahl (2005) article is recommended. Space-based (cf. Earth-based) missions investigating various features of general relativity and issues fundamental to physics are outlined and categorised. Interestingly, he mentions³⁰⁵ in the summary: “... [observational] searches for anomalous couplings of spin-particles with gravity.” This type of ‘interaction’ is closely related to, although subtly different from, the mechanism modelled herein.

³⁰¹This spacecraft lacks the navigational accuracy and precision of Pioneer 10 and 11 due to thermal radiation effects, arising from the proximity of the power supply to the spacecraft, but nevertheless its motion allows certain hypotheses for the Pioneer anomaly to be ruled out.

³⁰²Occasionally referred to as an Earth ‘swingby’.

³⁰³The gravitational field of the Earth is much more accurately known, as compared to other planets such as Mars.

³⁰⁴Based upon an article by Will Gater in *Astronomy Now*, August 07, p.12.

³⁰⁵In relation to experiments examining possible violations of Local Lorentz Invariance.

6.2.4 A conceivable extension of the model to explain the Earth flyby anomaly

There are two points of view regarding the “Earth flyby anomaly”. Firstly, a sceptical stance as favoured by P.G. Andreasian and S.G. Turyshev³⁰⁶, involving an error in the computer code used to shift between Earth-bound and space-based coordinate systems. This is done quickly in the flyby and they believe flawed modelling results in a velocity-dependent and (geocentric) latitude-dependent error. Alternatively, Anderson et al. (2008) report that: “...the Earth flyby anomaly is a real effect inherent to the tracking of spacecraft”. The difference of opinion is stark.

The empirical relationship proposed by Anderson et al. (2008) describing six Earth flyby events (for five spacecraft) is simple and engaging. The remainder of this subsection outlines how the rotating space-warps (RSWs), proposed herein to explain the existence of a real Pioneer anomaly, are compatible with a real Earth flyby anomaly. What is required is that the far-field (roughly) ecliptic plane orientation of the RSWs be locally (i.e. near field) *distorted* so as to be aligned with the Earth’s equatorial plane. Considering the gravitational acceleration at the Earth’s surface (9.81 ms^{-2}) is over ten billion times stronger than the acceleration (amplitude) associated with RSWs, local deformation/refraction of the plane of the RSWs in the vicinity of a high mass body is conceivable. Also, by aligning with the Earth’s equatorial plane, the RSW is not inclined to the orientation of the Earth’s Lense-Thirring effect, and a “least action” circumstance prevails.

A distinctly local distortion of the RSW’s orientation in the vicinity of the Earth might also explain the apparent distance/altitude dependence of the flyby anomaly, and (thus) why an anomalistic effect might be acting over perhaps ± 10 hours of perigee — as discussed in Section 1 of Anderson & Nieto (2010).

Appreciating that the Pioneer anomaly (as modelled herein) is associated with both a speed *and* distance shortfall per cycle time of a RSW, and thus a distance travelled by a spacecraft (S/C) approaching (and exiting an) Earth flyby encounter, we can establish a length ratio ($\Delta L/L_\infty$). This dimensionless ratio equates to the speed ratio³⁰⁷ ($\Delta V/V_\infty$) at a standard time rate, as utilised in Equation 2 of Anderson et al. (2008). The ratio ($\Delta V/V_\infty$) is proportional to an (Earth-based) proportionality constant and a *difference* in (the cosine) of geocentric inbound and outbound inclination angles.

This difference may be related to a spacecraft’s inclination to (both the RSWs and) the equatorial plane of the Earth (at sufficiently low altitudes). For in-plane S/C motion the *observed* (Pioneer-like) shortfall is maximised. With increasing angles to the plane of the RSWs (and Earth equatorial plane) the speed shortfall, as measured by a line-of-sight observation, is

reduced — in proportion to the cosine of the (geometric) inclination angle. Thus, for a flyby encounter, a reduction in geocentric *equatorial* latitude inclination (post- vs. pre-encounter) leads to a greater (measured) shortfall and thus an apparent anomalous *reduction* in osculating velocity *at* encounter is observed³⁰⁸. This change takes the pre- and post-encounter change of (measured) far-field geodesic motion circumstances into ‘account’. In the literature anomalous velocity *increases* are highlighted, indicating a reduction in the *measurable* (Pioneer-like) anomalous speed shortfall at higher (post- vs. pre-encounter) inclination angles. Note that in the far-field — sufficiently far away from the Earth’s gravitational field — a RSW’s orientation refracts/reverts back to its lunar-based plane.

In planetary flybys the ability of the rotating space-warp hypothesis to provisionally accommodate: the sudden speed variation ‘around’ encounter; the existence of both anomalous increases *and* decreases in osculating velocity; and the simple *geometric* (directed) motion relationship presented in Anderson et al. (2008) is a promising, but by no means (exhaustive or) conclusive, account of the situation. A systematic effect may still easily explain away the flyby anomaly, but the preceding explanation, based upon a quirk relating to line-of-sight observations rather than a (kinetic) energy conservation defying change in speed, could satisfy both sceptics and believers/accepters alike.

This application of our Pioneer anomaly based model appears to fill the explanatory vacuum that currently exists with regard to the Earth flyby anomaly.

6.2.5 A brief outline of solar system-based predictions arising from the model

- i) A sharp increase in a_p (for Pioneer 11) post-Saturn encounter, cf. an (initial) onset of the anomaly beyond Saturn — recall subsection 3.5.4.
- ii) In reply to oscillatory perturbations in acceleration/gravitational field strength arising from rotating space-warps, the LISA Pathfinder spacecraft should experience both: a tiny (anomalous) reduction in speed, together with some (tiny) quasi-stochastic changes in location around the Lagrange point equilibrium value — due to the underlying (superpositioned effects of) variations in field strength and hence LISA spacecraft speed.
- iii) A comparison of paths involving two asteroids of different mass, where one is experiencing the Pioneer anomaly and the other (larger asteroid) is of too great a mass to be affected. This relative comparison would suffice to indicate if a “cut-off” mass exists in support of the model³⁰⁹. See sub-

³⁰⁸With $\Delta t \rightarrow \text{small}$ (for this process), this gives the impression that the temporal duration is instantaneous.

³⁰⁹Without the benefit of an Earth-pointed antenna, it is not possible for the navigational tracking of single asteroids to match the precision and accuracy of ‘in situ’ spacecraft navigational tracking. Onboard the spacecraft the (second) ‘in situ’ antenna (by way of a transponder) is in ‘phase lock’ with an Earth-based (transmitting and receiving) antenna, which is referenced to a very accurate Hydrogen maser frequency standard. This makes a world of difference to nav-

³⁰⁶As reported by Richard A. Lovett in Issue 21 of the Australian popular science magazine “Cosmos” (pp.78-81); (titled) “Magical mystery tour: The Pioneer anomaly”.

³⁰⁷Note the difference in the order of magnitude of these quantities: ΔV is measured to the order of [mms^{-1}], whereas V_∞ is in the order of [km s^{-1}].

section 6.6.10 for further discussion.

6.2.6 A brief list of other (possible) ramifications arising from the model

The proposal of something in our solar system as noteworthy as a (gravito-quantum) rotating space-warp — that extends to infinity or the end of the universe³¹⁰ — (unavoidably) has significant effects upon numerous things, including the galactic and cosmological motion of low mass bodies (and photons). The space-warp's influence upon a variety of issues is now listed and some brief comments are made. A fuller exposé shall not be given herein. These implications remain underdeveloped and largely unsubstantiated at this stage.

- i) **Solar system formation.** Recall section 2.6.
- ii) **Asteroid crater size distribution.**
- iii) **Near-Earth object (NEO) risk assessment.**
- iv) **Spiral galaxy density waves.** An appreciation that there may be more than simply Newtonian Mechanics (and dark matter) 'at work' (gravitationally) in the galaxy, opens the way for other energy expressions of/upon gravitational fields. There is a distinct similarity between a spiral density wave and a rotating space-warp³¹¹ — in terms of (both of them) being a *rotating* perturbation upon a stable gravitational field.
- v) **Dark matter (DM).** Once again going beyond Newtonian Mechanics (NMs), it may be that (after stripping out DM effects) the gravitational potential well of a galaxy is not as per NMs. A galaxy comprises approximately 100 billion stars fairly evenly distributed throughout. The extrapolation of 2-body NMs from the solar system [with a dominant central Sun (comprising 99.86% of the total mass of the solar system)] to galaxies, may not be valid. Hitherto unforeseen field interaction effects may exist³¹², and thus dark matter might simply indicate a misconceiving of galactic gravitational field strength. This is speculative, but the lack of direct detection to date of (diffuse) DM, in either Earth-based laboratories or the outer space of the solar system, implies that DM may not necessarily 'be' a missing non-baryonic 'particle'.
- vi) **Cosmic microwave background radiation (CMB radiation).** Recall subsection 3.5.3. In particular, that a foreground effect may arise from the anisotropic dipole *motion* of the solar system

together with the existence of the model's (largest possible scale) 360°-rotation symmetric rotating space-warps (RSWs). Note that the latter, by way of Jupiter's Galilean moons, are primarily aligned with the ecliptic plane of the solar system. Furthermore, the wavelike nature of the RSWs (with amplitude Δa and energy $\propto \Delta a^2$) could account for the unexplained alignment of the quadrupole ($l = 2$) and octopole ($l = 3$) modes with each other and the ecliptic plane.

- vii) **Dark energy and cosmological accelerating expansion.** In Section 7 a further ramification is discussed, pertaining to electromagnetic radiation/photons, rather than the retarding effect of oscillatory acceleration perturbations (Δa) upon a mass in motion. Unlike the anisotropic *motion* based influence upon CMB radiation previously discussed, this further effect is *energy* based and isotropic — and pertains to EM radiation received from type 1a supernovae (standard candles). The (spherically symmetric) *non-local mass* (and hence energy) distribution 'surrounding' a GQ-RSW alters the energy (and hence frequency) of approaching photons — with the acceleration perturbations playing no role whatsoever. The implication is that dark energy may be a misinterpretation of the (redshift-based) observational evidence, thus resurrecting the possibility that the expansion of the universe is actually decelerating — a far from outrageous suggestion, albeit at odds with some aspects of the prevailing "concordance model". Section 7 expands upon this brief synopsis; it is exclusively devoted to this issue.
- viii) **Change in the fine structure constant ($\Delta\alpha$).** On a lesser note, an admittedly speculative and brief conjecture regarding this open cosmological issue follows. Of relevance may be the finding herein that spin geometric phase, but not orbital phase, can be affected by the geodesic motion (of atoms/molecules) in an external gravitational field — specifically, where the atoms/molecules are (a part of) the third body of a rotating three-body (celestial) mass system. Under the appropriate geometric conditions, this circumstance might disrupt the (standard and cosmologically invariant) energy differences pertaining to QM spin-orbital transitions 'within' an atom/molecule; thus affecting emission and absorption events. The 'fractional' quantum energy change involved would (then) give the (observational) impression that the fine structure constant (α) has changed from its standard laboratory condition³¹³. A good background reference is Murphy, Webb, & Flambaum (2003)³¹⁴.

igational precision and accuracy.

³¹⁰Both in the plane of rotation and orthogonal to this plane.

³¹¹Jerry Sellwood (Scientific American 21st Oct 1999, Ask the Experts) says that: "Fortunately, nearly everyone agrees that spiral density patterns extract gravitational energy from the field of a galaxy." This view is not inconsistent with the contention of Prokhovnik (1978) that: "The gravitational property of matter is associated with an energy field imbedded in a cosmological substratum."

³¹²Supported by a belief that NMs and GR are not the 'last word' in gravitation — if (gravito-quantum) rotating space-warps and non-local mass distributions are valid physical phenomena.

³¹³This mechanistic approach to $\Delta\alpha$ regarding *atoms and molecules* is indirectly supported by a fairly recent null result (King, Webb, Murphy, & Carswell 2008) concerning the cosmological evolution of another major dimensionless constant, the proton-to-electron mass ratio (μ). Admittedly, the status of changes in dimensionless constants is made uncertain by way of the complexities involved in processing the data.

³¹⁴Their paper, (and other) "change in α " numerical

As can be seen from this list of implications, the influence of (cosmological size) rotating space-warps — each with an attendant (or conjoint) non-local mass distribution — is conceivably quite far reaching and significant.

6.2.7 Spacetime curvature energy & the model’s denial of a graviton particle

In analogy with electromagnetic radiation ‘waves’: in GR, a time-varying mass-energy distribution leads to a time-varying gravitational field, i.e. GR’s gravitation waves. Associated with this is a loss of angular momentum in binary pulsars. The latter can occur with no mass actually lost from the celestial bodies themselves, and thus we are dealing with a loss of *specific* orbital momentum and energy by way of specific energy ‘radiated’ into the gravitational field.

There is a clear distinction between saying that:

The curvature of spacetime is itself a form of energy, which produces its own gravitational field (Harrison 2000, p.229).

and the model’s stance that: the re-expression (or re-distribution) of non-inertial (QM) energy, in a universal system as a (macroscopic) curvature of space-time, does *not* then also produce its own gravitational field. Herein the first of these two stances is discouraged.

The assumption of a graviton particle with an energy flux and momentum flux, in analogy with quantum field theory and the photon in EM, remains to be verified. In the absence of a graviton particle, the transport of ‘momentum’ within the field is (technically) not possible. In subsections: 4.2.9, 5.1.2, 5.2.8, 5.6.2, 5.7.1 and 5.7.5 we have argued that the model actually depends upon the non-existence of the hypothetical graviton (elementary particle) — at least as far as it being a decoherence ‘agent’ in the model.

We are restricting the model’s ‘physicality’ to energy — free of any related ‘force’. In subsection 6.6 the nature of the mass unit or ‘dimension’, within the physical quantities of energy and force, is more thoroughly investigated. It shall become evident (later) that although a rotating space-warp and its associated non-local mass distribution has an energy representation, the model’s mechanism does *not* have (nor need) an associated local (particle) momentum aspect.

6.2.8 A brief speculation concerning the origin of inertial effects

The discussion in this subsection is decidedly *speculative*, but worthy of brief mention, due to the ongoing debate that surrounds the basis of/inertial effects.

That inertial effects require and relate to a global reference frame is herein considered unavoidable. Additionally, macroscopic mass is considered to be (in one sense) simply a summation of atoms and molecules. Thus, with regard to inertia, the *quantity* of mass in QM bound systems shall not specifically concern us;

results, are not inconsistent with a mechanistic approach that conjectures the (fractional quantum) “wobble room” is bounded by: $|\Delta\alpha| \leq (\alpha/\pi)^2$ where $(\alpha/\pi)^2 \approx 5.4 \times 10^{-6}$.

rather, it is the *number* of atoms/molecules (and their superposition) that is of major concern. The difference is subtle, but it is conceptually important. The aim here is to try and ‘reduce’ the macroscopic concept of inertia to non-macroscopic (i.e. QM) circumstances. Note that our concern is *not* with the “origin of mass”.

The content and discussion of Section 5 implies that ‘inertia’, i.e. a resistance to a deviation away from either rest or uniform motion, might have something to do with a many ‘microscopic’ body system (i.e. particles, atoms, molecules) ‘settling’ into some type of (minimum energy) global phase coherence — much in the manner that ocean waves, in the absence of wind and other disturbances, settle into a (single frequency and amplitude) “ground swell”³¹⁵. Consequently, local macroscopic inertial effects might be an indication of the global system’s resistance to any deviation away from a (long-term) ‘settled’ systemic phase-coherence (or phase-harmony). This situation is not immediately obvious, because the phase status of QM (fermion) ‘particles’ are hidden from observational physics.

If this is the case, then only an indirect appreciation of inertia has hitherto been recognised, hence its enduring mystery. This (very raw) hypothesis relies upon the continual (and non-local/instantaneous) outward ‘reach’ of (non-observed QM) particle mass effects in an ‘entangled’ global system. It is consistent with our stance on cosmological temporal evolution/progression outlined in Section 4.

6.2.9 Virtual phase and energy, imaginary numbers and spatial dimensions

Our use of the ‘virtual’ ties in with our (complementary and noumenal) stance upon curved space involving ‘imaginary’ numbers in three spatial dimensions (discussed in subsections 4.4.16 and 4.5.14). There are strong mathematical ties (or links) between complex numbers and geometry; see for example work by Emeritus Professor David Hestenes on this issue. Additionally, note that the standard notion of SR’s and GR’s (‘phenomenal’) spacetime is unable to incorporate non-local quantities.

6.2.10 Some similarities in/of the model to magnetism

That an external (lunar orbital or spin axis) orientation, and the number of atoms/molecules, should dictate the orientation and strength of the new effect (respectively) has a number of similarities to magnetism.

1. Macroscopic magnetic properties arise by way of many component/constituent atoms/molecules having the same magnetic moments.
2. As with paramagnetism, an *external* field ensures the direction of the macroscopic field.
3. With ferromagnetism, the magnetic field of a physical “magnet” selects a dominate/special direction in space. Analogous to this is the planar nature of the space-warp’s (initial) formal representation.

³¹⁵At least upon the ocean surface.

4. Magnetism involves angular momentum, both directly and indirectly (i.e. only in the field). Quantum mechanical (intrinsic) angular momentum is (arguably) the foundation stone of the model.

Finally, note that we are told to not accept either orbital, nor spin, *rotation* in an atom; and yet magnetic effects necessarily pertain to the ‘rotational’ motion of charged particles, and hence magnetic dipole moments (in some sense at least). A somewhat similar conceptual leeway is apparent (and necessary) in our model (recall subsection 5.4.4).

6.3 The physical model quantified: internal (virtual) energy

The model has been labouriously conceptualised, and its (potential) fecundity displayed (section 6.2). The emphasis now shifts from a conceptual emphasis to the model’s quantification, with further conceptualisation included if/as necessary. We shall derive ΔE_w — the total energy offset magnitude, or/and total non-inertial QM energy — by way of quantum mechanical (QM) considerations; then this (weighted) energy is expressed both physically, and formally, as a rotating space-warp (together/conjointly with an associated non-local mass distribution). In equation form, we will see that (for each individual rotating space-warp):

$$\Delta E_w = \frac{1}{2}\hbar(\Delta t)^{-1}N_m\eta = \frac{1}{2}m_1^*\Delta a_w^2\Delta t^2 \quad (16)$$

This (and following) sections shall explain the various quantities in this dual-equality. Only loop/cycle time (Δt) is common to the very different internal (LHS) and external (RHS) forms of the (equivalent) energy magnitude.

The nomenclature of ΔE is preferred over E , so as to retain the form of energy uncertainty in Heisenberg’s Uncertainty Principle, and to signify both: an energy offset over the cyclic duration time Δt ; and secondly, an energy ‘transfer’ (by way of re-expressing the internal *virtual* energy externally³¹⁶).

6.3.1 A crucial difference between moons and atoms as composite systems

An atom/molecule as a whole is described by QMs, whereas a moon *as a whole* is not, even though it is comprised of atoms/molecules (i.e. QM sub-systems) — recall subsection 5.1.3. An atom/molecule is a composite QM system, (mainly) comprised of QM elementary fermion/matter particles (parts). Regarding the model’s virtual spin phase offset we hypothesise that: unlike the case with a moon — which is (herein treated as) an additive sum of its (matter) parts — an atom/molecule as a whole ‘receives’ the *same* virtual spin phase offset as does each (and every one) of its component elementary fermion/matter particle parts.

³¹⁶As a rotating space-warp, together with a non-local mass distribution.

6.3.2 The total virtual non-inertial ‘spin’ energy (for a moon as a whole)

We have argued that in suitable curved spacetime conditions, the projection of atomic/molecular (spin) angular momentum in a plane can become slightly non-inertial by way of a geometric phase offset — but only in a minimal/virtual manner, and only from a systemic (or global) perspective. With orbital angular momentum unaffected we have: $\Delta J_{\max} = (\Delta S_z)_{\max} = \frac{1}{2}\hbar$.

Electron’s, and also some other fundamental particles (protons, neutrons) have a spin whose magnitude is $\frac{1}{2}\hbar$. This is found from experimental evidence, and there are theoretical reasons showing that this spin is more elementary than any other, even spin zero. The study of this particular spin is therefore of special importance (Dirac 1958).

In keeping with subsections 4.2.2 and 5.6.7, and Section 5 in general, $(\Delta S_z)_{\max}$ corresponds to a (spin-based inertial vs. non-inertial/actual ‘frame’) geometric phase offset of $\beta = \pi$, indicating an offset of one quarter of one fermion wavelength (4π). For $\beta = 0$ or 2π , $\Delta S_z = 0$. Indeed, for $\beta \geq 2\pi$ we have $\Delta S_z \rightarrow 0$ by way of decoherence, thus denying any (global presence of) virtual QM energy.

With the angular momentum offset determined over a finite cyclic process time (Δt) the *maximum* virtual spin energy offset, for a *single* atom/molecule, is simply:

$$\Delta E_{\max} = \frac{\Delta J_{\max}}{\Delta t} = \frac{1}{2}\hbar(\Delta t)^{-1}$$

so that $\Delta E_{\text{virtual}} \leq \frac{1}{2}\hbar(\Delta t)^{-1}$. Compare this expression to one form of Heisenberg’s uncertainty principle: $\Delta E_{\text{real}}\Delta t \geq \frac{1}{2}\hbar$. The latter (HUP) expression concerns the non-definite energy of a quantum mechanical *state*, whereas the former concerns a virtual energy (i.e. a hidden systemic variable) — of a definite/exact magnitude.

Unlike the case with $\Delta p\Delta x \geq \frac{1}{2}\hbar$, Δt is *not* an operator belonging to a particle; it is an evolution parameter. Thus, in addition to (non-reversible) measurement, the mechanism proposed herein is effectively a second (and new type of) non-reversible time evolution of a quantum mechanical “system”.

Note that the ‘mass’ dimensional aspect of the non-inertial energy offset has its basis in Planck’s constant, which in turn is indicative of the (‘internal’) ‘mechanics’ of atoms/molecules — which herein (recall subsection 5.1.3) are considered to be the largest (bound) quantum mechanical system, and the *only* types of QM *system* directly affected by the new mechanism.

For a non-optimal effect an efficiency factor η is introduced, where $0 \leq \eta \leq 1$, such that $\Delta J = \frac{1}{2}\hbar\eta = \frac{1}{2}\hbar_w$. We shall refer to $\frac{1}{2}\hbar_w$ as the weighted intrinsic angular momentum offset (see Figure 9).

Further, with the offset applying (concurrently) to every (single) atom/molecule in a moon (N_m), this gives an (additive based) *total* ‘weighted’ energy of:

$$\Delta E_w = \frac{1}{2}\hbar(\Delta t)^{-1}N_m\eta \quad (17)$$

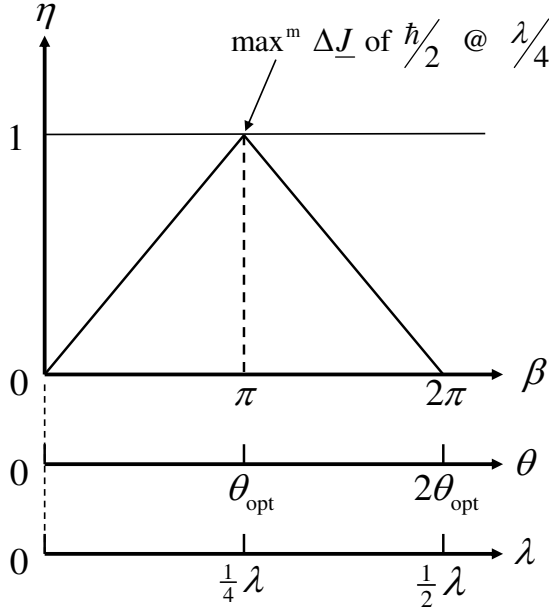


Figure 9: Diagrammatic representation of the variation in the proportion of (or efficiency with which) the smallest possible *quantum* intrinsic angular momentum ($\frac{1}{2}\hbar$) offset (occurs). Angular momentum magnitude is functionally related to a QM geometric phase offset (β), which coexists with a planetary progression angle (θ) (also see Figure 10). Full efficiency (i.e. $\eta = 1$ and an angular momentum offset of $\frac{1}{2}\hbar$) occurs at: $\beta = \pi$ radians, and $\theta = \theta_{\text{opt}} = \frac{1}{2} \tan^{-1}(8\pi)^{-1}$; with this value also representing a $\frac{1}{4}$ of a (fermion) wavelength offset. For values where: $\theta > 2\theta_{\text{opt}}$, $\beta > 2\pi$ — representing more than a $\frac{1}{2}$ quantum wavelength offset — quantum decoherence occurs and thus $\eta = 0$ (effectively).

This represents the total virtual energy imbalance; it is locally hidden, but resident within the (wider) system. The maximum (or optimum) available energy for a given *moon* is ΔE_o so that $\Delta E_w = \Delta E_o \eta$.

Note that with $\Delta E_w \propto N_m$ the largest moons of the solar system (in general) have the largest energy (excess), and their acceleration amplitudes dominate the response of the Pioneer spacecraft. This response is due to undulations in the strength of the acceleration/gravitational field, arising from a *number* of rotating space-warps, that are themselves centered upon a moon-planet system³¹⁷. Rotating space-warp acceleration amplitudes associated with the large moons of Uranus are too small to make any significant contribution to the Pioneer anomaly; see subsection 6.5.3 for further comment. Consequently, the remainder of this Section's quantification excludes data based upon

³¹⁷It is unclear as to whether the mechanism's 'centre' is: the moon's centre, or host planet's centre, or at the pair's centre of mass. Most likely, it is at the moon's centre.

Helical lunar orbit around planet-moon centre

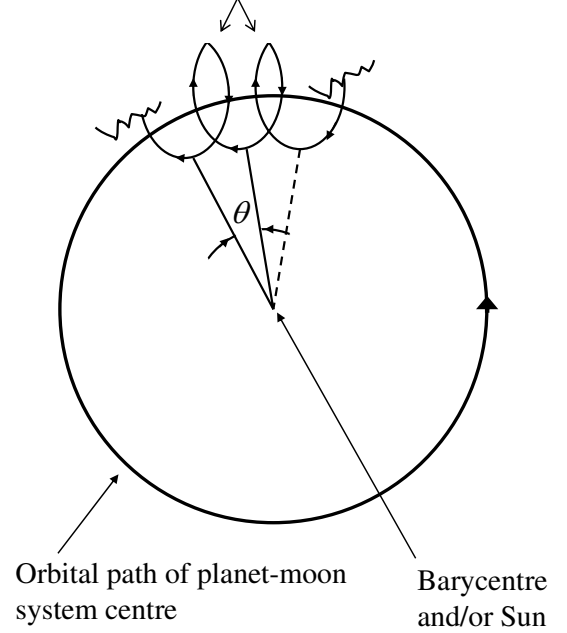


Figure 10: Celestial (curved space) geometric constraints of the model are indicated by this diagrammatic representation of planetary progression angle (θ), i.e. the angular progression of a planet around the Sun — over the duration of one lunar spin-orbital cycle. Note that the view is (downwards) from the north ecliptic pole. Theta i.e. θ (cf. θ_{opt}) both: establishes (and coexists with) the geometric phase offset (β), and the proportion (or efficiency) of a minimum angular momentum ($\frac{1}{2}\hbar$) that is (virtually) affected — previously illustrated in Figure 9.

Uranus' large moons, particularly Tables 2, 3, 4 and 5.

6.3.3 The relationship between: phase, angular momentum, and efficiency

A simple *triangle function* is used to relate the (continuous/analog) phase offset (β) to its (discrete minimum and/or maximum *virtual*) angular momentum 'counterpart' $\frac{1}{2}\hbar$ (see Figure 9). When $\beta = \pi$ we have $\eta = 1$; thus, when $\beta = \pi/2$ or $3\pi/2$ we have $\eta = 1/2$, and when $\beta = 0$ or $\beta \geq 2\pi$ then $\eta = 0$. This triangle function is seen to indicate a wave-to-wave effect representing QM (fermion) wave mechanics involving an inertial to non-inertial intrinsic angular momentum (offset). Recall that for motion along a geodesic, (QM 'systemic') intrinsic angular momentum exists in relation to an orbital angular momentum — which itself remains unchanged and thus also acts as a stable (i.e. unchanged) reference frame or datum.

Table 2: Angular progression of planet, around the Sun, for one lunar orbit.

MOON ^a	Units	Luna ^b	Io ^c	Europa ^c	G'mede ^c	Callisto ^c	Titan ^d	Triton ^e
Moon orbit frequency	(10 ⁻⁶ s ⁻¹)	0.42362	6.5422	3.2592	1.6177	0.69351	0.72586	1.9694 ^f
Moon's orbital period	(days)	27.3217	1.76914	3.55118	7.15455	16.68902	15.94542	5.87685
Host's orbital period	(days)	365.256	4332	4332	4332	4332	10759	60190
Orbital time ratio	(%)	7.480	0.0408	0.0820	0.1652	0.3852	0.1482	0.0098
Angular progression	(deg)	26.93	0.1470	0.2951	0.5946	1.3869	0.5335	0.0351

^aLunar data is taken from NSSDC (National Space Science Data Center web site), with lunar orbital time/period rounded off to 6 or 7 significant figures. ^bEarth's moon. ^cLarge moon of Jupiter. Note that 'Ganymede' is abbreviated to G'mede. ^dLarge moon of Saturn. ^eLarge moon of Neptune. ^fRetrograde motion.

Table 3: Geometric effectiveness/efficiency of various Sun-planet-moon motions.

MOON	Symbol	Units	Luna	Io	Europa	Ganymede	Callisto	Titan	Triton
Angular progression	θ	(deg)	26.93	0.1470	0.2951	0.5946	1.3869	0.5335	0.0351
Efficiency ^g	η	(-)	0.00	0.1290	0.2590	0.5219	0.7826	0.4683	0.0309

^g Efficiency by way of optimum (planet) progression angle of $\theta_{\text{opt}} \approx 1.13926$ deg [i.e. $\frac{1}{2} \tan^{-1}(8\pi)^{-1}$]. A triangular relationship is utilised to determine efficiency — indicative of a (quantum) wave-to-wave offset.

6.3.4 A non-metric approach, but very much a geometric approach

The model's external re-expression of the energy 'asymmetry' (ΔE_w) is not expressible by way of the exactitude of a (dynamic) metric theory, and *geometry* (necessarily) becomes the model's guiding aspect or 'principle'. A 'frictionless' equation of motion is fundamental to Newtonian and relativistic gravitation theory; (but) herein the equation of motion merely defines the (geodesic) path of celestial bodies, and hence the *path* of their constituent atoms and molecules.

The supplementary (i.e. perturbation) curvature, relative to the pre-existing gravitational field (i.e. the field already established by way of GR), causes a 'dissipation' of a moving body's kinetic energy; (or) in other words, the rotating space-warp leads to a shortfall vs. predicted motion — recall section 3.2. The field's cyclic nature can still be represented by a field energy, but (recalling Section 4) it is a field that in its idealised representation utilises a background (Eulerian) space continuum (and formalism) rather than a spacetime continuum and GR's formalism. Hence a secondary non-Euclidean geometry is (also) conceivable, but it cannot be quantified by way of a metric theory. Our (supplementary) continuum mechanics approach is further discussed in subsection 6.4.3.

6.3.5 Celestial geometry and $\frac{1}{2}\hbar$ efficiency

The geometry of three-body celestial motion determines θ , the planetary angular progression angle (around the Sun, per lunar orbit). This was discussed previously in subsection 5.6.7 and the results for major

solar system moon-planet systems appear in Table 2.

By way of comparing this angle to the optimum planetary progression angle (θ_{opt}) of $\frac{1}{2} \tan^{-1}(8\pi)^{-1} \approx 1.14^\circ$ — which also represents a QM relative spin phase offset of π radians — we may establish the efficiency of this planetary progression angle, i.e. the proportion of $\Delta J_{\text{max}} = \frac{1}{2}\hbar$ 'in play' systemically. The efficiency values for different moons are presented in Table 3. Figures 9 and 10 provide visual assistance, (respectively) clarifying the *efficiency* value (η) and planetary *angular progression* per lunar orbit (θ).

6.3.6 On the effect of lunar and planetary orbital motion eccentricity

Note that in this paper both lunar and planetary eccentricities are idealised as zero, i.e. $e \rightarrow 0$ and orbits are thus treated as circular. If $e \neq 0$ then the relationship between angular progression and time is variable. It is not the case that variable angular progression leads to variable efficiency (η), and hence variable space-warp amplitude (Δa_w) and energy (ΔE_w); because a variable rate of lunar and planetary angular progression simply leads to a non-constant rate of angular frequency/velocity for the rotating space-warp.

Examination into the possible existence of this second-order temporal variation in $a_p(t)$ is complicated by the coexistence of multiple RSW 'signatures' in the $a_p(t)$ data, and subsequently this minor issue shall not be pursued in this paper. Importantly, the acceleration/gravitational field amplitudes and the rate of speed shortfall (*per cycle*) are *unaffected* by orbital eccentricities. Similarly, a very minor second-order tem-

Table 4: Weighted QM ‘non-inertial’ energy per orbit (equaling space-warp energy) and associated values.

MOON	Symbol	Units	Io	Europa	G’mede	Callisto	Titan	Triton
Moon orbit frequency (Δt^{-1})	f_m	(10^{-6} s)	6.5422	3.2592	1.6177	0.6935	0.7259	1.9694
Mass	M	(10^{21} kg)	89.32	48.00	148.19	107.59	134.55	21.4
Number of atoms/molecules ^h	N_m	(10^{49})	5.379	2.891	8.924	6.479	8.103	1.289
Optimum Energy ⁱ	ΔE_o	(10^9 J)	18.555	4.968	7.612	2.369	3.101	1.338
Efficiency	η	(–)	0.1290	0.2590	0.5219	0.7826	0.4683	0.0309
Weighted Energy	ΔE_w	(10^9 J)	2.395	1.287	3.973	1.854	1.452	0.041

^h Based on carbon 12 molar mass and Avogadro’s number per gram mole (6.0221415×10^{23} mole⁻¹). ⁱ A full unit ($\eta = 1$) of excess (virtual) spin energy ($\frac{1}{2}\hbar\Delta t^{-1}N_m$) applies — where $\hbar = (2\pi)^{-1}(6.6260693 \times 10^{-34})$ J s.

poral variation of a_p over Jupiter’s orbital time (11.86 years), due to the orbital eccentricity of Jupiter (and its four large attendant Galilean moons) alone, could also be present in the Pioneer data; but with just under 12 years of data in the most accurate Pioneer analysis (Anderson et al. 2002, Figure 14), the ‘noise’ of the Pioneer data³¹⁸ means that this (conceivable but) very minor effect is (also) beyond observational recognition.

6.3.7 Total (idealised) internal energy

Having determined the efficiency of the internal (spin) energy offset, the total virtual offset energy or weighted energy $\Delta E_w = \frac{1}{2}\hbar(\Delta t)^{-1}N_m\eta$ (over time Δt) pertaining to each moon may be found — see Table 4. Note that the number of atoms/molecules (N_m) in each moon is idealised so as to be based on carbon 12 molar mass, with Avogadro’s number (per gram mole) (also) playing a vital role. *Significantly*, this idealisation has no effect upon the all-important acceleration perturbation amplitudes determined in section 6.5 and presented in Table 5; whereas the accuracy of non-local mass (and mass cut-off) values, established in section 6.6 and presented in Table 7, *are* affected by the inaccuracy arising from this all-inclusive idealisation³¹⁹. Fortunately, the low mass of the Pioneer spacecraft — much lower than any “cut-off” mass — makes this inaccuracy “of no consequence”.

6.4 The physical model qualified

6.4.1 Two types of equivalence

GR’s curved spacetime makes Newton’s gravitational “force” an inferior and approximate approach to gravitation. Herein, we ‘decompose’ the classical quantity “force” into its separate mass and acceleration parts/components. From subsection 3.2.12 we recall

³¹⁸Arising from *both* measurement noise, and the inherent variation/wandering of $a_p(t)$ around the long-term mean value ($a_p(t) = a_p$), proposed/hypothesised by the model (recall subsection 3.6.6).

³¹⁹Not so much Ganymede, Callisto and Titan, but more so the relatively denser moons of Io and Europa.

that (model-based) acceleration (herein) refers to either: a (monotonic) rate of change in (or rate of loss of) translational velocity (δa), and/or a sinusoidal/cyclic field curvature perturbation (with amplitude Δa) ‘at’ the spacecraft (over time Δt). In the model, mass and acceleration have/play distinct and separate roles, and force (*per se*) plays no (physical) role whatsoever.

Standard gravitation (GR) employs two types of equivalence *together*: primarily, (inertial and gravitational) *acceleration*; and secondly, (inertial and passive gravitational) *mass*³²⁰ equivalence. The former, and historically more recent equivalence, represents something of a “dematerialisation of gravitation”, in the sense that the free-fall motion of a (point) mass in a gravitational field is independent of the amount of (passive gravitational) mass; i.e. we deal solely with spacetime curvature, which can be conceptualised as a gravitational acceleration acting upon a body. Section 6.7 further examines these equivalence principles.

6.4.2 Introducing non-local mass

The non-local probabilistic nature of a QM mass (preceding its observation) — for example, Schrödinger’s equation applied to an unobserved free particle — is clearly distinct from both: *condensed/compact* (macroscopic) matter, e.g. a planet, a spacecraft, a comet, or a piece of chalk; and from gaseous and plasma matter. The model requires a *new* type of mass that is similar and yet different to the case of an (unobserved) free particle. We hypothesise that if the *virtual* nature of the model’s total QM energy offset (ΔE_w) — via its basis in energy uncertainty — is externalised as an energy field, then the mass aspect of this energy field is always “non-localised”; in the sense that the probability of finding a particle with mass anywhere in the field goes to *zero*. This field ‘mass’ is (thus) effectively non-local. The model requires this external (to atoms/molecules) distributed mass effect³²¹, without any associated external particle; i.e. a *non-local*

³²⁰With this equivalence dating all the way back (at least) to Isaac Newton and experiments by Galileo Galilei.

³²¹Possibly, “non-observable non-particle effect” is a preferable term.

mass effect — that will always be at a ‘subliminal’ level as far as direct physical observation is concerned.

Furthermore (re: the preceding paragraph’s discussion), there is *insufficient* energy for a point mass to ‘materialise’ anywhere in this external field, i.e. anywhere in the (spherical) volume associated with this non-particle-like (non-local) mass (m^*). Later (in section 6.6) we shall see that an invariant spherical volume to non-local mass relationship is important. The model proposes that: the mass aspect of the (virtual offset) energy is spread out *evenly* over a volume, thus making this mass aspect ‘quintessentially’ *non-local*. The basis for this new type of mass is the new relationship: $(\Delta E_{\text{virtual}} \Delta t) \leq \frac{1}{2} \hbar$ (recall subsection 6.3.2).

The extension of the non-local to local mass distinction (outlined above) to ‘gravitational’ field theorisation³²² requires we accept that: (in certain unique circumstances) microscopic/QM matter in ‘motion’ can induce an (external) field curvature/deformation, and that (virtual QM) energy plays a vital role in the model. To appease the dimensions of energy, and to respect the requirements of a physical quantity³²³ existing throughout a field, we unavoidably require this non-local mass to have a magnitude, at all points in the field.

In a way, the non-local nature of this new type of QM-based mass introduces a further stage in the *dematerialisation of gravitation*; in the sense that matter is involved, but in a non-compact *insubstantial* form. Non-local mass is not a ‘substance’ *per se*, although in subsection 6.6.8 we explain the circumstances whereby it does or does not influence the motion of (ordinary celestial) matter — be it: solid, liquid, gas or plasma.

6.4.3 A second and different type of non-Euclidean geometry

The model’s explanation requires a distinction between two different types of curvature involving space and time. Firstly, there is standard GR, where (active) mass, momentum and energy lead to spacetime curvature, and the acceleration of a (passive gravitational) mass is independent of its mass magnitude/amount. In spacetime we describe physical *events*, with these often occurring at a specific time or between specific times, rather than over a period or duration of time.

The (new) supplementary (i.e. cyclic perturbation) curvature, arising from the coexistence of quantum mechanical systems and celestial (geodesic) motion obeying GR, introduced by the model is markedly different. There is no metric, nor is a new equation of motion meaningful. To appease GR’s invariance the (gravitational) acceleration amplitude, associated with a rotating space-warp, is *constant* throughout space³²⁴, which is in stark contrast to GR’s gravita-

tional fields; whereas, to allow for energy dispersion, the non-local mass at a point in the field *varies* (with enclosed spherical volume) throughout space. Additionally, we are discussing an energy-based inherently non-instantaneous physical *process* cf. singular or multiple ‘events’.

With the introduction of non-local or distributed mass (elaborated upon in section 6.6), there is no violation of an *equivalence principle* involving point-like ‘condensed’ inertial mass and (passive) gravitational mass (see subsection 6.6.8). Neither is standard (i.e. general relativistic) inertial acceleration to gravitational acceleration equivalence violated³²⁵. We simply have an additional (and non-standard) contribution to the overall ‘gravitational’ field, in conjunction with the spatial distribution of (a new) *non-local* mass ‘quantity’ (denoted as) m^* (see subsection 6.6.5).

6.4.4 The Biot-Savart law, and extending continuum mechanics field theory

The rotating space-warp proposed is considered to have similarities to the magnetic (induction) vector field \underline{B} induced by a (steady) electrical current in a wire, and the velocity field ‘induced by’ a vortex line/filament³²⁶. Both of these physical phenomena are described mathematically by the Biot-Savart law. A (three-dimensional) circulatory aspect, which may be restricted to a planar effect, is common in all cases. The rotating space-warp is seen to be an external effect induced by an ‘internally’ inexpressible QM energy — with this energy based upon a new form/application of Heisenberg’s uncertainty principle (and the quantum or discrete nature of atomic/molecular angular momentum) — as discussed in subsection 6.3.2.

Recall that we are now discussing space curvature independently of Special Relativity, in conjunction with a model that prefers a *space* continuum, cf. a spacetime continuum. Note that both Mechanics of Solids and Fluid Mechanics utilise a *mass* continuum³²⁷. This is clearly an unorthodox approach to ‘gravitation’, or rather the supplementation of gravitation, but it is actually (merely) a simple extension of classical continuum mechanics into the celestial realm³²⁸ — albeit now incorporating delocalised mass (i.e. non-local mass).

6.4.5 Extending the model’s kinetic energy shortfall into three dimensions

The derivation of the kinetic energy ‘shortfall’ of spacecraft motion (cf. currently predicted motion) by way of

geometric) characteristics, and/or the *number* of (lunar) atoms/molecules involved.

³²⁵In GR this equivalence is between *uniformly* accelerated reference systems and homogeneous gravitational fields. A *sinusoidal* variation in acceleration around a mean value is a very different circumstance.

³²⁶Possibly, the phrase “induced by” could be replaced by “coexisting with”.

³²⁷From a microscopic/QM perspective, macroscopic matter is also largely comprised of ‘empty’ space.

³²⁸Recall that Newton unified terrestrial and celestial (condensed matter) gravitation over 300 years ago.

³²²So as to be a supplementary or further feature of gravitational field energy.

³²³That is, having a magnitude *and* a unit/dimension(s).

³²⁴Note that very slow changes in amplitude over *time* may occur and these are seen to propagate at the speed of light, but the amplitude (in the absence of change) is constant throughout *space*. Such a change only arises from changes in either lunar or planetary orbital (and hence

the undulatory acceleration/gravitational field(s) (outlined in section 3.2), was restricted to a planar (space-warp) phenomenon — albeit a distortion of a Euclidean plane³²⁹, with intrinsic curvature assumed³³⁰. The planar nature of the model lends itself to a more visual (and geometric) understanding of curved space.

The *specific* energy of the undulation, over cycle time Δt , is $\Delta e = \frac{1}{2}\Delta a^2\Delta t^2$. The extension of this space curvature into three dimensions is quite simple, resulting in a universe long axis about which an ‘infinitely’ wide space-warp rotates (recall subsection 6.2.2). Thus, the specific energy (Δe) relationship is upheld. This three dimensionality of the rotating space-warp is satisfying, because the Rayleigh Theorem based quantification (beginning at subsection 3.2.8) was initially restricted to simple *one-dimensional* (in-line) motion, and then extended by way of an (observationally demanded) hypothetical conceptualization to a *two-dimensional* rotating space-warp (see subsection 3.5.2).

Recall that a rotating space-warp leads to the same (path-based) loss of kinetic energy (per cycle) regardless of the (in-plane) direction of motion of a celestial body or spacecraft. Radial and circumferential motion in the solar system are equally affected. For three-dimensional motion, i.e. motion inclined to the plane of the space-warp’s rotation, the same variation in Δa over duration Δt is (or will be) ‘experienced’. Thus, speed shortfall (δv over duration Δt) along the path of motion — i.e. along the velocity vector — is the same as for (two-dimensional) in-plane motion.

Even though the kinetic energy ‘shortfall’ of moving (low mass) bodies is path-independent, there remains a need to make corrections for *line-of-sight* Doppler *observations* (geometrically) inclined to the Pioneer 10 path vector. This correction is quantified in subsection 6.5.8.

6.4.6 General remarks concerning the rotating space-warps (GQ-RSWs)

The simplicity of these moon-planet based (gravito-quantum) rotating space-warps is suitable for (i.e. not incompatible with) a field based *systemic* representation, that itself is (inevitably) centered at the solar system barycentre. A systemic representation is useful for determining speed shortfall cf. predicted speed (in the absence of rotating space-warps).

We shall see that non-local mass, in the field, varies with distance from each individual source (section 6.6), whereas acceleration amplitude is fixed throughout the field — although this (perturbation) acceleration does vary cyclically/sinusoidally (over time at a point or point mass) as the space-warp rotates.

In this “constructive theory” (cf. principle theory) approach, (non-local) mass and (RSW-based) accel-

ation are physically distinct variables, with the concept of *gravitational* force of no significance. Energy is the model’s primary physical quantity and conservation of energy its linchpin. Further, we shall see that the (initial) external field energy (ΔE_w) for an individual RSW cannot be completely localised (i.e. spatially *and* temporally) in the formalism; this is because, even though the specific energy of each RSW is constant throughout space³³¹, the distribution (and nature) of non-local mass is incompatible with a ‘point’ mass based determination of energy values.

In spite of the fact that the energy’s microscopic QM basis requires a closed (circumferential) loop (and hence a finite process time) for its quantification, a systemic representation of non-local mass (m^*) and acceleration undulation amplitude³³² at every point in the field (through time) is feasible in principle³³³. Herein, the model deals primarily with an average Pioneer anomaly value (a_p)³³⁴ arising from several coexisting RSWs. A (classical) continuum mechanics approach to the formalism of the (global) system is also required to achieve the formalism’s (in principle) localisation, that (separately) involves constant acceleration (amplitude) *and* non-local mass at points throughout the field. Note that the new/supplementary gravitational energy, like the gravitational energy in GR, can *never* be spatially or temporally localised, because the quantification of ΔE_w is process based, i.e. requiring $\Delta t \gg 0$.

It shall become apparent (in subsection 6.6.8) that the non-locality of what was originally inertial mass in two different atomic/molecular QM angular momentums (i.e. spin and orbital), and the coexistence of rotating space-warps and (non-local) mass in the field, has lead to the observational interpretation of an *apparent* violation of GR’s (mass) equivalence principle — in that ‘high mass’ bodies such as: Halley’s comet, Vesta (asteroid), Europa (moon), Pluto (dwarf planet), and the planet Saturn (for example) do not respond to the rotating space-warps, whereas the (‘low mass’) Pioneer spacecraft *do* respond.

Actually, in the case of comet Halley this *total* absence of any anomalous effect is not necessarily *always* the case, because in the unlikely event of comet Halley (with its current mass of $\approx 2.2 \times 10^{14}$ kg) coming within approximately: 2 AU of Jupiter, or 0.8 AU of Saturn, or 0.55 AU of Neptune, there will be ‘some’ influence — only one Sun-planet-moon system in the case of Saturn and Neptune — because the distance dependent mass cut-offs of the host planet’s moons are no longer less than the comet’s total (‘condensed’)

³³¹ Except for a (relatively very small) central near-field ‘hole’.

³³² Either side of an equilibrium acceleration value (that is) based upon general relativistic gravitation.

³³³ Knowing the timing of each phase of the rotating space-warps’ amplitude is required. Unfortunately, these are not currently known, because only their superposition is easily accessible to observations.

³³⁴ This average anomalous acceleration or speed shortfall rate, acting upon all (celestial) bodies of sufficiently low mass, physically coexists with the (on-going) resultant ‘summation’ of the acceleration/gravitational field (perturbation) amplitudes of the various rotating space-warps.

³²⁹ If we neglect the presence of GR’s gravitational field.

³³⁰ The ontological understanding of *intrinsic* curvature deserves further discussion. For our purposes we have envisaged an imaginary (number based) space dimension, into which space effectively deforms. This *conceptualisation* is not inconsistent with GR, and it is related to the discussion in section 4.5.

mass. Note that these aforementioned distance values were determined by way of interpolating the data (presented later) in Table 7 of subsection 6.6.6.

6.5 The physical model quantified: external (real) energy

In this section the acceleration/gravitational amplitudes of the rotating space-warps (RSWs) are determined, and the nature and coexistence of these accelerations are discussed. Issues concerning (non-local) mass shall be discussed in section 6.6. Unless otherwise specified a *single* RSW is the basis of any (rotating space-warp based) discussion in this section.

6.5.1 Geometry's other role in the (semi-empirical) model

In addition to determining internal geometric phase offset (β) and virtual angular momentum ($\frac{1}{2}\hbar_w$), geometry plays a further important role in the model. This is unavoidable for two main reasons. Firstly, the rotating space-warp needs to exhibit dispersion with increasing distance away from the warp's lunar-planetary source. Secondly, the energy of a rotating space-warp comprises both: a *non-spherical* (planar-based) rotating space-warp of (constant) amplitude Δa , and a presumably *spherical* (non-local) mass distribution. We shall need to appease, or at least understand, this (spherical vs. non-spherical) situation which ostensibly involves a geometric conflict.

The external expression of total internal virtual QM energy (ΔE_w), as a (real) rotating space-warp and non-local mass distribution, is a 'hybrid' energy, involving mass and specific energy ($\Delta e = \frac{1}{2}\Delta a^2 \Delta t^2$) *separately* — although not independently.

We shall now argue that both: the magnitude of Δa , and the non-local (initial) mass (associated with a given initial volume) m_1^* , are dependent upon a reference (or 'yardstick') radius from a moon-planet system, and hence the volume enclosed within this radius — with this reference radius determined by geometric circumstances. Additionally, cycle (or closed loop) time Δt acts as a reference (or 'yardstick') time. The determination of this (new) reference radius follows.

Geometry and differentiation give us the fact that: the derivative of spherical surface area (S) with respect to radius is: $dS/dr = 8\pi r$. If we set r equal to lunar orbital semi-major axis radius³³⁵ (r_o), the radius $8\pi r_o$ describes a unique radius. In the model this (geometrically unique) radius sets the 'initial' (non-local) mass value (for an initial volume and surface area). In subsection 6.5.2 we see that this radius also establishes the *constant* Δa_w value associated with each moon. The (overall) energy of a rotating space-warp, and the (initial) non-local mass associated with it, is:

$$\Delta E_w = \frac{1}{2} m_1^* \Delta a_w^2 \Delta t^2 \quad (18)$$

where m_1^* represents non-local mass at $8\pi r_o$ ($= r_1$ say), Δa_w is a weighted acceleration amplitude, and ΔE_w is

³³⁵We use the variable r_o in preference to a or r_a to denote the length of the lunar semi-major axis.

the total supplementary field energy over the course of a single lunar loop/cycle. Note that use of Δa previously (especially in Section 3) is replaced by Δa_w from here on — see subsection 6.5.2 for justification.

It can also be said that a ('defining' radius or) reference radius $8\pi r_o$ effectively represents a 'dividing-radius' between the near-field and far-field regions of the model, with our interest dominated by the far-field³³⁶. Speaking of a rotating space-warp in terms analogous to a fluid mechanical vortex: we have a space-warp external to an (inner) tube rather than external to a linear (one dimensional) filament.

The geometry-based assumptions presented in this section are necessary if we are to make the model match the awkward observational evidence. Once again the small angle $\tan^{-1}(8\pi)^{-1}$ is important; this time in conjunction with two sides of a right triangle: i.e. lunar semi-major axis length r_o , and the (geometrically unique) reference length $8\pi r_o$.

6.5.2 The use of a weighted acceleration

Previously (section 3.2) we used δa to denote the spacecraft's speed change over time (in response to a single space-warp) and Δa to denote the warp amplitude. The latter is now expressed as Δa_w , because it is a weighted acceleration: $\Delta a_w = \Delta a_o \eta$ where Δa_o is the optimum acceleration amplitude, i.e. the acceleration amplitude corresponding to $\beta = \pi$ where $\eta = 1$. Importantly, we recall (Equation 9, subsection 3.2.11) that $\Delta a = |\delta a|$, which is now written as: $\Delta a_w = |\delta a|$.

Just as $\frac{1}{2}\hbar$ represents the (maximum) internal uncertainty or 'wobble' room (of a single atom/molecule's intrinsic angular momentum), (so) the amplitude Δa_o is seen to represent the corresponding (maximum) external (acceleration/gravitational field) 'wobble' room of a single rotating space-warp. Further, the same internal efficiency factor (η) is reproduced externally upon Δa_o , rather than with regard to the total energy ΔE_w . The empirical model demands this 'physical' connection between $\frac{1}{2}\hbar$ and Δa_o or $\frac{1}{2}\hbar_w$ and Δa_w — i.e. (internal) intrinsic angular momentum and (external) acceleration field amplitude. This is further discussed in subsection 6.5.6.

6.5.3 The geometric determination of weighted acceleration (Δa_w)

We now turn our attention to the (geometric) determination of Δa_o , and hence Δa_w where $\Delta a_w = \eta \Delta a_o$.

A right triangle comprising sides: r_o and $8\pi r_o$, and angle: $\phi = 2\theta_{\text{opt}} = \tan^{-1}(8\pi)^{-1}$ is used to establish Δa_o . We utilise a reference point that is $8\pi r_o$ distant from a moon-planet's orbital centre³³⁷. Thus, the moon's centre, during its orbital motion, subtends a (maximum) angle ϕ relative to this external (geometrically established) reference point — see Figure 11.

³³⁶How to treat the near-field (i.e. $r < 8\pi r_o$) remains far from clear, and is not considered herein. One possibility is as a hole in an infinite field.

³³⁷Dominating the model are the large moons of the gas and ice giant planets, whose planet centres are very nearly the moon-planet (reduced mass) orbital centres.

Table 5: Weighted lunar space-warp acceleration amplitude and associated values.

MOON	Symbol	Units	Io	Europa	G'mede	Callisto	Titan	Triton
Mass	M	(10^{21} kg)	89.32	48.00	148.19	107.59	134.55	21.4
Semi-major axis ^j	r_o	(10^9 m)	0.4218	0.6711	1.0704	1.8827	1.2219	0.3548
Optimum acceleration	Δa_o	(10^{-8} cm s ⁻²)	39.821	8.4537	10.2590	2.4076	7.1485	13.487
Efficiency	η	(-)	0.1290	0.2590	0.5219	0.7826	0.4683	0.0309
Weighted acceleration ^l	Δa_w	(10^{-8} cm s ⁻²)	5.139	2.190	5.354	1.884	3.348	0.416 ^k

^j Semi-major axis data, usually denoted as a , is taken from the “Planetary Satellites Mean Orbital Parameters” page of the JPL Solar System Dynamics web site (<http://ssd.jpl.nasa.gov>). Data for Titan (1221865 km) and Triton (354759 km) exceed the significant figures in the table. ^k Although Triton’s retrograde motion ‘opposes’ the other major moons’ rotations, its contribution to Δa_w is considered ‘positive’. ^l Note that square root of the summation of squared (weighted) acceleration is 8.64×10^{-8} cm s⁻² without Triton, and 8.65×10^{-8} cm s⁻² with Triton’s contribution; as compared to the Pioneer anomaly’s quoted magnitude of $a_p = 8.74 \pm 1.33 \times 10^{-8}$ cm s⁻².

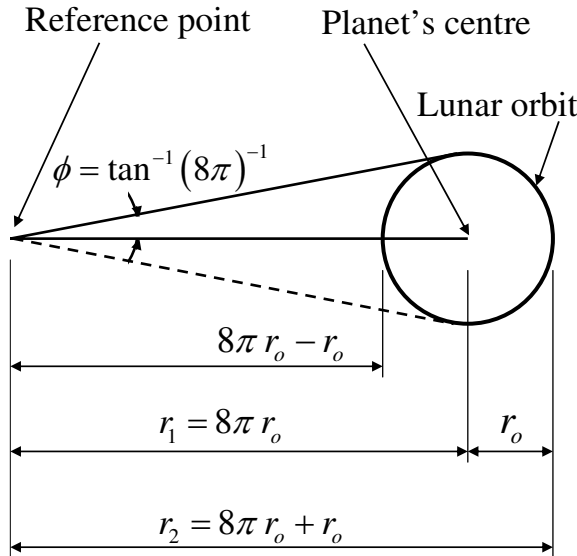


Figure 11: Schematic diagram showing the model’s (new) angular based reference point which is used to determine rotating space-warp (RSW) acceleration amplitudes. This reference point is $8\pi r_o$ distant from a planet’s centre, where r_o is the lunar semi-major axis (length). Note that the eccentricity of (large moon) lunar orbits is very small. The variation in gravitational acceleration over a $2r_o$ range is used to determine the rotating space-warp acceleration amplitudes (Δa).

From the Newtonian gravitation approximation:

$$\frac{F}{m} = a = \frac{GM}{r^2}$$

where m is a test mass (e.g. a spacecraft) and M is lunar mass, we may determine (to good accuracy), at the reference distance, half the total variation (i.e. $\frac{1}{2} \times 2\Delta a$) in acceleration due to the moon’s orbit.

We define the maximum variation in gravitational acceleration (Δa_o), around a mean value, as occurring between $r_1 = 8\pi r_o$ and $r = 8\pi r_o \pm r_o$. Remembering that r_o (cf. standard notation r_a or a) is the lunar semi-major axis, and letting $r_2 = 8\pi r_o + r_o$ the variation, or wiggle room, in acceleration is:

$$\Delta a_o = GM \left(\frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \approx GM \left(\frac{2r_o}{r_1^3} \right)$$

or alternatively:

$$\Delta a_o = 1.18843 \times 10^{-4} \left(\frac{GM}{r_o^2} \right) \approx \frac{2G}{(8\pi)^3} \left(\frac{M}{r_o} \right) \quad (19)$$

Our constructive empirical model asserts that this acceleration magnitude has a correspondence to both: the maximum intrinsic virtual angular momentum offset, i.e. $\frac{1}{2}\hbar$; and also a one quarter QM (fermion) wavelength offset (i.e. π rad). Table 5 presents the various Δa_o and Δa_w values of the moons dominating the Pioneer anomaly. Note that the efficiency values are as per Table 3, and $G \approx 6.67428 \times 10^{-11}$ m³ kg⁻¹ s⁻².

Regarding the moons of Uranus, the largest weighted acceleration value is 0.132×10^{-8} cm s⁻² for Titania, the eighth largest moon of the solar system. All the other moons of Uranus have values less than 0.10×10^{-8} cm s⁻². Thus, their quantitative contribution to the root sum squared based determination of the Pioneer anomaly (a_p) is negligible.

6.5.4 The model’s acceleration as compared to MOND’s acceleration

The ‘fixed’ value of the space-warp’s optimum amplitude (Δa_o) should not be confused with the additional fixed acceleration ($a_0 \approx 1.2 \times 10^{-8}$ cm s⁻²) associated with a modified Newtonian dynamics (or MOND) based approach to spiral galaxy equilibrium — which presumes the non-existence of dark matter (Milgrom 2006). Acceleration drops off as r^{-2} in NMs, and as r^{-1} in the MOND low acceleration regime of spiral galaxies; whereas in our model the supplementary acceleration is constant (i.e. it ‘drops off’ as r^0).

Unlike MOND's force based approach to gravitation, or rather gravitational modification³³⁸, the new model's spatially *constant* (sinusoidal) acceleration (amplitude) allows it to achieve a universality that is beyond GR's scope. This is because the comprehensiveness of GR's approach to gravitation is unavoidably local, whereas the model's local uniformity is compatible with non-locality — including quantum mechanical (spin entanglement based) *non*-locality.

6.5.5 Overall effective acceleration value arising from multiple space-warps

Subsections 6.2.1 and 6.2.2 discussed how spacecraft motion at a geometric angle to the plane of the rotating space-warps (RSWs), as determined by their lunar orbital planes, is quantitatively equivalent to a spacecraft or body moving in the plane of the (various) lunar orbits³³⁹. Thus, the weighted acceleration amplitudes of the model's (major) space-warps (Δa_w) applies without alteration, although a correction to the idealised model for the geometric inclination of the line-of-sight observations to the spacecraft path vector is required. This issue is further discussed in subsection 6.5.8.

The manner in which these space-warps act *together* upon a *moving* body's kinetic energy requires an explanation — which now follows.

We recall from subsection 3.2.12 that $\frac{1}{2}\Delta a^2\Delta t^2 = \frac{1}{2}\delta v^2$, where δv is the loss of a moving body's (spacecraft) translational speed in one cycle (i.e. rotation of the warp) — albeit for a *single* rotating space-warp.

In subsections 3.2.13, 3.5.2 and 3.6.2, for multiple rotating space-warps, a root sum of squares (RSS) approach was signalled. Recalling Equation 12:

$$a_p = \overline{a_p(t)} = \sqrt{\sum (\delta a_{\text{proper}})_i^2} = \sqrt{\sum (\Delta a_{\text{field}})_i^2}$$

and substituting the various Δa_w values of Table 5, which equal the Δa_{field} values — whilst noting that the δa_{proper} values are alternatively written (simply) as δa — we establish (for the idealised model) that:

$$a_p = \sqrt{\sum (\Delta a_w)_i^2} = 8.65 \times 10^{-8} \text{ cm s}^{-2}$$

which is within the error bars of the quoted value of the Pioneer anomaly, i.e. $a_p = 8.74 \pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$. Bear in mind that: $1 \times 10^{-8} \text{ cm s}^{-2} = 1 \times 10^{-10} \text{ m s}^{-2}$.

Note that in the absence of the rotating space-warps from the five other (primary) moons, Triton's RSW would necessarily (also) retard the motion of (sufficiently low mass) bodies. Thus, although Triton's retrograde motion (around Neptune) and associated retrograde RSW oppose the direction of lunar prograde RSWs, the root sum of squares approach (to quantifying a_p) is seen to remain valid. See subsection 3.5.4 for why Triton's largely negligible contribution may actually be one of destructive interference.

³³⁸This includes modification of a particle's inertia (under the action of a force) — whatever that physically entails.

³³⁹This is fortuitous because Jupiter and Saturn's equatorial planes are not parallel; additionally, Jupiter and Saturn's orbital inclination (to the ecliptic plane) are unequal.

The use of a (square) root (of) sum of squares (RSS) approach implies the (constant amplitude) space-warps are uncorrelated — which is non-problematic. Somewhat problematically, there is also the implication that the speed changes induced by the various space-warps upon a moving body, are in some other sense orthogonal to each other — as is the case with motion in three dimensions³⁴⁰, or a (quantum mechanical) Hilbert space. The issue of concern is that the various speed changes all act along the *same* velocity vector (or path vector) of a given spacecraft.

We shall hypothesize that this (elegant) RSS approach is how the superposition of the rotating space-warps is made physical for a moving body (of sufficiently low mass). This assertion is supported by:

1. the QM (energy) origins of the rotating space-warps and their associated non-local mass,
2. the agreement of the model with the observational evidence (see section 2.4 especially)³⁴¹.

6.5.6 On the hypothesised relationship between $\frac{1}{2}\hbar$ and Δa_o

The model's proposed relationship between a maximum/optimum virtual intrinsic angular momentum offset ($\frac{1}{2}\hbar$) and the magnitude of an optimum rotating space-warp's amplitude (Δa_o), or between (non-optimal/actual) $\frac{1}{2}\hbar_w$ and Δa_w is now examined. This subsection shall argue that it is a unique (and *new* type of) physical relationship.

This new relationship is the (non-local) backbone of the 'new' physics presented herein. It relates to the fact that in the model: collective (virtual) QM *spin* is (effectively) entangled with the (macroscopic) RSW's *acceleration*/gravitational field amplitude. This (one-way) non-local physical interrelationship is qualitatively distinct/separate from the model's quantitative overarching microscopic-to-macroscopic energy equality (Equation 16), with the latter pertaining to a (one-way) energy re-expression and (also) universal/systemic energy conservation.

The following discussion examines four primary physical 'quantities' and their units or (more specifically their) dimensionality, with the latter expressed in terms of the 'fundamental' (mechanistic) dimensions³⁴² of: mass, length, and time³⁴³.

1. Linear momentum (p), $\left[\frac{\text{ML}}{\text{T}}\right]$
2. Angular momentum (L), $\left[\frac{\text{ML}^2}{\text{T}}\right]$

³⁴⁰For example, an overall (or 'resultant') velocity magnitude is the square root of the squared values of speed in the x, y, and z directions — so that $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

³⁴¹Noting that temporal variation in a_p , around its very long-term constant value, are 'integral' to the model (recall subsection 3.6.8); and that the resonance of Jupiter's moons leads to only a minor smoothing effect (recall subsection 3.6.5).

³⁴²Units tend to be specific like speed in metres per second, whereas the word 'dimensions' is better suited to non-specific and generalised 'units' such as: mass, length, and time.

³⁴³Distinguishing between vector and scalar quantities is not of importance in this discussion.

3. Force (F), $\left[\frac{ML}{T^2}\right]$
4. Energy (E) and Torque (τ), $\left[\frac{ML^2}{T^2}\right]$

In physics, especially electromagnetism and particle physics, (instantaneous linear) particle momentum (p) is related to particle energy (E) by way of (multiplying/dividing) the particle's speed — which has units/dimensions $\left[\frac{L}{T}\right]$. Relationships between the quantities: p and F , and L and E involve a differentiation/integration with respect to *time*, whereas a relationship between: F and E involves an integration/differentiation with respect to *distance* (i.e. a length). Further, the relationship between p and L (and F and Torque) utilises a cross product involving a radius arm vector. Thus far we have discussed five permutations involving pairs of the above four 'primary' physical quantities (p , L , F , and E) — see Figure 12. The use of ' L ' and ' L ' in conventional nomenclature to denote angular momentum (as a vector) and the length dimension (respectively) is somewhat unfortunate and should be noted.

dimensions	$[ML]$	$[ML^2]$
$\left[\frac{1}{T}\right]$	$p \longleftrightarrow L$	
$\left[\frac{1}{T^2}\right]$	$F \longleftrightarrow E$	

Where: p is linear momentum,
 L is angular momentum,
 F is force, and E is energy/torque.

Figure 12: Tabular schematic representation of the *dimensional* contributions of mass, length and time with regard to the quantities: linear momentum, angular momentum, force and energy. For example, energy has dimensions of: $\left[\frac{ML^2}{T^2}\right]$. This facilitates an understanding of the dimensional relationships existing between these quantities. A new relationship between (a QM offset of) angular momentum (L) and Force (F) — or rather acceleration (F/m) — is central to the model proposed in this paper. Unfortunately, standard nomenclature dictates that ' L ' and ' L ' represent the (vector) physical 'quantity' of angular momentum, and the length dimension (respectively); this subtle difference should be noted.

The only one of six permutations omitted from a conceivable relationship, is a (direct) relationship be-

tween angular momentum (L) and force (F)³⁴⁴. In the model we have needed to relate angular momentum, in the form of (a proportion of) $\frac{1}{2}\hbar$ to an acceleration Δa_w , with the latter's dimensionality, i.e. $\left[\frac{L}{T^2}\right]$, equivalent to a specific force (F/m). Thus, this is a new and unique relationship; a systemic (i.e. global) relationship unique to the model, that is implied (indirectly) by way of "best-fitting" the observational evidence. This relationship coexists with, facilitates, and supports the use of conservation of (systemic/global) energy as the main guiding *principle* of the model.

Note that in the model, initial or starting non-local mass (m_1^*) is a fixed quantity, and thus the relationship between angular momentum and a constant acceleration (amplitude) allows the establishment of a pseudo-force term ($F = m_1^* \Delta a_w$) for each moon — not that it is a physically relevant quantity. The primary quantities in the model's mechanism are: 'initial' non-local mass (m_1^*), constant acceleration amplitude (Δa_w), total (virtual) angular momentum ($\frac{1}{2}\hbar_w N_m$), and total (re-expressed) energy (ΔE_w). Throw in the predominantly linear momentum (p) of the (point mass-like) Pioneer spacecraft, at least over relatively short time spans (Δt), and the broad scope of the physical quantities comprising the model is evident.

6.5.7 A comment upon dimensional variation and 'reference' quantities

We note that to change a quantity from a *linear* momentum (p) to an energy expression (E) involves multiplying by a quantity with dimensions $\left[\frac{L}{T}\right]$ (i.e. speed, e.g. speed of light). By way of contrast, the physical link from a (microscopic) angular momentum to a (macroscopic) pseudo force (mass times an acceleration, $m^* \Delta a_w$) involves altering the dimensions by $\left[\frac{1}{LT}\right]$. The former change is well suited to the physics of particle motion, whereas the latter 're-expression' — also involving (only) one length and one time dimension — is unique to the new (non-graviton-particle, non-local, systemic) model proposed herein.

Once again, we note that the model involves an unusual re-expression of (systemic) atomic/molecular angular momentum based energy, and has no place for a particle momentum based 'force' — as embraced and prioritised by the standard model and standard physics.

The coexistence of the physical re-expression of (virtual) intrinsic angular momentum as a (constant amplitude) rotating space-warp with a non-local mass component, in conjunction with the three particle-based forces of the standard model does not deny the model's implementation of an additional (hidden) background Euclidean frame (as discussed in section 4.4). This ('idealised') frame represents the flat space circumstances that exist in the *absence* of all matter and measurable energy effects. Although not physi-

³⁴⁴ Admittedly, this discussion is fast and loose, with (more specific) quantities such as impulse, tension, and heat (for example) not mentioned; but our interest lies in the sixth (and final) permutation — involving any two of these four primary physical quantities — rather than further instantiations of the permutations already recognised.

cally ‘realised’, it is conceivable as well as formally and conceptually useful. In a nutshell, we employ a global ‘pseudo-frame’ with an associated (global) pseudo-time, in order to allow us to quantify the (equal) energy of both: a (coherent and collective) QM fermion ‘wave’ (phase) offset (achieved over a given cycle time), and a macroscopic rotating space-warp.

It can be argued that for the dimensional ‘multipliers’ of $[\frac{L}{T}]$ and $[\frac{1}{LT}]$ to *coexist* non-problematically in nature, a systemic reference frame and a systemic time unit must exist, and upon this stage (hidden from observations) particle motion (also) occurs. It is ultimately the observation of quantum non-locality, and the role of non-locality in the model, that demands a (‘noumenal’) simultaneous background/hidden time be instantiated — both theoretically and ontologically.

Our geometry based model has required the introduction of a (geometrically significant) reference length ($8\pi r_o$) and a (physically significant) reference cycle time (Δt), as compared to the standard model’s use of (what may be called) a “reference speed” — i.e. the *speed* of light (c) which, along with Planck’s constant, is a *universal* (i.e. global) constant. The model also requires a reference *angle* $\tan^{-1}(8\pi)^{-1}$, which can also be thought of as a ‘dimensionless’ ratio of two lengths: $8\pi r_o$ and (lunar semi-major axis length) r_o — the non-hypotenuse sides of a right triangle. Interestingly, the standard model has no ‘universal’ angle, i.e. an angle of particular importance; and it is *geometry*, with the assistance of energy conservation, that has primarily allowed us to relate a (supplementary) gravitational phenomenon to a quantum mechanical circumstance.

Recalling section 4.3, hopefully the reader now accepts that to simply assume “measurements are reality (without remainder)”, and hence to embrace GR’s general covariance as indicative of a characteristic of reality-in-itself³⁴⁵, is conceivably a non-progressive stance — at least in the specific case of the Pioneer anomaly. To only accept GR’s stance on time necessarily leads to a complete denial of the new model hypothesised. In the case of a *real* Pioneer anomaly, it appears that only by way of enriching Nature’s ontology, and appreciating the limitations of locality and observational evidence, can a viable and progressive scientific explanation be achieved.

6.5.8 Extending the modelling of the Pioneer spacecraft to three dimensions

In this paper (and generally) the word ‘dimension’ is used both as a (non-specific/generalised) unit, and as an ontological (and conceptual) aspect of reality. We now return to its (latter and) more common meaning, especially regarding space.

When visualising a (macroscopic) rotating space-warp, it has been useful to think in terms of an (idealised two [then actually three] dimensional) perturbation upon either: an (idealised) uncurved flat space, or the pre-existing general relativistic gravitational field. We now quantitatively extend the model’s planar gravitational/accelerational (perturbation) influence to spacecraft (S/C) motion inclined to a plane

that is orthogonal to the space-warp’s axis of rotation. In short, the model is extended to the motion of (passive ‘gravitational’) mass in three spatial dimensions.

Earlier (subsection 6.2.2) we were able to extend the space-warp’s (presence and) influence to having a more than minimal thickness — i.e. we went beyond its idealised planar instantiation. Regardless of how far the (moving) Pioneer 10 and 11 spacecraft (S/C) lie from the ecliptic plane, and regardless of their velocity vector’s inclination to the ecliptic plane, the model suggests they will ‘receive’ the rotating space-warps *full* influence³⁴⁶ (recall subsection 6.2.1). Similarly, within a given plane, e.g. the ecliptic plane, there is no variation of anomalous speed shortfall arising from the direction of motion; radial and circumferential motion are equally affected.

The ubiquitous nature of the Pioneer anomaly (motion shortfall rate³⁴⁷) means there is no correction required for S/C motion inclined to each specific lunar orbital plane. But, this shortfall is attenuated if the spacecraft’s path/velocity vector is inclined (i.e. not aligned) with the line-of-sight Doppler observations. This is inevitably the case with Pioneer 10 and 11 and thus the observed Pioneer anomaly will be different to the full path only based effect (see Table 6).

Since Jupiter’s Galilean moons dominate the anomaly, and Jupiter’s equatorial plane is inclined at only 1.3° to the ecliptic plane, we shall (when loosely speaking) refer to the ecliptic plane when using a single plane to ‘represent’ a plane to which the majority of various lunar orbits, and hence rotating space-warps, are closely parallel.

6.5.9 Lesser importance ascribed to the Pioneer 11 navigation experiment

There are two (interconnected) reasons for assigning the Pioneer 11 results a considerably lesser degree of confidence (and significance) than the Pioneer 10 data; the Pioneer 10 data is discussed in subsection 6.5.10. Firstly, by way of quoting Anderson et al. (2002, p.23) “... the [Pioneer 11] data was relatively noisy, was from much closer to the Sun, and was taken during a period of high solar activity. We also do not have the same handle on spin-rate change effects as we did for Pioneer 10.” Secondly and *significantly*, the final value given by Anderson et al. (2002, pp.39-40) for the Pioneer anomaly (a_P) omits the Pioneer 11 data completely, and *only* uses the Pioneer 10 data.

Anderson et al. (2002, p.37) mention that: “... the [annual variation] is particularly large in the out-of-ecliptic voyage of Pioneer 11 ...”; whereas one can (just as easily) retort, by way of Olsen (2007, p.396), that: “... the results for Pioneer 11 show no clear annual variation of the anomalous acceleration.” These two opposing viewpoints, regarding Pioneer 11’s ‘annual’ variation in anomalous acceleration (a_P), highlight a lack of consensus amongst different data anal-

³⁴⁶For example, the Ulysses spacecraft.

³⁴⁷For ‘low mass’ bodies, whose mass is below a “mass cut-off threshold”, and recalling that the ‘shortfall rate’ is relative to a predicted motion (that assumes no anomalous/additional physical effect).

³⁴⁵Or alternatively, “all there is” to reality.

Table 6: Approximate corrections to the idealised Pioneer acceleration (a_p) for spacecraft path inclination (i.e. angular offset) relative to Doppler line-of-sight observations. The corrections are approximate because the angles for the line-of-sight observations are idealised, in that they are considered to be taken from the barycentre/heliocentre cf. the Earth’s surface (at 1 AU). We seek to *correct* the (one-dimensional) total (gravito-quantum rotating space-warp) acceleration amplitude $[\sum(\Delta a_w)_i]^{\frac{1}{2}} = 8.649 \times 10^{-8} \text{ cm s}^{-2}$, which [via $(\Delta a_w)_i = (\delta a)_i$] is also idealised a_p , for both solar ecliptic latitude and longitude. Only the longitude is significant, arising from the hyperbolic nature of the spacecraft’s path out of the solar system (recall Figure 2). With the Pioneer 10 and 11 path inclination to solar ecliptic latitude (as measured from the heliocentre) being of the order of: less than 1 degree and about 1.5 degrees respectively, the cosines of these angular differences are effectively negligible; subsequently, they are of no significant consequence in the post-correction “observable a_p ” results given in the last column of the table.

Spacecraft	Date ^m	Radius ⁿ	SE Latitude ⁿ	SE Longitude ⁿ	Angle ^o $[\varphi]$	$\cos \varphi$	Observable a_p
<i>Units</i>	[D–M–Y]	AU	degrees	degrees	degrees	—	$\times 10^{-8} \text{ cm s}^{-2}$
Pioneer 10	01 Jan 80	20.54	3.141	59.67	23	0.9205	7.96 ^p
Pioneer 10	01 Jan 87	40.01	3.105	70.99	12	0.9781	8.46
Pioneer 10	22 Jul 98	70.51	3.044	76.45	7	0.9925	8.58
Pioneer 11	01 Jan 87	22.39	16.58	254.93	33	0.8387	7.25 ^q
Pioneer 11	01 Oct 90	31.70	16.07	265.66	23	0.9205	7.96 ^q

^m Time of day is 00:00 UT (i.e. Universal Time). ⁿ Information is derived from JPL’s HORIZONS system, and is consistent with l_o and b_o given in Table III of Anderson et al. (2002, p.45, Appendix). ^o Further approximation arises because the (in plane) ‘inclination’ angles were determined (by the author, simply) by drawing the S/C path and the Sun-to-spacecraft line and then measuring the angle of inclination. This (approximate) approach gives angles that are only accurate to the nearest whole degree. ^p This slightly lower value is well within the noise of the observations — as evidenced by the variation in a_P results for the three intervals I, II and III comprising the 1987 to 1998 data set (Anderson et al. 2002, Table 1, p.22). ^q The model’s low Pioneer 11 values are of concern but there are strong mitigating circumstances (see discussion within subsection 6.5.9).

yses of temporal variation in Pioneer 11’s a_P value. Such a difference of opinion supports this subsection’s main contention that: considerably lesser importance should be (and indeed has been) ascribed to the Pioneer 11 navigation experiment (cf. the Pioneer 10 navigation experiment).

The recent paper by Turyshev et al. (2011) relies heavily upon results from Pioneer 11. Subsequently, their findings that the Pioneer anomaly is: non path-based, and exhibits a reduction/decay in its magnitude over time — so as to be supportive of a heat-based explanation — is not seen as a definitive rebuttal/refutation of the hypothesis (and model) of a real Pioneer anomaly pursued throughout this paper.

6.5.10 Corrected acceleration

By way of Table 6 the model’s average value for the *observed* anomalous acceleration, of Pioneer 10 between 1987 and 1998, is:

$$(a_p)_{\text{model}} = (a_{p10})_{\text{model}} = 8.52 \pm 0.66 \times 10^{-8} \text{ cm s}^{-2}$$

which displays the modelling error³⁴⁸ argued for in subsection 6.1.3. This magnitude is a sound quantitative match to the “headline” Pioneer anomaly acceleration:

$$(a_P)_{\text{observation}} = 8.74 \pm 1.33 \times 10^{-8} \text{ cm s}^{-2}$$

³⁴⁸Possibly “uncertainty of the model” is a better expression.

Note that the model’s average value for the *observed* anomalous acceleration of Pioneer 11 between 1987 and late 1990 is³⁴⁹:

$$(a_{p11})_{\text{model}} = 7.61 \pm 0.66 \times 10^{-8} \text{ cm s}^{-2}$$

This *average* (additional/‘anomalous’) roughly sunward and line-of-sight based ‘acceleration’ value (for Pioneer 10) is most appropriately understood as a *measured* (cf. actual path-based) total *loss* of speed over a duration of time, rather than as a (constant) force based acceleration. Actually, the full acceleration ($8.65 \times 10^{-8} \text{ cm s}^{-2}$) is directed against the *path* of the Pioneer 10 (and 11) spacecraft — which happens to be a hyperbola whose direction is predominantly radial, i.e. away from the solar system’s barycentre.

For non-radial spacecraft motion it is conceivable that the spacecraft’s rate of spin rotation might change with $a_p(t)$, as implied by the observational evidence — recall subsection 3.5.5.

6.5.11 A possible minor correction for the partial non-solidity of lunar bodies

The analysis (thus far) has assumed/idealised moons to be 100% *solid* bodies, although not necessarily *rigid* bodies — (the latter) so as to allow tidal effects to

³⁴⁹The orbit determination program of Toth (2009, p.18) generates a similar difference between the Pioneer 10 and 11 values: 9.03 vs. 8.21 ($\times 10^{-10} \text{ m/s}^2$) respectively.

achieve tidal locking (i.e. moon-planet orbital resonance). The non-solid aspects of a lunar body, e.g. molten core and/or mantle, vulcanism, and the presence of water, have not been fully appreciated. For example, Io and Ganymede are considered to have (partial) molten and liquid cores respectively, whereas Europa, Callisto and Titan are considered to have (or possibly have) an internal (relatively thin) layer of liquid water³⁵⁰ deep beneath their ice crusts. Nevertheless, the vast majority of matter in these five large moons central to the model *is* solid (surely $> 95\%$), generally comprising rocky material (especially silicates) and water ice.

Internal and/or surface motions of material (comprising atoms/molecules) within or upon a moon, i.e. non-gravitational lunar-based motions, will (conceivably) fail to keep the (atomic/molecular) geometric phase shifts (per spin/orbital cycle) to the minimal virtual level so crucial to the model — recall the numerous and tight constraints outlined in subsection 5.6.3. Thus, the number of molecules/atoms (N_m) in a bulk lunar body contributing to the energy of the (gravito-quantum) rotating space-warps will (in all likelihood) be reduced from the model's ideal values given in Table 4 — which assumed a completely (100%) solid (and considerably rigid) bulk lunar body³⁵¹.

Fortunately, because the amplitude of the various rotating space-warps (Δa_w) is dependent upon total lunar mass (M) rather than the total number of atoms/molecules comprising a 'solid' body (recall subsection 6.5.3), any correction arising from lunar non-solidity is restricted to only altering values of m_1^* and ΔE_w . Corrections to the (Pioneer anomalous) acceleration value a_p — based upon a path to observational line-of-sight angle (see Table 6 and subsection 6.5.10) — are also independent of N_m . The model's pre-correction 'acceleration' value (or rather speed short-fall rate) $a_p = [\sum(\Delta a_w)^2]^{\frac{1}{2}} = 8.65 \times 10^{-8} \text{ cm s}^{-2}$ — as quoted in subsection 6.5.5 and determined from weighted acceleration values in Table 5 — is effectively a fixed (or immutable) quantity.

Unlike many other scientific models applied to anomalous observations in need of an explanation, the model has no 'flexible' parameters that can be easily adjusted or 'tweaked'. In other words, the model's *primary* quantity (a_p), i.e. the full anomalous acceleration magnitude acting along the *path* vector of a moving body (not corrected for the observational line-of-sight)³⁵², is essentially closed to (theoretical or) model-based parameter adjustments. Further, we note that N_m is the (celestial physical) quantity whose magnitude is known with (by far) the least accuracy. With a_p not directly dependent upon 'participating' N_m , its

'component' $\Delta a_w (= \delta a)$ values are determined (in the model) with better accuracy than either ΔE_w , m_1^* , or the various $m^*(r)$ distribution values discussed in section 6.6 and given in Table 7.

6.6 Non-local mass distribution

This section continues the quantification of the macroscopic physical model, involving a number of rotating space-warps (RSWs) in superposition, and the non-local mass distribution associated with each particular RSW. The following discussion of non-local mass needs to clearly distinguish mass from (ordinary/non-exotic) matter, in the sense of *matter* being a tangible or observable physical 'substance' that: occupies space, has rest mass, and is comprised of "building blocks" (or is itself a building block) — e.g. a moon, a lake, a grain of sand, an atom, an electron, or a quark.

6.6.1 Background to non-local mass

To distinguish non-local mass from standard (baryonic) mass we indicate it as: m^* . The non-particle-like nature of the model's non-local mass effect³⁵³, means that it is associated with both: conditions *at a point* in a (space) field; and also a *distribution* effect throughout space [$m^*(r)$]. We shall see that $m^*(r)$ exists at every point upon a spherical surface that encloses a volume (of specified radius r). As is the case with time, the 'essence' of m^* shall not be of major concern herein; rather, our major concern is with the model's quantification of m^* , particularly its spatial variation.

Note that m^* has its basis in the mass 'dimension' of Dirac's constant \hbar , which appears in the (total) virtual QM energy expression (Equation 16, section 6.3).

The geometric basis of the model ensures that non-local mass is best understood in association with the total volume *enclosed* by the 'establishment' (or reference) distance from the source ($r_1 = 8\pi r_o$). With its volume dependence, m^* is somewhat like the density of standard matter — which has dimensions: $[\frac{M}{L^3}]$. In the spirit of QMs, the introduction of m^* means that 'mass' is exhibiting a (local-nonlocal) duality — rather than (i.e. as distinct from) a wave-particle duality³⁵⁴.

The distribution of non-local mass (at any point) in the field is assumed to be continuous, and change very gradually between neighbouring regions. Of special importance is the change in the magnitude of non-local mass as one moves away from its 'source' (or initial/establishment) surface and volume, and thus, there is a need for a (non-local) mass distribution function. This is addressed from subsection 6.6.5 onwards.

³⁵⁰ Assisted by the presence of antifreezes, e.g. ammonia.

³⁵¹ Although, if the external/extrinsic imposition upon 'internal' *geometric* phase, by way of its virtual 'nature', is completely independent of quantum mechanical *dynamical* phase, then the macroscopic entanglement effect (associated with it) may well remain at 100%, and as such there would be no correction required for (partial) non-solidity.

³⁵² That is, a_p as given is subsection 6.5.5, as compared to the (Pioneer 10 based) 'observational' (a_p)_{model} value given in subsection 6.5.10.

³⁵³ I am reluctant to say non-local mass has a (classical) 'wave-like' nature because 'wave' implies a *transport* of energy, whereas we only require the wave characteristic of being a de-localised phenomena, i.e. one spread out in space.

³⁵⁴ Possibly, a (non-reductionist) local-nonlocal duality is more general than the wave-particle duality. Such a duality or complementarity has been discussed in relation to the Aharonov-Bohm effect by Aharonov & Reznik (2000, Abstract) — albeit with a different emphasis 'in mind'.

6.6.2 Comments regarding the governing energy expression

In the expression for the total supplementary field energy of a rotating space-warp (and its conjoint mass):

$$\Delta E_w = \frac{1}{2} m_1^* \Delta a_w^2 \Delta t^2$$

(i.e. Equation 18, section 6.5) the mass and acceleration components are separate (recall subsection 6.4.1). The total scalar energy ΔE_w can be understood as simply a ‘mass’ multiplied by specific energy. That is:

$$\Delta E_w = m_1^* \Delta e_w$$

The specific energy (Δe_w) is comprised of (a sinusoidal) acceleration amplitude and cyclic duration (Δt) and it is essentially always fixed, because of the dynamically ‘stable’ moon-planet-Sun celestial systems in our solar system³⁵⁵. Note that m_1^* is actually non-local mass *at* the (initial) ‘establishment’ radius, i.e. the reference radius, $r_1 = 8\pi r_o$; further, we signify V_1 as the spherical volume enclosed within this radius. Thus, upon establishment of the space-warp, all quantities: ΔE_w , m_1^* , and $\Delta e_w = \frac{1}{2} \Delta a_w^2 \Delta t^2$ may be considered *fixed*, i.e. (long-term) constant values.

6.6.3 Total energy constancy, and the conceivable dispersion of non-local mass

The external counterpart of the mass aspect of the total excess (non-inertial) QM (intrinsic angular momentum based) energy (ΔE_w) is m_1^* . The (total energy) equality: $\Delta E_w = m_1^* \Delta e_w$ is assumed to (only) apply at *every* ‘point’ on the initial (spherical) *surface* and not throughout the (spatial) field exterior to the initial radius³⁵⁶, i.e. $r > r_1$. Further, the magnitude of m_1^* is associated with an initial (enclosed) volume (V_1).

Note that if we (were to) propose an equality, involving a surface energy, such that (at $r = r_1$):

$$\Delta E_w = (E_w)_{\text{surface}} = \gamma_1^* S_1$$

then γ_1^* has dimensions $[\frac{M}{T^2}]$. Interestingly, both spring stiffness (K) and fluid mechanical surface tension (γ) also have dimensions $[\frac{M}{T^2}]$. This notion is not pursued.

Physical circumstances pertaining to the space-warp’s energy description when $r > r_1$ are quite distinct from the ($r = r_1$) establishment conditions. Due to the very slow evolution of moon-planet-Sun celestial systems, and the constancy of the various Δa_w values, the various (lunar) specific energies Δe_w can fortunately be treated as a fixed quantity at *every* point *throughout* their respective rotating space-warp fields. Thus, for $r > r_1$ our concern lies with how m^* varies with r , or rather (enclosed) spherical volume. We denote *variable* (and distributed) m^* as $m^*(r)$ which is abbreviated to m_r^* ; with $m^*(r_1)$ written as m_1^* .

³⁵⁵At least over human time scales, with these being of the order of tens, hundreds, or thousands of years. The proportion of non-solid matter in a moon also remains essentially fixed (recall subsection 6.5.11).

³⁵⁶The conditions interior to $r = r_1$ shall not concern us in this subsection, nor have they been of any significant importance (to the model) elsewhere in this paper.

For a given space-warp the magnitude of the *total* field energy ΔE_w is independent of position (r) in the field, even though (in the formalism) a specific radius ($r = r_1$) is implicitly associated with m_1^* . This is because non-local mass — which is unlike standard (inertial, passive and active gravitational) mass — requires an initial reference radius (recall section 6.5). Nevertheless, rather than a *total* field energy, we can still conceive of a (non-local) field energy at *points* in the field: (say) $E_{\text{field}}^*(r) = m_r^* \Delta e_w$, which varies with m_r^* . We also recollect that this energy — which pertains to a *single* rotating space-warp and non-local mass distribution — is process based, i.e. occurring over Δt . Thus, equating this energy $E_{\text{field}}^*(r)$ [or rather $(E_r^*)_{\text{component}}$, using the nomenclature of Section 7] to the notion of gravitational energy (at a point in the field) is nonsensical; as is (the case with) gravitational energy at a point in the field (at a given time) in general relativity. In stark contrast to this, we have the well defined *specific* energy of a (360°) rotating space-warp — in two and three dimensions — i.e. $\Delta e = \frac{1}{2} \Delta a_w^2 \Delta t^2$, which is invariant throughout ‘space’ and effectively fixed/constant throughout (cosmologically recent) ‘time’.

6.6.4 Global or universal aspects of the wave-like non-local mass distribution

Just as Δe_w fills the whole of space (i.e. the universe), the mechanism’s non-local mass (m_r^*) also fills the universe. For points $r > r_1$ (i.e. in the far-field) the associated (spherical) enclosed volume and surface area increase in magnitude relative to the initial reference conditions (where $r = r_1$). In analogy with classical mechanics, we hypothesise the notion of *energy dissipation*, and hence (non-local) *mass dispersion*, as we examine conditions at points progressively further away from the source, or rather ‘core’, of the non-local mass distribution. This core is the (initial and) minimum volume (and surface area) of the mass component of the virtual QM energy’s externalisation.

With Δe_w constant, a far field (pseudo)-energy at a point in the field $[(E_r^*)_{\text{component}}]$ is seen to vary in proportion to m_r^* . The different physical ‘natures’ of non-local mass and acceleration/gravitational (field) warp amplitude in the model means that this far-field energy cannot be localised. What can be asserted with confidence is that *total* (QM-based) scalar energy ΔE_w is fixed/unchanged/constant (i.e. invariant) regardless of the radius r , surface area S , or enclosed volume V considered. This *scale* independence of total (non-local mass and space-warp) energy arises from its (closed loop) QM virtual energy basis, and its real exterior expression of universal extent or size — in that the space-warp extends to infinity in the plane, and its rotation axis also extends to infinity. It is (only) non-local *mass* that varies with (the) scale (considered) — thus incorporating all scales from an minimum/‘initial’ reference (sub-universal) system, all the way up to the full (universal) system (where $r^3 \rightarrow \infty$ and $m_r^* \rightarrow 0$). This variation in *non-local* mass magnitude is necessarily subject to another *geometric* constraint of the model — see discussion in subsections 6.6.5 and 6.6.6.

Note that these (spatially based) changes (in non-local mass) can only be apprehended by way of the existence of a systemic (i.e. global and/or universal) space continuum or space substratum³⁵⁷. Variation in m_r^* [or alternatively $m^*(r)$] by way of a variation in the enclosed (spherical) *volume* (and indirectly surface area) is our prime concern; we note/recall that m_r^* (itself) describes conditions on the surface of the (enclosed) volume. The magnitude and distribution of m_r^* has been constrained (and indirectly implied) by observation evidence (recall section 2.6) concerning small comets of approximately 1 km diameter (with mass of $\sim 2.6 \times 10^{11}$ kg) apparently being (of the order of) the largest size/mass of bodies ‘anomalously’ influenced.

6.6.5 Quantifying the variation of non-local mass with the enclosed volume

The inception, i.e. establishment, of a (single/specific) rotating space-warp and non-local mass distribution involves both: a reference (or minimal) *enclosed* volume V_1 , and m_1^* — which is non-local mass *at a distance* $r = r_1 = 8\pi r_o$ from the space-warp’s central point³⁵⁸, where $r_o = r_a$ i.e. lunar semi-major axis.

Note that a rotating space-warp (slice in two dimensions) has a point-like centre of rotation (i.e. an origin), but there is no ‘point’ source (*per se*) for the space-warp’s (associated) non-local mass. The term ‘source’, rather than ‘origin’, shall designate the finite: radius (r_1), volume (V_1), and surface area associated with the initialising/establishment of non-local mass. The relevance of this distinction is lessened when large radii from the centre of rotation are considered.

The model has proposed/argued that: although the non-local mass m_1^* in the expression for ΔE_w is a fixed quantity, the value of the non-local mass *at a point* in the far-field (m_r^*) diminishes as r^{-3} (i.e. m_r^* reduces with increased volume enclosed) for radii $r \geq 8\pi r_o$ (i.e. $r \geq r_1$) away from the source of a rotating space-warp. This is based upon observational evidence and theoretical compatibility with GR, the latter because the specific energy (ΔE_w) of the rotating space-warp is the *same* throughout the field — at least for stable lunar orbital conditions, and also because (cosmologically very short) time scales (Δt), of the order of weeks, are involved. Recall that the quantity ΔE_w — i.e. the virtual QM energy requiring external ‘re-expression’ — only applies to non-local mass (and rotating space-warp) *establishment* — in that it contains the ‘initialising’ non-local mass value (m_1^*).

Herein we have been able to treat specific energy ΔE_w as a constant. Slow changes in specific energy ΔE_w , i.e. changes in either orbital time and/or acceleration amplitude, propagate (and spread) into the field at the speed of light. In contrast, the apparent simultaneity associated with quantum non-locality means that, in ‘*measurable time*’, non-local mass ‘propa-

gates’ (and spreads) into the field/universe instantaneously. In other words, non-local mass is coexistent throughout the field at any given (‘noumenal’) ‘*systemic time*’³⁵⁹. In accord with section 4.4 we may say that systemic time is a theoretical second time, for global (hidden) processes occurring between the moments grasped by measurements — such that some processes (especially entanglement) *appear*, from an observational/‘phenomenal’ perspective, to involve instantaneous ‘interactions’ (and “action at a distance”).

We have seen that the *dissipation* of total (non-local mass and gravitational/accelerational) field *energy* lies solely in the (spatial) *dispersion* of the non-local mass term (m_r^*). Subsection 6.6.6 shall argue that at greater distances/volumes from a source there is a similar reduction in the (passive) mass of bodies (m_p) that the space-warp’s conjoint/associated m_r^* can affect³⁶⁰. We shall see that *non-local* mass m_r^* is markedly distinct/different from inertial mass m_i and gravitational mass (both active and passive) m_g , in that the equality $m_i = m_g$ is not extended so as to include m_r^* . In general, the non-local mass field value does not equal a body’s (local) mass value, i.e. $m_r^* \neq m_i$ and $m_r^* \neq m_g$.

The model proposes a (non-local mass) *distribution function*, with dimensions $[ML^3]$, of the form:

$$m^*(r) V(r) = \text{constant} = m^*(8\pi r_o) V(8\pi r_o)$$

or more succinctly:

$$m_r^* V_r = \text{constant} = m_1^* V_1 \quad (20)$$

where: $m^*(r)$ or m_r^* is the non-local mass value at (spherical) radius r , and $V(r)$ or V_r is the *volume enclosed* within r . Once again we note that this distribution function is restricted to $r \geq 8\pi r_o$, i.e. $r \geq r_1$.

6.6.6 Distribution equation ramifications and values for the different moons

We can compare and contrast this non-local-mass distribution function/equation in *space* to one form of the continuity equation in fluid mechanics for a steady flow: $\rho_n A_n u_n = \text{Constant}$ ³⁶¹. This is a conservation of mass equation associated with (mass) motion in *time*, and has dimensions $[\frac{M}{T}]$. It follows that in one sense Equation 20 — with its dimensions/units of $[ML^3]$ — can conceivably be described as a *spatial non-local-mass continuity equation*.

In Table 7 the variation of non-local mass with distance from its source is detailed. Each of these non-local masses (m_r^*) are referred to as a “cut-off mass”, because if the mass of a moving body (m_p

³⁵⁹Except for a small central area (or tube in three dimensions) of radius $8\pi r_o$.

³⁶⁰“Passive mass” or “passive ‘gravitational’ mass” is now being used in the sense of a mass (that also) responds to the new acceleration/gravitational fields proposed herein — in addition to standard gravitation.

³⁶¹Where: $n = 1, 2, 3, \dots$, ρ is fluid density, A is flow area, and u is speed. Usually, continuity equations are expressed as a differential equation, and thus are a (stronger) *local* form of conservation laws — paraphrased from Wikipedia: *Continuity equation*, 2010-12.

³⁵⁷In the sense discussed in subsection 4.4.10.

³⁵⁸The central point of the space-warp generator/emitter is probably the moon’s centre, but possibly it is the centre of the moon-planet system. This conceptual inexactitude has a negligible quantitative effect upon the model (as presented herein).

Table 7: Mass cut-offs at various distances for large (effective) solar system moons.

MOON	Units	Io	Europa	Ganymede	Callisto	Titan	Triton
Weighted Energy (ΔE_w)	(10^9 J)	2.395	1.287	3.973	1.854	1.452	0.041
Moon orbit frequency (f_m)	(10^{-6} s $^{-1}$)	6.5422	3.2592	1.6177	0.69351	0.72586	1.9694
Weighted acceleration ^r (Δa_w)	(10^{-10} m s $^{-2}$)	5.139	2.190	5.354	1.884	3.348	0.416
Mass cut-off at $8\pi r_o$ (m_1^*)	(10^{15} kg)	692	507	64.6	44.7	12.2	165
Semi-major axis (r_o)	(10^9 m)	0.4218	0.6711	1.0704	1.8827	1.2219	0.3548
Transition radius ($8\pi r_o = r_1$)	(AU) ^s	0.0709	0.1127	0.1798	0.3163	0.2053	0.0596
No. of transition radii in 1 AU	(-)	14.11	8.87	5.56	3.16	4.87	16.78
Mass cut-off at 1 AU	(10^{12} kg)	276	817	422	1590	118	39.2
Mass cut-off at 2.5 AU ^t	(10^{12} kg)	17.7	52.3	27.0	101.7	7.56	2.51
Mass cut-off at 10 AU ^t	(10^9 kg)	276	817	422	1590	118	39.2
Mass cut-off at 100 AU	(10^9 kg)	0.276	0.817	0.422	1.590	0.118	0.039
Mass cut-off at 100 000 AU ^u	(kg)	0.276	0.817	0.422	1.590	0.118	0.039

^r Non-corrected values. ^s Astronomical Unit (149.598×10^9 m). ^t Compare to comets of various diameters (comet density assumed equal to 0.5×10^3 kg m $^{-3}$): 1 km $\rightarrow 0.26 \times 10^{12}$ kg, 3 km $\rightarrow 7.1 \times 10^{12}$ kg, 5 km $\rightarrow 33 \times 10^{12}$ kg.

^u 100 000 AU is a distance equal to 1.58 light years or approximately half a parsec.

say) is above this (lunar specific) mass, i.e. $m_p > m_r^*$, then the body cannot respond to the model's (rotating space-warp based) additional perturbation of acceleration/gravitational field strength — see subsections 6.6.7 and 6.6.8 for further discussion. Clearly, the Pioneer S/C with a mass of approximately³⁶² 259 kg lies well below the various cut-off masses of the various ('active') moon-planet-Sun systems — regardless of the relative distance between the moon and spacecraft. By contrast, the large mass of the planet Uranus for example, at 8.68×10^{25} kg, is always immune from any influence. Thus, the *apparent* presence of a violation of the weak principle of equivalence with regard to (local inertial and/or gravitational) mass in a gravitational field, as associated with the Pioneer spacecraft anomaly, is (i.e. has become) non-problematic.

6.6.7 Relating non-local mass to local 'bulk' physical mass (at a point)

Obviously, matter-comprised bodies are not point masses; they have (three-dimensional) extension in (three dimensional) space. In this subsection we enquire: how do the model's distributed "at-a-point" non-local mass field values [$m^*(r)$ or m_r^*] — that extend throughout space — relate to a finite size macroscopic physical body (m_p)? We seek to address this issue fully aware of the ambiguity surrounding just what non-local mass (m^* , or m_r^* at a distance r from its source) actually is.

In (classical) Newtonian gravitation we can usually assume that the extended mass of a macroscopic

body acts as if it is (totally) located at a *central point*. This idealisation relies on two things: firstly, the spherical nature of (macroscopic) gravitational influence, and the fact that the field values of interest lie well beyond the surface of the 'extended' (gravitational source) mass itself; and secondly, that a passive (gravitational) mass in a gravitational field is treated as a point mass. In the model, quite different reasons shall result in a similar (point-mass) circumstance. Clearly, we need to go beyond classical physics (alone) to elucidate the nature of an interaction/relationship concerning m_r^* and the motion of an (isolated in space) macroscopic physical body (m_p).

In physics, the correspondence principle states that: the behavior of systems described by the theory of quantum mechanics reproduces classical physics in the limit of large quantum numbers. In subsection 4.1.6 we mentioned that: "whenever the correspondence principle holds, the centre of mass of a quantum wave packet (for either a single particle or for an entire quantum object) moves according to Ehrenfest's theorem along a classical trajectory."

Obviously, any collection of neighbouring atoms/molecules in a bulk-matter body has a centre of mass. A bulk/macroscopic body may alternatively be conceived (of) from a quantum mechanical (wave-like) 'perspective'; as such, each and every atom/molecule (only) has a quantum wave packet associated with it. Importantly, these numerous wave packets are distributed (over the "whole of space"). This (corresponding and complementary) conception of a macroscopic body is analogous to the inherently *distributed* (throughout the universe) 'nature' of non-local mass. Subsequently, we note that, just as non-local mass (in the model) can have *both* a point-like value in a field, as well as a distribution of

³⁶²The Pioneer spacecraft launch mass comprised a total module "dry weight" of 223 kg and an additional 36 kg of hydrazine propellant for the three thruster pairs (Anderson et al. 2002, pp.2-3).

values throughout the field; so the mass of a macroscopic body (e.g. a spacecraft) may be conceived of in both a (distributed/non-local) wave-like manner as well as in a corresponding (local) particle-like manner — with both conceptions ‘appreciating’ the usefulness/relevance of a (point-like) centre of mass.

Taking this (dual) correspondence one step further, we hypothesise that: as far as the interaction between m_r^* and m_p and the model’s quantification is concerned, the point-like ‘existence’ of non-local mass in a field has a ‘correspondence’ to bulk/macroscopic matter existing effectively at a *point* in the field, and thus we assume that m_r^* interacts with the macroscopic matter m_p “as if” the m_p existed solely/totally at its centre of mass. Importantly, we (further) conjecture that m_r^* is *concurrently* interacting with (the QM ‘face’ of) macroscopic/bulk matter m_p in a (corresponding) quantum mechanical wave-like manner, and as such the ‘at-a-point’ (in space) relationship between m_r^* and (total) m_p is not an idealisation — as is the case in classical gravitational physics — but a true representation of their (field point-to-‘particle’) interrelationship. Note that conceptually ‘we’ — meaning people and scientists — have a strong tendency/bias towards a particle-like (cf. a wave-like) perspective on things.

Planets, moons, comets, asteroids, and spacecraft are all (considered) sufficiently compact/condensed (so as) to obey this (m_r^* to m_p) interactive feature, but not so a (whole) galaxy (for example). In subsection 6.6.8 we build upon this hypothesis/conjecture and further describe how the m_r^* upon m_p ‘interaction’ is perceived and treated in the model.

6.6.8 How non-local mass influences physical objects with (passive) mass

If the passive (condensed) mass (m_p) of (for example) a spacecraft ($m_{s/c}$) or celestial body (m_{body}), is *less* than the non-local mass (m_r^*) — at a given distance from a *particular* rotating space-warp ‘generator’ — then that mass *is* considered to be influenced by the new cyclical acceleration field variation, otherwise it is not. Thus, the Pioneer anomalous acceleration affects spacecraft (because $m_{s/c} \ll m_r^*$), but not planets, moons, and ‘large’ asteroids, where $m_{\text{body}} > m_r^*$ — at all times and distances³⁶³. See Table 7 for the different ‘cut-off’ masses (associated with each particular moon-planet-Sun system) and typical comet size to comet mass relationships; (and) for comparative purposes see/recall Table 5 for “large moon” masses.

The physics behind this ‘interactive’ relationship between (each specific) m_r^* value and a body’s m_p is indeed hypothetical. The physical nature of this relationship is considered to be somewhat analogous to the mathematical (and physical) concept of *convolution*, in the sense of: “the mathematical technique for determining a system output given an input signal and the system impulse response”.

In the analogy: m_r^* (and a space-warp undulation/sinusoid of amplitude Δa_w) are the input signals; the system impulse response is determined by a body’s m_p ; and the system output is the additionally perturbed (or non-perturbed) motion of the spacecraft³⁶⁴. Although usually associated with the frequency response of a physical observing instrument (that possesses inertia), Bracewell (2000, p.24) notes that:

Later we show that the appearance of convolution is coterminous³⁶⁵ with linearity plus time or space invariance, and also with *sinusoidal* response to sinusoidal stimulus.

With the (multiple) sinusoidal stimuli/inputs being the multiple/various oscillatory RSWs (of amplitude Δa_w) over their respective cycle times Δt , and the response being the spacecraft’s various speed (Δv) variations/sinusoids and speed losses (δv) per cycle. In the model (only) five moon-planet systems dominate the Pioneer acceleration anomaly; Io, Europa, Ganymede, Callisto with Jupiter; and Saturn’s Titan. The Earth-Moon system, with its collision based heritage, is (for geometric reasons) not a space-warp ‘generator’ — as indicated in Tables 2 and 3.

The physical nature of this (proposed) m_r^* to m_p interaction appears to implicate some kind of inherently quantum mechanical wave-like aspect of m_p in the interaction (recall subsection 6.6.7), that can only (at this stage) be figuratively appreciated and formally represented in a rudimentary manner. The conceptual intractability (or ‘weirdness’) of QMs provides little assistance in elaborating upon this coarse description.

How the model’s non-local mass might ‘interact’ with the Higgs field (and mechanism) is not discussed.

6.6.9 Commenting upon local and global spatial quantities in the new model

The constancy of the (non-local) mass-volume product and distribution relationship of Equation 20 in subsection 6.6.5, with dimensions $[ML^3]$, ensures this new *mass-spatial* product/relationship describes a *global* quantity. In contrast to this, the non-local mass at a point in the field (m_r^*) is a *local* quantity, albeit of a distributed non-local ‘pedigree’. By its very nature, the density of standard mass, which has dimensions $[M/L^3]$, is also a (spatially) distributed quantity³⁶⁶. Density is usually treated as a non-global (i.e. body-specific) quantity — although it (along with total baryonic mass) also has one extremely global application, i.e. its application to the universe as a whole.

The constant quantity $m_r^* V_r$, where V_r is an *enclosed* spherical volume (of radius r), is: new, unnamed, and without a representing symbol. Tentatively, we could call this quantity a “volumetric non-local mass distribution constant”, in that it establishes

³⁶³Thus, one can create a demarcation between large and small asteroids (and comets) based upon this criteria. Interestingly, small comets (< 1 km diameter) are unusually rare (recall section 2.6).

³⁶⁴Like the photoelectric effect, this is an “all or nothing” system output.

³⁶⁵Meaning: being the same in extent; coextensive in range or scope.

³⁶⁶Indeed, density is (arguably) ‘undefined’ at a point.

the volumetric distribution of the non-local mass component — of the externally re-expressed (QM spin offset) energy ΔE_w . Dimensionally, this new quantity is peripherally ‘related’ to both: first moment of area [L^3], and (mass) moment of inertia [ML^2] — both of which involve *rotation* in some sense, with the latter also involving a *distribution* of matter.

Interestingly, the hypothesised existence of (each) space-warp involves a rotation; further, each (gravito-quantum) rotating space-warp is most readily idealised as an (*acceleration/gravitation* based) phenomenon, with a tubular core³⁶⁷, which exists in *conjunction* with a spherical non-local mass distribution that has a spherical ($r = r_1$) core region³⁶⁸. Thus, this new quantity’s constancy, and its (unique) dimensionality of [ML^3], might be related to the geometric conflict between (a pure) spherical volume and a rotating tubular-centered volume discussed in subsection 6.5.1.

Finally, it appears that any further rotation of a moon-planet-sun system, e.g. within a galaxy (and so forth), is not problematic to the model — in that $m^*_r V_r = \text{constant}$ (Equation 20) applies regardless of this motion. On a more speculative note, there is a possibility that the constancy of a distributed mass times volume product/relationship could have application to the (formation and) evolution of spiral galaxies, and/or the very early formation of the solar system.

6.6.10 1862 Apollo: an asteroid exhibiting intermittent anomalous behaviour?

The mass and orbital characteristics of the asteroid “1862 Apollo” makes it an ideal ‘exhibiting body’ of, and ‘test body’ for examining, the model’s proposed (distance dependent) non-local mass cut-offs [$m^*(r)$, or alternatively m^*_r] associated with the four Galilean moons of Jupiter. Saturn’s greater distance ensures that the rotating space-warp, associated with its large moon Titan, fails ‘at all times’ to influence this asteroid’s motion, i.e. $m^*(r) < m_{1862}$ for all possible distances (r) between 1862 Apollo and Saturn-Titan.

By way of various sources/websites³⁶⁹ 1862 Apollo has a mass of $\approx 5.1 \times 10^{12} \text{ kg}$ (denoted m_{1862}), a mean diameter of approximately 1.7 km ³⁷⁰, an aphelion of 2.294 AU , a perihelion of 0.647 AU , an orbital inclination of 6.354° , and an orbital period of 651.4 days. It is a Near Earth Asteroid (NEA) that crosses the orbits of Mars, Earth and Venus. These physical and orbital characteristics ensure that ‘Apollo’ can receive either: most, some, or none of the retardation effect stemming from the individual (lunar-‘driven’) rotating space-warps (RSWs) — depending on the location of Jupiter (with its four Galilean moons) and the asteroid itself, and the separation distance between them.

³⁶⁷Planar in two dimensions, tubular in three dimensions.

³⁶⁸This tubular core (around the space-warp’s axis of rotation) vs. spherical core distinction further discourages the possible implementation of a local energy (E^*_{field}) throughout space (as mentioned in subsections 6.6.3 and 6.6.4).

³⁶⁹Including: IAU Minor Planet Center, JPL Small-Body Database Browser, Johnston’s Archive, and Wikipedia.

³⁷⁰Associated with a mean density of $\approx 2.0 \text{ g/cm}^3$, i.e. $\approx 2.0 \times 10^3 \text{ kg/m}^3$.

With Jupiter having an aphelion of 5.458 AU and a perihelion of 4.950 AU , it turns out that the values of $m^*(r) = m_{1862}$ for Io, Europa, Ganymede and Callisto all lie at distances *between* their closest possible approach of 2.656 AU , and a maximum possible separation distance of 7.752 AU .

By way of using the mass cut-off values at 2.5 AU (or/and 10 AU) as presented in Table 7, we may extrapolate (or interpolate) to determine the distance (r) where $m^*(r) = m_{1862}$, i.e. where the (non-local) mass cut-off value equals the asteroid’s mass. These values, in ascending order of distance, are: Io 3.78 AU , Ganymede 4.36 AU , Europa 5.43 AU , and Callisto 6.78 AU ³⁷¹. Thus, none, one, two, three, or all four RSWs associated with Jupiter’s Galilean moons can be actively ‘causing’ the (type of) motion retardation (cf. predicted motion) that also affects the Pioneer 10 and 11 spacecraft³⁷².

Assuming that the true mass and density values of asteroid 1862 Apollo are near their currently estimated values, the distance dependent activation and deactivation of the RSW-based motion retardation mechanism (upon the asteroid) then becomes an ‘in principle’ key prediction of the model. In practice, the observation of this (four ‘geared’) engaging and disengaging behaviour will be extremely difficult to measure, due to the ‘smallness’ of the effect and the asteroid’s position in the inner solar system where radiation pressure effects are not insignificant³⁷³.

Our ability to directly observe asteroid 1862 Apollo from Earth over an extended period of time (albeit intermittently) is a positive; but the tiny motion variation effects³⁷⁴ and the cumulative (per cycle) motion retardation effect involved appear to be beyond the combination of our best models and best observational techniques — both now and in the near/foreseeable future. Methods involving: an accurate radiation pressure model, Doppler effects, and angular triangulation can be envisaged, but they are ‘futuristic’. Conceivably, this would involve (for example) either: (1) a transmitter/transponder (of some kind) upon the asteroid itself, in conjunction with observing equipment upon: Earth, Mars and a ‘large’ (unaffected) asteroid; or (2) an observing device upon the (unfortunately rotating³⁷⁵) asteroid itself, utilising the accurately known locations of three or more planets³⁷⁶.

³⁷¹Note that the value for Saturn’s large moon Titan is a mere 2.85 AU .

³⁷²Albeit without the contributions arising from ‘distant’ Titan and Triton.

³⁷³Note that if the model’s non-local mass cut-off values were out by a factor of two (for example), due to some minor mathematical oversight, the choice of the asteroid 1862 Apollo would still be of some use (but no longer ideal).

³⁷⁴As the strength of each supplementary gravitational field, associated with each individual RSW, undulates the asteroid’s speed responds sinusoidally around an equilibrium speed — assuming $m^*(r) > m_{1862}$. Note that in practice only a superposition of these individual effects actually exists.

³⁷⁵We also note that the YORP (Yarkovsky-Okeneff-Radzievskii-Paddack) effect upon this asteroid has been successfully modelled.

³⁷⁶If this motion variation effect were measurable with a high degree of accuracy, and the proposed model is valid,

A more qualitative approach involving comparing the errors accumulated over time in the predicted paths of asteroids (of various masses), e.g. (re: Apollo asteroids) the more massive 1620 Geographos and/or the less massive 1566 Icarus, appears to be the best option *currently* available. Furthermore, a somewhat step-like discrepancy in predictive accuracy involving the orbit determination of asteroids whose mass is either side of an ‘intermittent zone’ — spanning $\approx 1.0 \times 10^{12}$ kg to $\approx 2.7 \times 10^{13}$ kg (roughly corresponding to a mean diameter range of 1.0 km to 3.0 km) — would indirectly support the existence and quantification of the non-local mass cut-offs proposed by the model. Subsequently, such a result would (also) provide good support for the model “as a whole”. Note that ‘large’ asteroids (i.e. > 3 km in diameter) will not display the additional/anomalous ‘deceleration’ and thus they should have a lesser discrepancy in their orbit determination — all other things being equal.

6.7 The Pioneer anomaly, GR, and the three Equivalence Principles

This section investigates the new model argued for herein in relation to two principles associated with the establishment of general relativity: the principle of equivalence and the general principle of relativity. Such an analysis serves to clarify the veracity of the gravitational supplementation that has been proposed.

6.7.1 Introduction

Currently, general relativity (GR) is generally regarded as the only theory (and “final word”) on gravitation. If we assume that all systematic causes of the Pioneer anomaly are ruled out, and embrace the rotating space-warps (RSWs) — or more formally the ‘gravito-quantum’ rotating space-warps (GQ-RSWs) — implied by the Pioneer anomalous observations, then a standard appreciation/conceptualization of gravitation is necessarily incomplete.

In this section (i.e. 6.7) we more fully examine the relationship of the Pioneer anomaly to different forms of the equivalence principle (EP)³⁷⁷. Of particular interest is the apparent violation of the weak EP, also known as the principle of universality of free fall, with regard to the motion of low mass bodies³⁷⁸ (e.g. the Pioneer spacecraft). The ramifications of a ‘real’ Pioneer anomaly upon our understanding of gravitation in its widest sense is also investigated, with an emphasis given towards understanding how our new model is *not* ruled out by GR’s general principle of relativity (GPrR).

Previously, it was argued that: the Earth flyby anomaly is (merely) an observational artifact (subsection 6.2.4) that arises indirectly from the (gravito-quantum) RSWs associated with (and instigating) the

then the mass and average density of ‘small’ asteroids could conceivably be determined with significantly greater accuracy than is currently the case.

³⁷⁷Weak, strong, and the Einstein equivalence principle.

³⁷⁸Below the cut-off mass at a given position [i.e. $m^*(r)$ or alternatively m_r^*].

Pioneer anomaly. Consequently, our discussion of equivalence principle violation, within the solar system³⁷⁹, can be solely restricted to the (idiosyncratic) ‘Pioneer anomaly’ case.

6.7.2 General remarks regarding a violation of the Equivalence Principle

This subsection provides a preliminary guide as to how a violation of the equivalence principle (EP) is handled by respected authors. We briefly explore this issue in terms of the Pioneer anomaly specifically, and also the general conceivability of an EP violation.

Firstly, the type of violation is unexpected.

Neither case [Pioneer anomaly, nor Earth flyby anomaly] matches expectations for an EP violation; for example, the directions of the anomalous accelerations do not match [the authors] Equation (1) (Anderson & Williams 2001, p.2449).

Secondly, the violation itself is based upon an extremely minor additional acceleration/gravitational field occurring in the solar system’s *weak* gravitational field. Such a minor violation³⁸⁰, especially one associated with quantum mechanical energy in ‘partnership’ with a curved spacetime field, is not a cause for alarm — and indeed something like it has been ‘heralded’ by Clifford Will.

...there is mounting theoretical evidence to suggest that EEP [Einstein equivalence principle] is *likely* to be violated at some level, whether by quantum gravity effects, by effects arising from string theory, or by hitherto undetected interactions. Roughly speaking, in addition to the pure Einsteinian gravitational interaction, which respects EEP, theories such as string theory predict other interactions which do not (Will 2006, p.23).

Thirdly, our approach of considering the GQ-RSWs as additional/supplementary to general relativity’s (approach to) gravitation is not without some loose ‘ideological’ support.

...the only theories of gravity that have a hope of being viable are metric theories, or possibly theories that are metric apart from very weak short range non-metric couplings (as in string theory). ... In principle, however, there could exist other gravitational fields besides the metric, such as scalar fields, vector fields, and so on (Will 2006, p.26).

³⁷⁹Subsection 6.2.6 raised the issue of a measured/apparent ‘change’ in the fine structure constant. This implies a violation of the Lorentz Position Invariance aspect of Einstein’s equivalence principle (EEP). Due to the provisional nature of this observation, and the great distances involved, this issue has not been pursued.

³⁸⁰At least over fairly short (celestial) time intervals, i.e. of the order of days, weeks, or months — and even years.

In our case/model the situation is somewhat different, with the proposed very weak (non-metric-based) ‘coupling’ being (non-local and) long range, and taking the form of two scalar fields: a constant amplitude (Δa) sinusoidal/oscillatory field (also representing non-Euclidean geometry), and a variable non-local mass [$m^*(r)$] scalar field.

Thus, to simply dismiss the Pioneer anomaly on the grounds of it indicating a violation of the EP is too quick and inappropriate; although there appear to be (*prima facie*) sound reasons for this dismissal, not the least of which is how (multiple instantiations of) these scalar fields (particularly Δa) coexist with standard general relativistic spacetime curvature. To fully appreciate how a real (i.e. non-systematic based) Pioneer anomaly may be contravening GR’s equivalence principle (in an acceptable manner), we need to reconsider (more deeply) what our supplement to (GR’s) gravitation really entails and its relationship to the various guises of the EP. The latter is pursued in subsections 6.7.3 and 6.7.4, the former from subsection 6.7.5 onwards.

6.7.3 Pioneer anomaly as regards the weak and strong Equivalence Principles

For the weak EP (WEP), Jammer (2000) distinguishes between a kinematic version, the principle of universality of free fall, where at a given location all bodies fall with the same acceleration; and a dynamic version where the gravitational acceleration of (or acting upon) a body is independent of its structure (i.e. composition and the amount of mass).

The long temporal and (long) distance propagational requirements, as well as the very accurate measurements necessary to recognise the very tiny (low mass only) Pioneer anomaly, are exceptional. Comparatively *short* temporal/duration torsion balance and terrestrial ‘free fall’ experiments involve masses (m_{body}) on the *same* side of all of the (lunar specific) cut-off mass values [$m^*(r)$]; and gravitational redshift experiments by way of the Mössbauer effect are unsuited to recognising this (Pioneer) EP violation. Similarly, we note that the Earth-Moon (strong EP) ongoing ‘experiment’ has (for both the Earth and Moon) $m_{\text{body}} \gg [m^*(r)]_{\text{max}}$.

The strong EP (SEP) assumes the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system, which then implies that there is no physical difference between inertial motion without a gravitational field, and free fall in a gravitational field. Regarding the ‘felt’ aspect of the SEP³⁸¹, we extend Einstein’s lift/elevator-based thought/gedanken experiment, where $\underline{a} \equiv \underline{g}$, to a sinusoidal/undulatory scenario with amplitude $\Delta a \equiv \Delta g$; recalling that the latter is associated with a speed/motion undulation of amplitude Δv (recall section 3.2) — in the case of low mass bodies i.e. $m_{\text{body}} < [m^*(r)]_{\text{minimum}}$.

³⁸¹As compared to the acceleration of a *reference system* in GR, which then allows the scientifically rigorous (quantitative) theory of general relativity to be formulated.

Interestingly, within the Pioneer spacecraft (itself) a ‘felt’ (temporal sinusoidal) acceleration variation³⁸² [$a_{\text{proper}} = a = a(t) = -\Delta a \cos(\omega t)$], whether delivered by way of a RSW [$\Delta g \sin(\omega t) = \Delta a \sin(\omega t)$] or conceivably by a mimicking variation in thrust [$-\Delta a_t \cos(\omega t)$], causes a purely oscillatory/sinusoidal variation in motion [$v_{\text{proper}} = -\Delta v \sin(\omega t) = \Delta v \sin(\omega t - \pi)$] around a steady mean speed that is pendulum-like or swing-like in nature³⁸³. *Locally*, i.e. within the (windowless) spacecraft, there would be no physiological sense of an unmodelled monotonic motion ‘shortfall’ over extended intervals of time, i.e. (no sense of) an additional/supplementary ‘constant’ (Pioneer anomaly-like) deceleration³⁸⁴. A number of provisos (concerning this statement) need to be appreciated.

We recall (from section 3.2) that the motion shortfall is relative to *predicted* motion. We also note that the *Pioneer* anomalous acceleration arises from (the superposition of) a *number* of these (coexistent) rotating space-warp based sinusoids.

Further, it is only in the global/barycentric frame, and by way of having/using a theoretical model to *predict* a spacecraft’s trajectory and motion, that a small redistribution of S/C translational/propagational kinetic energy (down from 100%) into undulatory/oscillatory kinetic energy has physical validity. Predictions of S/C motion that omit this feature, inevitably notice a motion shortfall (over time) that is ‘declared’ the Pioneer anomaly. Importantly, ‘locally’ we could (still) in principle transform away gravitation by choosing a particular reference frame, with the additional terms being ‘understood’ as inertial force terms. Thus, (in this sense) the strong equivalence principle, associated with Einstein’s (windowless) lift thought experiment (or a windowless moving spacecraft), is *retained* when extended to a pure undulatory/oscillatory gravitational field — i.e. a first order cyclic perturbation (of *constant* amplitude) superimposed upon a (locally³⁸⁵) *steady* gravitational field.

We note that a global/barycentric perspective is necessary, in addition to a local ‘felt force’ perspective, to appreciate the (counter to predicted motion) Pioneer anomaly³⁸⁶. By comparison, GR is/was constructed/established upon a *local* principle of equivalence (and ‘felt-force’, or lack thereof) perspective. This need for the coexistence of two (or dual) per-

³⁸²Using nomenclature consistent with subsection 3.2.4.

³⁸³We note that ω is the rotating space-warp’s angular frequency, and that the (proper) velocity variations are 180 degrees out of phase with acceleration/gravitational field variations (recall Figure 1). For a spacecraft radially exiting the solar system at negligibly relativistic speeds, we can figuratively conceive (i.e. imagine) the forward looking *space* curvature ‘encountered’ as acting like an inclined ramp. The undulatory variation in acceleration/gravitational field strength can be imagined as a smoothly varying cyclic change in the angular slope of such a (curved space) ‘ramp’.

³⁸⁴Regardless of the spacecraft’s direction of motion: circumferential, radially inward, or radially outward, etc.

³⁸⁵Both spatially and temporally — in relation to one’s location ‘onboard’ the spacecraft.

³⁸⁶Adding windows to the spacecraft (or elevator) would in principle (i.e. conceivably) ‘allow’ anomalous (i.e. unpredicted/unmodelled) barycentric position and/or speed variations to be *indirectly* ascertained.

spectives, i.e. local and systemic as regards this new (supplementary) gravitational effect, is symptomatic of the new model's approach.

It is not unreasonable to surmise/conclude that: both the kinematic and dynamic versions of the weak EP 'can' be violated (in some sense), although in practice this violation is observationally both: exceptional (or even unique), and exceedingly small in magnitude. A comparison of planetary motion and Pioneer spacecraft motion confirms a violation (of some kind) — albeit requiring a global/barycentric perspective, as well as a local Earth-based observer. It becomes a question of (semantics and) one's own conceptual perspective as to whether one considers the *acceleration* (equivalence to gravitation) *aspect* of the strong equivalence principle (SEP) to have been violated by the Pioneer S/C. If we *restrict* ourselves to a local physiological perspective, as Einstein did with his windowless lift/elevator experiment, then the SEP is *not* violated — even in our undulatory/cyclic gravitational field circumstance (i.e. subject to the effects of RSWs). The SEP *is* violated when it subsumes the existence of a mass-based violation of the WEP. Further, the proposed existence of a non-local mass field [$m^*(r)$] (or fields) — related to the non-local implications of (lunar-based) quantum mechanical systems — is supplementary to the scope of the assumed 'equivalence' of inertial mass and passive gravitational mass: $m_i \equiv m_p (= m_g)$.

'Large-mass' solar system bodies such as: planets, moons, large comets, and large asteroids do not respond to the additional acceleration/gravitational field associated with GQ-RSWs, whereas the Pioneer 10 and 11 spacecraft (and other 'low mass' bodies) do respond; for the latter we can 'figuratively' write that: $\underline{a}_{\text{body}} \equiv \underline{g}_{\text{total}}$. Consequently, by way of *not* indicating the existence of the supplementary additional acceleration/gravitational field, it is (surprisingly) these large-mass celestial bodies that are all in *violation* of the 'SEP', when the SEP is extended to first order field perturbations — albeit a very minor violation (hitherto unappreciated). In other words, the motion of larger/more-massive bodies does not 'completely' exhibit/indicate the (entire) non-Euclidean geometry of the solar system, such that (for them): $\underline{a}_{\text{body}} \neq \underline{g}_{\text{total}}$.

6.7.4 The Einstein Equivalence Principle and exclusivity of the metric in GR

Extending the SEP, Einstein made the Einstein equivalence principle (EEP) part of the bedrock of his general theory of relativity (GR). It acts (somewhat) as a correspondence principle between SR and GR, by ensuring that locally SR's Lorentz invariance and position invariance are enforced. Clearly, the model's supplementary acceleration/gravitational field — associated with non-Euclidean geometry — needs to be in denial of the 'exclusivity' of the metric tensor(s) arising from GR's field equations, in the sense that we need to question the following 'exclusive' statements:

1. If SEP is strictly valid, the (components of the) metric *alone* determines the effect of gravity.
2. Matter and non-gravitational fields respond *only* to the spacetime metric.

3. The gravitational field is *entirely* describable by a universal coupling of the (macroscopic) mass-energy contents of the 'world' to the metric tensor.
4. Test bodies follow geodesics of *the* metric.

As mentioned previously, there is nothing wrong with GR; rather our concern is necessarily with the scope or domain of application inherent in GR's conceptualisation and formalism, and hence the idea that GR is the 'final word' on gravitation — such that GR 'completes' gravitational theorisation. Supporting this concern we recognise the exclusions and idealised circumstances associated with GR outlined in subsection 4.1.5, and the various restrictions to gravitational theorisation discussed in subsection 4.3.3. We reiterate a few pertinent examples. Firstly, tidal effects, which arise from non-uniformity in gravitational fields, illustrate why there is a need to restrict the acceleration/gravitational/curvature aspect of the SEP to a *local* effect; this is in stark contrast to the cosmological scale amplitude uniformity (Δa) associated with the unsteady/undulatory field of the model's GQ-RSWs. Secondly, effectively *only macroscopic* bodies are considered, with all ('isolated' cases of) microscopic/quantum mechanical contributions to gravity (including energy effects) effectively assumed negligible; note that this negligibility includes the additive sum of a QM angular momentum rate (i.e. energy), externalised as GQ-RSWs, in our model. Thirdly, in subsection 4.4.6 we briefly discussed how the non-locality of QM entanglement³⁸⁷, and a finite communication rate (i.e. light speed) in SR and GR, sit together uneasily (Albert & Galchen 2009)³⁸⁸.

Furthering our concern, Einstein, in creating and developing GR, (as late as 1916) sought to *generalise* the relativity of motion in special relativity (SR) to accelerated motion³⁸⁹. The resultant General Theory of Relativity is astounding, but this (aforementioned) 'elegant' objective was not entirely fulfilled; nevertheless its 'footprint' remains in GR's (approach to its) formalism.

Furthermore, recalling subsection 4.3.4, acceleration and rotation (in at least one sense) resist a purely relativistic understanding, even though *observations* of accelerated motion are relative. We note that (coincidentally and fortuitously) rotation *and* acceleration/gravitational field undulations together, in the form of (gravito-quantum) rotating space-warps (GQ-

³⁸⁷Also known as "the quantum non-local connection"; (and) which herein involves a 'transmutation' of QM spin *energy* into 'gravitational' energy cf. (simply involving) an equal spin to spin (i.e. angular momentum) relationship.

³⁸⁸Contemporary physics 'accepts' a situation whereby: non-local correlations do occur, but because they cannot be used to transmit information, they do not violate causality. Our 'uneasiness' stems from a lack of explanatory depth, in that an (additional) process/mechanism to account for the ongoing 'sustenance' of these correlations is not considered absolutely necessary when explaining entanglement.

³⁸⁹Additionally, he also sought, and eventually abandoned (as late as mid 1918), a 'Machian' relativity of inertia, thus resigning himself to a physics in which spacetime has independent existence and physical qualities (Hofer 1994).

RSWs), are central to the model³⁹⁰.

Finally, but on a lesser note, it can be argued that the ‘extension’ of SR to GR has a ‘downside’, in that if a weakness and/or alternative interpretation exists in our physical understanding of (all that is within) the Theory of Special Relativity, then this flaw will somehow ‘express’ (or even compound) itself in GR. Our concern lies with the ‘extension’ of SR’s (very successful use of) Minkowski/flat *spacetime* into GR’s (equally successful use of) curved spacetime. Specifically, the latent presumption that only four-dimensional spacetime is associated with gravitation (in its broadest sense); this (ontological feature) is contrary to what our model requires, i.e. a perspective where space and time (need to) retain some degree of independent existence. *Three* features, associated with ‘diminishing returns’ or restrictions as we move from SR to GR, loosely support this concern. Firstly, *global* Lorentz covariance goes to *local* Lorentz covariance. Secondly, GR requires (more complicated) covariant derivatives and metric tensors cf. the partial differentials of non-relativistic macroscopic physics. Thirdly, GR’s formalism, particularly the (non-linear) Einstein field equations, have very few simple/elegant solutions. These features of localisation and complication, can be interpreted as merely signifying *restrictions* necessary for the establishment of a ‘general’ theory of relativity, but additionally/alternatively these restrictions may be interpreted as providing indirect support for our model’s supplementation of gravitation, with its ‘supplementary’ use of (independent) space and time cf. spacetime — as argued for in Section 4.

This subsection has sought to illustrate how general relativity, by way of its ‘make-up’, is actually *incapable* of describing the very minor additional gravitational phenomenon associated with the Pioneer anomaly. We have needed to cast doubt upon the necessity and exclusivity of a (spacetime) metric-based approach, partly by way of an exceptional circumstance in our new model; i.e. a constant amplitude (rotating) ‘warped’ acceleration/gravitational field — albeit undulatory/sinusoidal at a ‘point’ in space, or rather upon a fixed direction radius ‘arm’ (centered at a lunar GQ-RSW ‘generator’). Further signs of a restrictiveness within GR’s approach, especially regarding its domain of application, include: a non-relative aspect pertaining to (the motion-based concepts of) rotation and acceleration; the existence of QM *non-local* relationships/behaviour; and the fact that GR is necessarily a *relativistic* theory of (and approach to) grav-

itation — requiring special relativity and Minkowski spacetime as a ‘foundation’. Aspects of GR, such as localisation (regarding the Einstein equivalence principle), and the inherent difficulty in solving the Einstein field equations are loosely supportive of these concerns; as is the failed agenda/goal to ‘relativise’ accelerated motion and/or inertia (in the final form of the theory). Citing these ‘imperfections’, within and peripheral to GR, indirectly acts to support our new model; this *modus operandi* (i.e. method of operating) is furthered in subsections 6.7.5 and 6.7.6.

6.7.5 Discussing the new model’s need to supplement General Relativity

A real Pioneer anomaly cuts to the core of gravitational conceptualisation, although it does *not* imply that general relativity is in error. Instead, GR is seen as an incomplete account of ‘gravitation’ — in the broadest sense of the word — in need of a quantum mechanical (and Heisenberg uncertainty principle-based) supplementation; with quantum mechanical entanglement and non-locality, in conjunction with geometric phase, also playing a vital role. It remains the case that (i.e. the new model respects that):

1. The only theories of gravity that can fully embody the EEP (by default) are those that satisfy the postulates of metric theories of gravity.
2. In local Lorentz frames, the non-gravitational laws of physics (describing observations of physical phenomena) are those of special relativity.
3. The strong equivalence principle (SEP) implies that gravitational acceleration is (in one sense) an entirely geometrical (natural) phenomenon.

The model’s sinusoidal gravitation/acceleration field supplement (of amplitude Δa), in conjunction with non-local mass $[m^*(r)]$, is *inconsistent* with both the EEP and the exclusivity of GR’s metric, even though removing gravitational effects locally regains SR³⁹¹. Our aim here is not to pursue a drawn-out scholarly review of GR; rather, we have sought to highlight how GR’s non-Euclidean (spacetime-based) curvature/geometry needs to coexist with the RSW-based *supplementary* non-Euclidean curvature/geometry. Recalling section 4.4, this supplement needs to (additionally cf. alternatively) treat space and time as distinct ‘features’ of reality — albeit requiring an additional conceptual level of ‘reality’, separate from (and ‘prior’ to) observational/phenomenal time and reality. This ontological feature is introduced so as to be able to ‘deal with’: QM non-locality and entanglement, (an externalisation of) non-inertial QM energy, and the new gravitational effect, *concurrently*.

The primary difference, between GR’s standard gravitation and our supplementary gravitational effect, lies in there being two distinct *source ‘classes’*

³⁹⁰Recall that (each): lunar, atomic/molecular, self-interference, and geometric phase based, supplementary rotating space-warp [of undulatory acceleration amplitude (Δa)], coexists with a non-local mass distribution $[m^*(r)]$. Together, they express the total non-local externalised (‘fractional’ and inexpressible) quantum mechanical energy — associated with the many atoms/molecules within each: geometrically suitable, spin-orbit coupled, (large) moon of the solar system. The contribution of the various RSWs (with perturbation amplitude Δa_w) to non-Euclidean geometry is necessarily a superposition of *secondary* ‘gravitational’ effects, in that they ‘piggy-back’ upon, and require the *pre-existence* of, standard general relativistic (spacetime) non-Euclidean geometry throughout the solar system.

³⁹¹Such that the theoretical/formal requirements of local Lorentz invariance and local position invariance are *observationally* upheld, and such that SR remains a limiting case of both GR and ‘gravitation’ in its widest sense — with the latter additionally involving the RSW supplementation.

and two distinct theoretical representations of non-Euclidean geometry. In GR the sources are formally represented by the stress-energy tensor, whose components involve: (macroscopic) energy density and flux, momentum density and flux (as well as shear stress and pressure). In our model a physically inexpressible sub-quantum mechanical-based energy³⁹², in the form of a (quantifiable) *process*-based per cycle rate of (intrinsic) angular momentum (shared by a great many atoms/molecules), is necessarily ‘re-expressed’ as (macroscopic) non-Euclidean geometry (i.e. as a GQ-RSW)³⁹³ with an accompanying non-local mass distribution — so as to ensure that global conservation of energy is maintained ‘through’ time. This conservation of energy involves both: microscopic *and* macroscopic aspects of reality (with seamless functional coordination); as well as a form of functional separation of energies between these two domains/realms³⁹⁴, so as to ensure a (micro-to-macro) re-expression/‘transfer’ of (a given/equal quantity of) energy³⁹⁵. Note that the ‘macroscopic’ in our model unavoidably requires a systematic/global (i.e. non-purely-relativistic) perspective.

In a manner of speaking, the inertial *force* terms of GR (cf. SR), need to be supplemented by, and *coexist* with, a (systemically/globally relevant) inertial *energy* offset (established) at the microscopic level (recall subsection 5.7.4). This supports our assertion of *two* quite distinct types/sources of curvature/non-Euclidean geometry, with two quite distinct types/methods of (theoretical) representation. Differences between GR and our new approach involve: the field types, the role of energy conservation, and the nature of space and time in theory formulation³⁹⁶. Unlike GR, the new model

³⁹²Arising (in part) from a conflict involving the *discrete* energy levels associated with atoms and molecules, and *analog* geometric phase offsets arising from the closed loop motion of these QM systems in analog curved spacetime.

³⁹³A loosely analogous situation would be the ‘expression’ of ripples on a pond when some additional energy is imparted to the water/medium.

³⁹⁴Note that electromagnetism is unique amongst physics’ ‘four forces’, in that it can be bifurcated in at least two special ways: macroscopic/classical level phenomena vs. microscopic/QM phenomena, and wave vs. particle aspects.

³⁹⁵Note that an *agenda* that seeks to ‘unify’ gravitation/general relativity and quantum mechanics — regardless of their generally disparate phenomena and disparate mathematical descriptions — into a “theory of everything” or “unified field theory” does not need to ‘subscribe’ to the ‘dual-conception’ stance alluded to herein.

³⁹⁶Einstein’s SR drew upon Poincaré’s light signal based operationalist methodology, so as to ensure that the speed of light between different inertial frames, and Maxwell’s equations of Electromagnetism, remain invariant. This required the implementation of Minkowski spacetime in the formalism. Our concern herein is with the further presumption, as based upon (fully valid) scientific *observational* methodology, that spacetime is ontologically real, and *completely* replaces the prior/earlier conceptualisation of space and time. As we have discussed previously (particularly in sections 4.4 and 4.5), our model requires a noumenal supplementation to this purely observational/phenomenal (spacetime) approach; in that outside of observations (and relativistic theoretical formulations), space and time as distinct ‘entities’ needs to maintain theoretical validity — especially/crucially when quantum mechanical energy and

makes use of: QM non-locality, QM self-interference, (hidden/background) ‘noumenal’ systemic/global aspects, and three-body orbital motion — with the latter being described by ‘phenomenal’/observational based ‘relativity’. Furthermore, non-locality appears to necessitate the use of a *non-observationally* perceptible global space substratum (or continuum) coexisting with an ‘idealised’ background (hidden) time simultaneity (recall Section 4); this suits the ‘weirder’ aspects of QMs, but it is incompatible with the foundation “principles” and ontological assumptions upon which general relativity was constructed and *is* built.

For the devotee of general relativity’s perfection and exclusivity, who is incapable of appreciating the possibility of a ‘holistic’ supplementation³⁹⁷ that coexists with a ‘relativistic’ approach to gravitation, the model’s new approach and ontological stance will necessarily remain ‘anathema’ to them; as will the idea that the Pioneer anomaly is physically real and indicative of a new physical phenomenon — as compared to an undetected/unappreciated mundane systematic effect. Note that in achieving our new model, we have employed an eliminative methodology guided by the highly constraining observational evidence (outlined in section 2.4) — subsequent to *assuming* the (non-systematic based) ‘reality’ of the Pioneer anomaly. This eliminative methodology involved proceeding much in the manner/spirit of Sir Arthur Conan Doyle’s fictional detective Sherlock Holmes, in that we have sought to adhere to his creed that: “...when you have eliminated the impossible, whatever remains, however improbable, must be the truth.” In our case, Doyle’s ‘truth’ is replaced by the model’s proposed/hypothesised ‘new physics’.

6.7.6 Post-production chinks in the general principle of relativity’s armour

The general principle of relativity (GPrR) is the requirement that the equations describing the laws of nature/physics have an equivalent form in all systems of reference (and observation) — i.e. inertial and accelerated frames.

The great power possessed by the general principle of relativity lies in the comprehensive limitation which is imposed upon the laws of nature³⁹⁸ ... (Einstein 1920)

On the proviso that: the $2\Delta a$ range of sinusoidal acceleration/gravitational field strength lies within the ‘wiggle room’ allowed by Heisenberg’s uncertainty

QM non-locality and entanglement are involved.

³⁹⁷Holistic, in the sense of needing to appreciate the universal/cosmological system as a whole at different given ‘moments’ (i.e. from a non-relativistic noumenal perspective), rather than only from a ‘sum’ of the parts perspective — involving for example: the metric at different points, different spacetime intervals between two events, and clocks ‘running’ at different rates.

³⁹⁸A view endorsed by philosopher of science Karl Popper, who believed (*The Logic of Scientific Discovery*, 1972, Addendum to Section 40): the more laws of nature prohibit, the more they say.

principle, the *compatibility* of our new and supplementary model with GR's GPrR is (trivially) assured because the undulatory field strength/amplitude (Δa) of the RSWs is *constant* throughout the universe. We note that the variable non-local mass [$m^*(r)$] accommodates energy dissipation with increasing distance from the source of a given GQ-RSW. Thus, [excluding the source-centered $m^*(r)$ distribution] all systems of (far-field) reference are (necessarily) equivalent with respect to the formulation of laws concerning (the model's supplementary) gravitation/acceleration/curvature — in the form of a number of gravito-quantum rotating space-warps (GQ-RSWs).

Our model accepts that there is no (justifiable) preferred reference frame for describing: SR, GR's gravitation³⁹⁹, and the *laws* of physics; and that the notion of an (observationally 'active') aether/medium is "without merit". Nevertheless, the cosmic microwave background radiation⁴⁰⁰ (CMB radiation) appears to "fit the bill" of a privileged universal frame (of sorts) in our expanding universe, in that it permits a local determination of global/absolute speed and/or direction of travel in space — i.e. with respect to the rest frame of the CMB radiation — e.g. Earth dipole motion/velocity. A thought/gedanken experiment involving two spacecraft (in inertial motion) fitted with equipment that can ascertain the spacecraft's speed and direction relative to this CMB 'rest' frame, is indirectly also ascertaining a common third 'frame' (cf. observer) which is both: at 'rest' and of cosmological extent. Such a circumstance, while not denying the special principle of relativity, is not (at all) in the spirit of special relativity as originally formulated. Similarly, quantum mechanical non-local behaviour⁴⁰¹ 'goes against the grain' of special relativity as originally conceived.

Drawing upon Lehner (2005), the general principle of relativity (GPrR) is meant to be a principle about the real world. As such, general covariance, which seems to be (only) a formal property of a theory, cannot alone imply GPrR. Possibly it can if: "Nature's laws are merely statements about temporal-spatial coincidences; ..."; but as John Norton has pointed out, this claim is far from trivial. Furthermore:

[Such a claim] is not even plausible without the fundamental reinterpretation of

³⁹⁹ "Just as one could formulate Newtonian mechanics or special relativity in generally covariant coordinates, so it is possible to formulate general relativity in a preferred coordinate system. Einstein does not argue that it is not possible to do so, but merely that such a formulation imposes a formal structure without physical relevance (Lehner 2005, p.106)."

⁴⁰⁰ First detected in 1964 by American radio astronomers Arno Penzias and Robert Wilson.

⁴⁰¹ Observationally well supported by Alain Aspect and his colleagues (including P. Grangier, G. Roger, and J. Dalibard) at Orsay, Paris in 1981-1982 with their 'Bell test experiments' (post-dating an 'initial' Bell-test of S. J. Freedman and J. F. Clauser in 1972). These experiments confirmed (John S.) Bell's theorem (or Bell's inequality), first published in 1964 and later refined [e.g. to the CSHS (Clauser-Horne-Shimony-Holt) inequality in 1969]. John Bell drew his inspiration from the Einstein-Podolsky-Rosen (EPR) paradox, originally published in 1935.

space-time properties that general relativity produced. Newton's [rotating] bucket [argument] is — on the face of it — a striking counterexample to this claim (Lehner 2005, p.105)."

Subsequently, Lehner regards the mature formulation of Einstein's principle of general relativity (*circa* 1921) that: "all states of motion are equivalent in principle, before a specific distribution of mass in the universe is specified (Lehner 2005, p.106)", as having its (real world) basis upon/within the principle of equivalence (cf. Mach's principle or general covariance) — which requires that there is no physical difference between inertial motion without a gravitational field and free fall in a gravitational field. Additionally, Einstein believed that:

If this equivalence is to be a fundamental principle rather than a coincidence in physical phenomena, there can be no structure like Newtonian absolute space that distinguishes a priori which motions are inertial and which are not (Lehner 2005, p.106)."

A statement we endorse, as far as regards *Newtonian* absolute space, but shall reject in response to our additional and idealised noumenal (beyond/prior to observation) perspective — hypothesised, discussed, and argued for in Section 4.

6.7.7 Summarising aspects of GR's principle approach and further discussion

A major objective of this section (6.7) has been to show how and why our model can supplement and coexist with GR. Interestingly, the new model is not in conflict with much of GR's underlying *conceptual* structure and (formative) *principles*. For example:

1. We endorse special relativity's stance that there can be no structure like a (physically independent/'inert') Newtonian absolute space and no (same *rate* for all observers) absolute time.
2. Special relativity maintains its role as a limiting case, in the absence of gravitational effects.
3. The acceleration aspect of the strong equivalence principle is maintained, so as to not preclude its extension to first order (undulatory/sinusoidal) variations upon a (pre-existing 'steady') gravitational field.
4. Furthermore, gravitational acceleration remains conceptually 'reducible' to, and concomitant with, a curvature-based (and non-Euclidean geometry-based) 'situation'.

A clear omission of our new model is that the discussion of curvature (i.e. non-Euclidean geometry) is only conceptual; no formal representation of the (supplementary) *curvature* itself is given. The model's major distinction (cf. GR) — as demanded by the awkward observational constraints and a macroscopic re-expression of (an externally imposed⁴⁰²)

⁴⁰² By way of three-body celestial motion in smoothly (cf. discontinuous) curved spacetime.

many atoms/molecules-based ‘fractional’ QM energy ‘offset’ — lies in the introduction of a new (variable, and source-based) non-local mass scalar field $[m^*(r)]$. This field, in conjunction with the RSW constant (Δa) amplitude undulations, has the ability to influence masses in celestial motion in a manner that lies beyond GR’s theoretical ‘coverage’ — both formally and (in some regards) conceptually.

The model’s nonconformity with the (empirical rule of) inertial mass to (passive) gravitational mass equivalence⁴⁰³, suggests that we (need to) further (conceptually) ‘detach’ gravitational acceleration from *passive* (gravitational) mass. This move (further) undermines the notion of gravitational ‘force’ [with dimensions ($\frac{ML}{T^2}$)] as a primary ‘gravitational’ concept — a feature initiated by GR’s curvature-based understanding of gravitation. Inevitably, this $[m^*(r)]$ based exception/violation to a general/all mass-type equivalence — that is generally considered to be restricted to $m_i \equiv m_g$, but now (newly) incorporates $m^* \neq m_g$ — means that the principle of equivalence no longer attains universal physical *principle* status. Recalling the discussion in subsection 6.7.6, this violation subsequently denies the (without exception) generalisation of SR’s relativity principle; although the general principle of relativity (GPrR) still applies when we restrict applications to solely macroscopic physics — but only regarding the type of theoretical *formalisms* that may be established, cf. the GPrR having watertight ‘real world’ *physical* validity. Actually, the Theory of General Relativity can be applied almost universally, except in the rare/exceptional and probably unique (macro- and microscopic/QM combined) circumstances exhibited by our modelling and explanation of the Pioneer anomaly by way of a QM energy *source* — which is an “exception to the rule” of general relativity’s gravitational ‘law’.

Importantly, our stance regarding the principles of: equivalence, general relativity, and general covariance mimics (and is compatible with) that of Einstein (*circa* 1920 and beyond⁴⁰⁴), in that these principles are seen to have played a historical and heuristic role. They are no longer seen to be an essential part of the ‘basis’ of the theory, although (in this author’s opinion) their formative ‘footprint’ remains. This legacy allows doubt to be cast upon the *completeness* of a general relativistic approach to gravitation, and allows concern to be raised regarding GR’s (scope or) domain of application. There is nothing like a gravitational “completeness theorem” for general relativity’s theoretical method and scope, merely a firm belief in its comprehensiveness.

Circa 1920 “... Einstein [now] postulates the metric field as the fundamental entity of his theory (Lehner 2005, p.105).” This is understandable, in that having completed the theory’s formulation, and appreciating its observational confirmation, it is the metric tensor (at each point) — i.e. the solution(s) of the Einstein field equations — that inevitably becomes the new and more appropriate focus of a relativistic

theory of gravitation⁴⁰⁵. Such a *change* in attitude/focus, although theoretically/epistemologically neutral⁴⁰⁶, nevertheless involves two major *implicit* assumptions that have needed to be disassembled.

Firstly, our new/supplementary model’s very existence has demanded that we find fault with the assumed ‘exclusivity’ of GR’s metric, as well as the finality/completeness associated with the Einstein field equations (see subsections 6.7.4 to 6.7.6). This exclusivity is based upon both: the need for a generally covariant approach, and the physical validity of the general principle of relativity. We argued that the latter requirement has not been achieved, and note that general covariance — a formal (rather than a physical) requirement/constraint for a relativistic *theory* of gravitational phenomena — is insufficient on its own to warrant this exclusivity.

A second implicit set of assumptions that needed to be disassembled concerns certain ontological assumptions/beliefs, related to the (long past and completed) theoretical *formative* process that led to GR, which have become ‘built into’ GR’s relativistic approach to gravitation. In Section 4 we discussed how a model for a ‘real’ Pioneer anomaly has no choice but to revisit the ‘standard’ *ontological* commitments/assumptions of special relativity (that are extended ‘into’ GR); especially the veracity/reality of the independent existence of Minkowski spacetime and its associated physical qualities. This specific concern was dependent upon our taking issue with the ‘standard’ (and *solitary/exclusive*) interpretation given to space and time in special relativity. In Section 4 we proposed a *alternative/complementary* (less simple, more subtle) interpretation that is consistent with *both*: the observational evidence relevant to the Special Theory of Relativity, and non-local behaviour in Quantum Mechanics. Only this secondary/supplementary interpretation could have facilitated the construction of the model presented in this paper, so that this model can now stand alongside Relativity Theory — upon which we have performed a partial *conceptual* ‘reconstruction’.

6.7.8 Brief summary and closing remarks regarding section 6.7

Our investigation of the Pioneer spacecraft’s ‘apparent’ violation of (at least one form of) the Equivalence Principle has required a fairly extensive investigation

⁴⁰⁵In a manner of speaking, the ‘metric’ in GR does the ‘heavy lifting’ of the theory. Aspects/features of the *metric* include: that the curved spacetime geometry around a star (for example) is described by a metric tensor at every point; it generates the connections which are used to construct the geodesic equation of motion and the Riemann curvature tensor; the Ricci tensor and scalar curvature depend on the metric in a complicated non-linear manner; it is used to raise and lower indices; (and finally) in the weak field approximation (applicable in our solar system) the metric tensor is closely related to ‘gravitational potential’.

⁴⁰⁶In that subsequent to GR’s formalism having been successfully established in 1915 (the Einstein field equations etc.), the nature of the formalism has remained fixed/unmodified — notwithstanding the contentious issue of whether or not to include a cosmological constant term.

⁴⁰³Based upon (macroscopic) experiments dating back to the 17th century.

⁴⁰⁴As discussed by Lehner (2005, p.105).

of general relativity’s conceptual foundations. Combining this investigation with an appreciation of the new model’s content, allows us to reach an understanding of how the motion of the Pioneer spacecraft is not in defiance of general relativity (i.e. a relativistic theory of gravitation).

This section (6.7) has argued that the ‘footprint’ left upon the Theory of General Relativity by both: Special Relativity’s use of Minkowski spacetime, and the somewhat antiquated notions of: the equivalence principle (in its various forms) and the general principle of relativity — by way of their use in the formative/development stage of general relativity (GR) — have acted to restrict (in a good way) GR’s theoretical formalism. Unfortunately, if the conceptual argument used to establish a *complete* and proper physical generalisation of (special) relativity is open to doubt, or indeed invalid, then the formative ‘footprint’ left by these conceptual ‘building blocks’ can also limit/restrict (in a negative manner) the scope/‘domain’ of General Relativity’s application. We have argued that this is the case. This less than perfect situation is indirectly supportive of our model, with its stated aim of establishing a viable ‘best’ explanation/hypothesis for the case of the Pioneer anomaly being ‘real’ cf. a systematic effect.

It has become apparent that the formal content of our (new and provisional) model is mutually exclusive from/to GR; the model provides a supplementation of gravitational theory that in no way disparages GR, nor finds fault with any of its formal/mathematical content. Whereas GR is applicable to all forms of macroscopic mass and energy, our new model is something of a ‘boutique’ model with only a few applications. The model’s underlying mechanism is unique, and involves a very specific set of microscopic *and* macroscopic circumstances coexisting in Nature. The model’s incorporation of QM non-local behaviour/phenomena, and an alternative ontological stance regarding space and time ‘underlies’ this segregation of Gravitational (approaches and) descriptions. Thus, our (boutique) model’s complementary approach and quantification of ‘gravitation’ (in its widest/new sense) is neither denied nor in direct conflict with GR’s ‘unique’ (and broad-ranging or ‘general’) approach to gravitation.

In Section 8 an overview summary and final discussion is given, drawing upon all Sections of the paper, but prior to that (in Section 7) a further and somewhat surprising ramification/application of the new model is presented.

7 Rotating space-warps, non-local mass distributions and type 1a supernovae results

This Section brings together a number of aspects of the preceding model to argue that the current interpretation of type 1a supernovae observations, implying *accelerated* cosmological expansion, may be misguided — subsequent to gravito-quantum rotating space-warps (GQ-RSWs), and (particularly) their

conjoint non-local mass distributions (NMDs), achieving recognition. Although a not insubstantial level of idealisation is used in this informal investigation, the main aspects of the discussion are not compromised.

Recently, the “standard candle” supernovae method has been supplemented and corroborated by a “standard ruler” method, involving baryon acoustic oscillations (BAOs)⁴⁰⁷ and (statistical features of) the large scale clustering of galaxies (Blake, Davis, Poole, Parkinson, Brough, Colless, Contreras, Couch, Croom, Drinkwater, Forster, Gilbank, Gladders, Glazebrook, Jelliffe, Jurek, Li, Madore, Martin, Pimblett, Pracy, Sharp, Wisnioski, Woods, Wyder, & Yee 2011).

7.1 Preliminaries

Previously (subsections 6.6.9 and 6.4.5), we have argued that *below* a certain mass cut-off threshold, GQ-RSWs (or more succinctly, RSWs) influence the *motion* of low mass moving bodies equally regardless of the body’s direction of motion; i.e. the influence (from the body’s standpoint) is *isotropic* — although the line-of-sight *measurement* of motion shortfalls, relative to predictions ignoring this influence, is not isotropic. We cannot simply extend this influence to photons because the speed of photons/light (in vacuum) is ‘physically’ invariable (or constant). Nevertheless, by way of the change in the non-local mass value with radial distance from a RSW source, a new and interesting (isotropic) effect upon the redshifted spectroscopic lines of distant type 1a supernovae (SNe 1a) may be hypothesised. Interestingly, this further effect⁴⁰⁸, involving photon propagation (and/or EM radiation energy propagation), is *obscured* by way of its isotropic nature; in that the observer moves *with* the solar system and the RSWs and non-local mass distributions (NMDs) it harbours.

7.2 Proposing a fourth type of ‘red-shift’ (of cosmological extent)

We hypothesise that a fourth (and new) type of general⁴⁰⁹ photon *frequency shift* exists in addition to: (1) a Doppler motion based redshift, (2) a scale factor (cosmological) redshift⁴¹⁰, and (3) a gravitational

⁴⁰⁷Baryon acoustic oscillations (BAOs) correspond to a preferred length scale imprinted in the distribution of photons and baryons by the propagation of sound waves in the relativistic plasma of the early Universe (Blake et al. 2011, Introduction).

⁴⁰⁸Previously (subsections 3.5.3 and 6.2.6), we discussed how RSWs moving with the solar system might anisotropically affect CMB radiation measurement.

⁴⁰⁹Pertaining to the equal shifting of spectroscopic lines regardless of their frequency or wavelength.

⁴¹⁰Some cosmologists prefer to interpret cosmological redshift as a (kinematic) Doppler shift (Bunn & Hogg 2009; Davis 2010). We shall refrain from this interpretation, citing concern with the downplaying of a wavelike aspect of electromagnetic radiation’s propagation, as evidenced in Young’s double slit experiment. This wavelike aspect is considered to be (unavoidably) affected by space (distance between two points) expansion — in an expanding universe. If we restrict Planck’s relation ($E = hf$) to emission and ab-

redshift⁴¹¹. Whereas the three standard redshifts can be understood by way of laws involving a transformation between reference frames, the manner of frame transformation for this proposed (fourth type of) redshift/blueshift is subtly different.

Conceivably, (4) an isotropic *blueshift* arises for photons as they travel *towards* the source of RSWs and NMDs in *our solar system*, which is also the location of an Earth based observer. We hypothesise that this occurs by way of the greater (local) value of non-local mass (and non-local energy), associated with the NMDs, closer to their source. This frequency blueshift is similar to, but quite distinct from, the gravitational blueshift experienced by photons as they move ('downhill') into a stronger gravitational field. The new blueshift, as measured upon Earth, pertains to far distant objects, and thus it is a blueshift of *cosmological* extent. As such, its contribution to the electromagnetic-based frequency shifts of the Pioneer spacecraft's observational Doppler data is negligible.

Note that ('now', i.e. present time ± 100 years) the stability of Sun-planet-moon motion in the solar system ensures that the specific energy of each RSW (Δe_w) is (effectively) constant throughout the universe. Subsequently, our primary concern in this Section is the effect of NMDs upon the propagation of electromagnetic waves/radiation (and photons).

Quantitatively, we *assume* (essentially by default) that this perturbation is observationally indicated by way of a relation similar to the energy to frequency proportionality in the Planck relation⁴¹² (or the Planck-Einstein equation): $E = h\nu$. As such, there will be a (hitherto unappreciated) distance dependent blueshift offset ($\delta\nu$) in the frequency of observed photons (cf. expectations) — regardless of the magnitude of their wavelength — due to the presence of (all) the solar system's (RSWs and) NMDs. Additionally, we shall define a path dependent *change* in this blueshift (offset): $(\delta\nu)_{\text{final}} - (\delta\nu)_{\text{initial}} = \Delta(\delta\nu)$.

An order of magnitude ('ball-park') investigation shows that Planck's constant (h) cannot be the proportionality constant in this new 'negative redshift' (i.e. blueshift) phenomenon. The introduction of a new (mechanism-specific) proportionality constant, denoted h^* , is proposed; it has dimensions $[ML^2/T]$ or units $[\text{kg m}^2 \text{s}^{-1}]$, as is the case with Planck's constant, angular momentum, and the action (of a path or trajectory). This proportionality constant relates the to-

tal (RSW and) NMD based energy *gain* to a frequency blueshift. It is unlikely that a quantum mechanical 'universal' constant (especially relevant to atoms and molecules in emission and absorption events) would be applicable at the macroscopic/cosmological level, i.e. external to (microscopic) QM systems. Possibly, h^* is somewhat like a *least* action constant that applies to photon/'light' propagation in the presence of a variable/dispersive non-local energy field.

The effects of the dispersion (with increasing radius from a RSW's core) of the non-local mass field, and hence energy field, is more fully discussed and algebraically formulated from section 7.6 onwards, but prior to this a case for the validity of this hypothesis is built.

7.3 Dark Energy transition redshift & the time from RSW initiation

Support for this hypothesis comes from consistency between the transition redshift of dark energy⁴¹³ (z_t), and the light travel time (also known as lookback time) 'back' to the *establishment* of lunar spin-orbit resonance in the early solar system's history.

Good quality recent values of z_t are: 0.43 ± 0.07 (Riess et al. 2007, Section 3.1, p.32) and 0.46 ± 0.13 (Riess et al. 2004, Abstract). Note that the underlying physics of z_t is generally considered unknown (Rapetti, Allen, Amin, & Blandford 2007); and that alternatives to dark energy, such as grey dust and/or the evolution of supernovae properties, appear to be without merit (Wood-Vasey et al. 2007).

Taking the age of the solar system as 4.567 billion years⁴¹⁴, and that the time to lunar spin-orbit resonance (i.e. synchronous rotation) of the Galilean moons is 2.5 *million* years (Peale 1999, p.560); RSW initiation or establishment is of the order of 4.56 billion 'lookback' years. By way of Edward (Ned) Wright's or Siobhan Morgan's (internet based) "Cosmic Calculator" one can generate a 'best' estimate of the redshift of this lookback time. Taking: $H_0 = 71.5$ (km/s)/Mpc, $\Omega_M = 0.265$, and assuming a flat universe, a z_t value of 0.44 is associated with a universe age of 13.64 billion years and a lookback time of 4.55 billion years. Thus, it is not beyond conception that RSW (and non-local mass distribution) establishment might be the 'cause' of (or reason behind) a perceived universal expansion inflexion point (in time) — dividing a decelerating universal expansion from a more recently (perceived) accelerating one.

7.4 A new energy, and questioning a pivotal cosmological assumption

As discussed previously (subsections 6.1.2 and 5.6.8), the *energy* 'driving' the GQ-RSWs (or just RSWs) and NMDs arises from an external (collective) expression of non-inertial (frame-based) atomic/molecular

sorption events, then concern with regard to photon energy loss during its propagation is not an issue. The Doppler interpretation of 'cosmological' redshift is not being denied; rather it is considered a secondary interpretation that may (possibly) be of benefit to our understanding of general relativity.

⁴¹¹The latter involves photons changing position in the gravitational field of an uncharged, non-rotating, spherically symmetric mass — as confirmed by the Pound-Rebka experiment.

⁴¹²There is no other simple way to link the energy of electromagnetic radiation to both: a frequency change/offset, and a QM mass-energy system. In the case of the latter, we are referring to both: the atomic/molecular basis of (conjoint) RSW and NMD energy, and the necessary externalisation of this (non-inertial) energy.

⁴¹³That is, when a deceleration in the expansion of the universe 'gives way' to an accelerating expansion.

⁴¹⁴Wikipedia: *Age of the Earth*, 2010-12.

energy⁴¹⁵, common to a great many atoms/molecules, that occurs below a minimum (internally) expressible energy value. Generally, regarding the expansion of the universe, and in the absence of (Doppler) peculiar motion and gravitational redshift effects, it is assumed that:

...there is a direct one to one relationship between observed redshift and comoving coordinate (Davis & Lineweaver 2004, p.107).

It is this assumption, underlying the case for dark energy, that we seek to question. In other words, we are questioning the validity of the currently conceived relationship between redshift (z) and (time-dependent) cosmic scale factor(s) at observation and emission, i.e:

$$1 + z = \frac{a(t_o)}{a(t_e)} = \frac{a_{\text{now}}}{a_{\text{then}}} = \frac{\nu_e}{\nu_o} = \frac{\lambda_o}{\lambda_e}$$

Note that comoving distance is (cosmological) proper distance (D) multiplied by this ratio of scale factors, and that in a ‘flat’ universe, line-of-sight comoving distance (D_C) equals transverse comoving distance (D_M). Further, comoving distance factors out the expansion of the universe whereas proper distance does not; it equals proper distance at the present time.

7.5 De-prioritising Dark Energy

The accelerating expansion of the universe, as implied by the type 1a supernovae (and baryon acoustic oscillations) results, is but one of *three* pillars upon which the theoretical concept of dark energy rests. The second pillar arises by way of a need to reconcile the cosmic microwave background (CMB) anisotropies, that indicate a flat/uncurved geometry of (cosmological) space, with the total amount of matter (baryonic and dark) in the universe⁴¹⁶ — implied by measurements of: the CMB, large-scale structure, and gravitational lensing. Lastly, we have the late-time integrated Sachs-Wolfe (ISW) effect, closely related to the Rees-Sciama effect (Granett, Neyrinck, & Szapudi 2008). With temperature variations of “about a millionth of a Kelvin⁴¹⁷”, the meticulous research associated with validating the ISW effect as a “direct signal of dark energy” is cutting edge but it is not beyond reasonable doubt.

The ontological alternative discussed in Section 4, regarding space and time, took issue with the assumed extension of General Relativity to the universe “as a whole”, in the sense that $k \neq 0$ is denied⁴¹⁸ — although this in no way denies the validity of (observations based upon) the Friedmann-Lemaître-Robertson-Walker (FLRW) metric with $k = 0$. Accepting this

⁴¹⁵Effectively analogous to an ‘inertial’ force (or more appropriately to avoid confusion, a ‘fictitious’ or pseudo force), but concerning energy. Herein, the expression “virtual energy” is preferred, in a bid to (fully) avoid confusion.

⁴¹⁶Between 25 and 30 percent of the critical density.

⁴¹⁷<http://physicsworld.com/cws/article/news/35368> (Jon Cartwright, Aug 8, 2008).

⁴¹⁸By way of ontological “boundary conditions”, i.e. additional features of the universe (as a whole) that go beyond a purely reductive (to general relativity) approach.

stance removes the logical implication that a ‘flat’ universe and sub-critical density necessarily implies dark energy.

Subsequently, the pivotal experimental evidence behind the presumed existence of dark energy is the type 1a supernovae (and BAOs) data, and any potential reinterpretation of the SNe 1a data is worth considering and potentially significant.

Furthermore, tests of the gravitational inverse-square law below the dark-energy length scale (Kapner, Cook, Adelberger, Gundlach, Heckel, Hoyle, & Swanson 2007) yield a “result [that] is a setback in the search for the gravitational effects of dark energy, which cosmologists believe should begin to appear at this length scale⁴¹⁹.” Finally, Frieman, Turner, & Huterer (2008) in their review paper — *Dark Energy and the Accelerating Universe* — highlight two other problems: (1) “the coincidence problem”, where $\Omega_{DE} \sim \Omega_M$ [pp.404-405]; and (2) the smallness of the energy density of the quantum vacuum [p.386].

7.6 Volumetric ‘dispersion’ of the ‘non-local mass & RSW’ energy

Subsections 6.5.1 and 6.6.2 gave the total energy of a single rotating space-warp (RSW) that coexists with non-local mass as:

$$\Delta E_w = \frac{1}{2} m_1^* \Delta a_w^2 \Delta t^2 = m_1^* \Delta e_w$$

where: m_1^* represents non-local mass at (the inception radius) $8\pi r_o = r_1$, Δa_w is the (weighted, i.e. actual) acceleration’s sinusoidal amplitude, Δt is lunar spin-orbit duration, and ΔE_w is the total supplementary field energy over the course of a single lunar loop/cycle. With our solar system quite stable (over the long-term), Δe_w (i.e. the weighted specific energy) is (essentially) a constant and thus:

$$\Delta E_w \propto m_1^*$$

Secondly, we recall the *non-local mass* distribution function of subsection 6.6.5 (Equation 20), which was alternatively conceived of as a spatial non-local mass continuity equation (in subsection 6.6.6):

$$m_r^* V_r = \text{constant} = m_1^* V_1$$

where: m_1^* and V_1 are (respectively) the *fixed* non-local mass, and enclosed volume, at inception. Note that m_r^* is both: the maximum (inertial or passive gravitational) ‘compact’ mass value at radius r (that can be affected by a NMD), and the non-local mass value at radius r in the field’s distribution, with V_r the (total) volume enclosed within this radius⁴²⁰. This non-local mass distribution relationship describes a type of (non-local) mass dispersion as we move away from the (single case) RWS and NMD inception radius, such that: $m_r^* \propto 1/V_r$ and

$$m_r^* \propto (1/r)^3$$

⁴¹⁹<http://physicsworld.com/cws/article/news/26826> (Hamish Johnston, Jan 17, 2007).

⁴²⁰The ‘ r ’ subscript simplifies the algebraic notation; its use mimics an elementary form of mass continuity equation. Recall (subsection 6.6.5) that: $m_r^* \equiv m^*(r)$ and $V_r \equiv V(r)$.

Although the energy of a (conjoint) RSW and NMD is ‘defined’ at inception (i.e. at $r = r_1$), the dispersion of non-local mass magnitude (for $r_1 < r < \infty$) allows us to (equally) conceive of the dispersion of this RSW and NMD *energy* for $r > r_1$ and $V_r > V_1$.

Let us propose and *define* a *total* energy (at cosmological distance/radius⁴²¹ r from an Earth-based observer) applicable to *all* ‘conjoint’ RSWs and NMDs:

$$E_r^* = \sum (m_r^* \Delta e_w)_i \quad (21)$$

$E^*(r)$, or alternatively E_r^* in our abbreviated notation/nomenclature, is the total energy (or work) ‘capacity’ of the RSWs and NMDs (collectively) at *cosmological* (‘comoving’) distance r . It comprises a summation of non-local mass carrying capacity terms multiplied by their respective wave-like $\frac{1}{2}\Delta a_w^2 \Delta t^2$ (specific energy) space-warp terms — both of which can act upon a (localised) ‘compact’ mass. Note that the ‘ i ’ subscript in Equation 21 refers to the index of summation applicable to the different RSW and NMD systems (involving for example: Sun–Jupiter–Ganymede, Sun–Saturn–Titan, etc.). At the inception (i.e. establishment) radius ($r = r_1$) of each RSW and NMD we have: $\Delta E_w = E_1^* = m_1^* \Delta e_w$. Thus, the dispersion of the field energy (and/or the ‘work’ capacity of a particular RSW and NMD), as r *increases* from r_1 , is reduced such that each: $(E_r^*)_{\text{component}} \propto 1/V_r$ or $(E_r^*)_{\text{component}} \propto (1/r^3)$.

To proceed without undue complication, it is appropriate to this Section’s goal that we idealise circumstances by way of three reasonable approximations. Firstly, we assume solar system Sun–planet–moon orbital stability over most of the solar system’s history. The $(\Delta e_w)_i = (\frac{1}{2}\Delta a_w^2 \Delta t^2)_i$ (component) terms are effectively constant at the present time (\pm thousands of years). We extend this treatment of e_w values as *constants* to billions of years. Secondly, with the e_w values being of similar magnitude (see Table 5 in section 6.5) we assume all e_w values are *equal*. Thus, $E_r^* \propto \sum m_r^*$, where $\sum m_r^* \equiv (m_r^*)_{\text{total}}$. Note that we are using E_r^* rather than writing $(E_r^*)_{\text{total}}$. Thirdly, with the m_r^* values being of similar magnitude (see Table 7 in section 6.6) we assume all m_r^* values are (also) *equal*. Thus, we have $E_r^* \propto \overline{m_r^*}$, where $\overline{m_r^*}$ is a mean NMD of the various m_r^* distributions; and (similarly to an individual/component m_r^*) we obtain:

$$E_r^* \propto (1/r^3)$$

A subtlety worth recalling/noting is that E_r^* is a total energy (that is) based upon the entire enclosed (cosmological scale) volume at a distance r ; but by way of the non-locality (or global nature) of non-local mass (m^*), this energy magnitude applies to *all* locations on/at the spherical *surface* of the NMD’s enclosed/spanned volume V_r .

In section 7.8 we begin to more fully examine how the presence of (constant acceleration/gravitational

amplitude Δa_w) RSWs with their (conjoint) m_r^* distributions may influence, i.e. do work on or alter the energy of ‘inbound’ electromagnetic radiation (and photons). Note that this paper has previously concentrated upon the influence of the RSWs’ acceleration undulations upon the motion of *low mass bodies*, whereas the wave-like propagation of photons as EM radiation (at c), and their *zero* (rest) *mass*, means that the propagation speed of photons cannot be influenced by RSWs in the manner that compact (low mass) bodies are — although in subsection 3.5.3 we argued for the possibility that RSWs may have left a ‘signature’ upon CMB radiation data (as measured by the Wilkinson Microwave Anisotropy Probe i.e. WMAP).

7.7 Circumventing the ambiguity of ‘light’ propagation distance

Section 7.6 discussed collective/total RSW and NMD based energy changes for increasing distance *from* their source ‘region’ (i.e. the solar system); whereas, the arrival of photons at the Earth’s surface involves propagation *towards* the solar system — i.e. *decreasing* distance. The arriving photons have had to climb into the dispersed total non-local mass $[(m_r^*)_{\text{total}}]$ and total energy E_r^* field of the RSWs and NMDs so as to reach the observer. In other words: a propagating photon/electromagnetic radiation *en route* to the solar system (inadvertently) ‘experiences’ a radius/distance dependent increasing scalar field. Note that a photon’s overall/total m_r^* (‘starting’) value is set at emission/absorption, with m_1^* (at r_1) being the maximum achievable value of any m_r^* component. The greater the distance travelled, the greater the difference/increase (is) between the m_r^* and m_1^* values, see Figure 13. Beyond the dark energy transition (and RSW and NMD initiation) redshift distance z_t (defined and discussed in section 7.3), the difference between these two values flattens out, i.e. no longer changes. Note firstly, that the discontinuity mentioned in Figure 13 (top diagram, maximum distance) refers to the extent of the RSWs’ and NMDs’ influence upon electromagnetic radiation (as observed ‘now’); and secondly, that the current (i.e. ‘now’) field of the (non-local, global/systemic) RSWs and NMDs actually fills the *entire* universe, regardless of its ‘size’, with even lower (individual) m_r^* and (collective) E_r^* values achievable (in the future) in a further expanded universe.

A major issue (until now unmentioned) is that the distance (r) from the various rotating space-warp (RSW) source radii has been treated as independent of the universe’s expansion — i.e. it has been effectively treated as a comoving distance — whereas the distance travelled by a photon is dependent upon the (metric) expansion of space. Thus, our discussion of distance travelled is inherently ambiguous. Fortunately, the inverse *cubic* dispersion of each $(m_r^*)_i$ and $(m_r^*)_{\text{total}}$ with increasing r means that: for small cosmological distances, cosmological expansion can be neglected. Additionally, for larger r , i.e. between (say) 2.0 and 4.56 billion light (travel time) years, the relative error in the m_r^* values, and hence E_r^* , is very small — even though a non-expanding distance scale is oversimplified.

⁴²¹Small differences in distance (r) between the different Sun–planet–moon (RSW and NMD ‘generating’) systems, by way of their ‘sources’ lying in different parts of the solar system (i.e. near Jupiter, Saturn or Neptune), are negligible when *cosmological* distances are being considered.

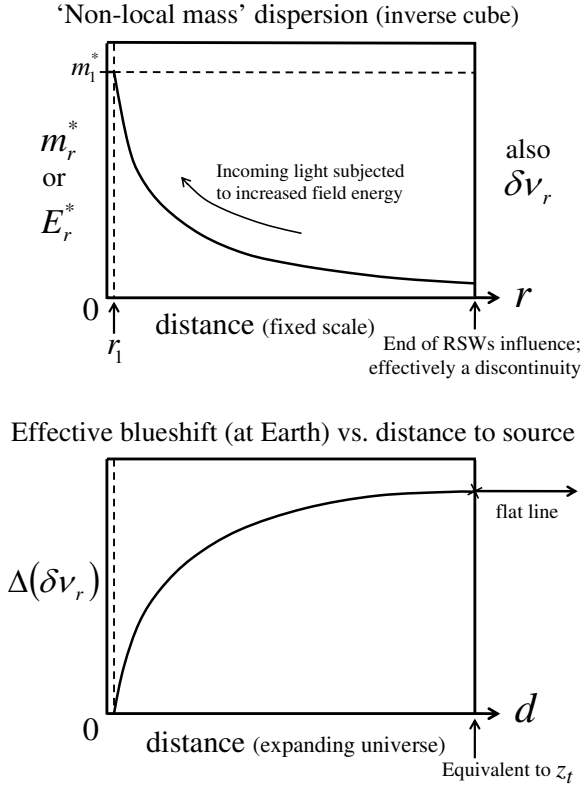


Figure 13: The top schematic diagram represents the dispersion of both: a single non-local mass distribution (m_r^*), and total/collective non-local energy (E_r^*), with increasing distance (r) away from the ‘source’ of the rotating space-warps (RSWs) and their conjoint non-local mass distributions (NMDs). We hypothesise that this inverse cube-like relationship also represents a distance dependent blueshift frequency offset ($\delta\nu_r$), relative to a case without RSWs and NMDs present. The lower schematic diagram illustrates the presently *observable* difference in this blueshift offset [$\Delta(\delta\nu_r)$], as experienced by light/electromagnetic radiation ‘during’ its propagation (towards the solar system) through the (non-local mass based) E_r^* energy field (arising from multiple RSWs and NMDs). Beyond the time of RSW and NMD initiation, i.e. > 4.56 billion years ago — which is effectively the (dark energy) transition redshift ($z_t \approx 0.44$) — there can be no further increase in this value.

For our approximate (i.e. idealised) purposes the expansion of the universe, although obviously significant for cosmological redshift $\Delta\nu$ (or $\Delta\lambda$), is fairly insignificant for the (RSW and NMD based) $\delta\nu$ (or $\delta\lambda$) offset values proposed in this Section. We shall proceed without correcting E_r^* , $\delta\nu$, or (the various individual) m_r^* (values) for the expansion of the universe. Additionally, it should be noted that we have not considered solar system (peculiar) motion, with respect to the type 1a supernovae (‘standard candles’), over the past (approximately) 4.56 billion years.

Finally, the non-locality of RSWs and their conjoint NMDs (implying ‘instantaneous’ field entanglement) makes comoving distance — i.e. the distance between two points measured along a path defined at the *present* cosmological time — equivalent to the RSW and NMD distance measure discussed throughout this paper (i.e. $D_C = r$), but only ‘now’/at the present cosmological time — a feature that suits the non-locality inherent to the model. As regards this/our idealised investigation, the distinction between comoving distance and proper distance (the distance scale that expands with the universe) is not a pressing concern.

7.8 An alternative interpretation of the type 1a supernovae data

The top schematic diagram of Figure 13 represents the spatial distribution of field strength for both: a single non-local mass distribution (m_r^*), and total/collective non-local (scalar) energy (E_r^*) — arising from (one or multiple, respectively) rotating space-warp(s) and (respectively) its/their conjoint non-local mass distribution(s) — that have influenced photons ‘received’ (in the present epoch⁴²²). In section 7.6, Equation 21 defined the total energy (or work) ‘capacity’ of multiple (coexisting) RSWs and NMDs as $E_r^* = \sum (m_r^* \Delta e_w)_i$. Subsequently, let us propose and define

$$E_r^* = h^*(\delta\nu_r)$$

or (alternatively) written more conventionally

$$E^*(r) = h^*[\delta\nu(r)]$$

as being indicative of the (radius/distance dependent) frequency *shift* associated with the *existence* of the new (total) scalar energy field. Note that each $\delta\nu_r$ [i.e. $\delta\nu$ at r] value represents the *offset* in cosmological ‘redshift’ due to the presence of RSWs and NMDs cf. the case without the presence of this scalar field effect. We also note that the use of ‘ δ ’ does *not* represent a (mathematical) infinitesimal change in the value of a variable (as is the case) in infinitesimal calculus.

We use this (unconventional) ‘ δ ’ nomenclature/notation for two reasons. Firstly, to indicate an offset from idealised (no E^* field) conditions; in this sense it is similar to δ as used in Engineering Mechanics to represent a deflection (e.g. of a beam from equilibrium conditions). Secondly, it represents an anomalous/second type of change in addition to a standard change/decrease (of frequency with distance, $\Delta\nu = \nu_e - \nu_o$), in the same manner that *two different* speed changes were required earlier (e.g. in subsection 3.2.9) — one for the speed oscillation amplitude (Δv) induced by a rotating space-warp, and the other for the associated monotonic speed loss (δv) of a moving body (cf. predicted speed without the RSWs’ presence).

Actually, neither E_r^* nor $\delta\nu_r$ can be directly measured; only a *change*/increase in their values is observationally relevant. Thus (via final and initial values):

$$E_1^* - E_r^* = h^*(\delta\nu_1 - \delta\nu_r) \quad \text{or alt}^y \quad \Delta E_r^* = h^*[\Delta(\delta\nu_r)]$$

⁴²²With “present epoch” being used in a cosmological sense, such that any given time in the past 50 years or so can be effectively considered ‘now’.

represents the difference/increase that exists between initial photon emission/absorption and (final) reception/observation. This blueshift effect is supplementary to the standard cosmological expansion effect (i.e. a ‘cosmic’ scale factor based *redshift* effect, $\Delta\nu = -(\nu_o - \nu_e)$). The spatial distribution of this difference $[\Delta(\delta\nu_r)]$, stripped of the proportionality constant (h^*), is shown in the bottom schematic diagram of Figure 13. This quantity is physically ‘real’ in that it represents a *quantifiable* blueshift of EM radiation arising during its propagation *toward* our solar system, as realised by measurements at the present ‘time’.

The variation of (i.e. change in) the proposed (frequency) blueshift offset $[\Delta(\delta\nu_r)]$ with distance, and (recalling section 7.3) the $z_t = 0.44$ transition (for $H_0 = 71.5$ (km/s)/Mpc, $\Omega_M = 0.265$, and assuming a flat universe), concur with the graphical supernovae representation of Davis, Mörtzell, Sollerman, Becker, Blondin, Challis, Clocchiatti, Filippenko, Foley, Garnavich, Jha, Krisciunas, Kirshner, Leibundgut, Li, Matheson, Miknaitis, Pignata, Rest, Riess, Schmidt, Smith, Spyromilio, Stubbs, Suntzeff, Tonry, Wood-Vasey, & Zenteno (2007, Figure 7), which plots redshift vs. ‘distance’ modulus⁴²³ (μ).

Physically, this (*non-local* mass based) frequency blueshift effect appears to be restricted to (i.e. only active upon) the wave-like nature of EM radiation. In other words, as light propagates toward the observer this hitherto unforeseen blueshift (energy increase) occurs. Note that nothing discontinuous is seen to occur suddenly upon measurement. Emission and absorption events and their associated spectra, as well as the generation of continuous spectra, and also measurement events, are all governed by standard quantum electrodynamics.

7.9 Ramifications of the NMD-based supplementary blueshift

Standard cosmological redshifts arise by way of comparing distant spectroscopic lines with the wavelengths of reference spectroscopic lines — as determined/calibrated in Earth-based laboratories. The measured brightness of a type 1a supernova (SNe 1a) indicates the distance (now) of the object, whereas the redshift illustrates the intervening expansion history of the universe⁴²⁴. Including this supplementary blueshifting ‘process’ makes for greater cosmic scale factor redshifts than is currently appreciated; i.e. the currently *perceived* relationship between measured redshift and cosmic scale factor is ‘significantly’ altered, as is the redshift-magnitude relation of type 1a supernovae, with the most significant effects at *low* redshifts.

Upon accepting the presence of NMDs, the (once surprising⁴²⁵) dimness of type 1a supernovae need not

be attributed to a recent acceleration of the universe’s expansion, and the hypothetical existence of dark energy. Correcting for the new blueshift effect, means that objects such as SNe 1a are actually more distant cf. distance (expectations) based upon (recalling section 7.4) an *assumed* “... direct one to one relationship between observed redshift and comoving coordinate.” Furthermore, the (possible existence of) foreground ‘contamination’ in the CMB radiation data — which possibly arises indirectly from the existence of RSWs (see subsection 3.5.3) — is supportive of the idea that a re-interpretation of the universe’s expansion history may be required.

This re-interpretation potentially allows a return to (the more logically simpler status of) a *decelerating* expansion. Certainly, issues such as the “age problem” of the universe will resurface; but the ramifications of the existence of rotating space-warps, with their associated non-local mass (and energy) distributions, deserve investigation. As such, the scenario proposed in this Section, albeit involving a not insubstantial degree of idealisation, conceivably explains why there has been a remarkable lack of progress in characterising the ‘essence’ of dark energy, and the physics underlying the transition redshift of dark energy (z_t). Supporting evidence for this conjecture is presented in section 7.10.

7.10 How the RSWs’ non-local mass distributions act to mimic effects attributed to dark energy

Detailed investigation has revealed that *dark* energy density (i.e. energy ÷ volume) remains constant with time (Wang & Tegmark 2004). Indeed, Max Tegmark told PhysicsWeb⁴²⁶: “I’m struck by the fact that the dark energy seems so ‘vanilla’.” Furthermore, the cosmological *constant* term in gravitational theory is considered to act like a constant (positive) vacuum energy density. The implication here is that: $\Delta E_{\text{dark}} \propto \Delta V_u$ where V_u is the volume of the universe; with temporal changes in V_u (i.e. ΔV_u) dependent upon changes in cosmic scale factor [$a(t) = a = a_{\text{scale}}$ (say), in order to avoid confusion with acceleration amplitude]. Thus:

$$\frac{\Delta E_{\text{dark}}}{(\Delta a_{\text{scale}})^3} \approx \text{constant}$$

For a photon *approaching* the solar system, section 7.6 argued that: $E_r^* \propto r^{-3}$, which then implies: $\Delta E_r^* \propto (\Delta r)^{-3}$. Alternatively, if we replace the model’s solar system (origin) based distance (r) with a distance (R) that *increases* with photon propagation duration, and whose ‘origin’ lies well beyond any photon emission points, then we have (roughly) $\Delta E_R^* \propto (\Delta R)^3$. Subsequently, noting that R is a (towards Earth) distance and *not* cosmic scale factor:

$$\frac{\Delta E_R^*}{(\Delta R)^3} \sim \text{constant}$$

ery’. Today, physicists (in general) are very accepting of this observation, although decidedly ‘less so’ the interpretation given to it.

⁴²⁶<http://physicsworld.com/cws/article/news/19736> (Belle Dumé, Jun 24, 2004).

⁴²³Distance modulus is the apparent magnitude (of an astronomical object) minus the absolute magnitude, i.e. $\mu = m - M$.

⁴²⁴Distances need to be determined in conjunction with some particular model of spacetime; usually this is a (standard) Lambda-Cold Dark Matter (Λ CDM) cosmology.

⁴²⁵The type 1a supernovae observations/results were a major surprise to physicists at the time of their ‘discov-

Over the past two billion years — the period of time where the change in dark energy is most noticeable — both a_{scale} and R evolve (approximately) linearly with time (i.e. the *same* time). Thus, $\Delta a_{\text{scale}} \propto \Delta R$ is feasible for recent (cosmological) times and low redshifts ($z < 0.16$). With a common *cubic* distance relationship, or volume-like relationship, present in the denominator of both the two (change in) energy based ratios (with numerators ΔE_{dark} and ΔE_R^*), the similarity of their (through time) variation is greatly enhanced. Indeed it is their definitive shared attribute.

For this reason, at low redshifts with R^3 proportional to a volume V_R we have:

$$\frac{\Delta E_R^*}{\Delta V_R} \sim \text{constant}$$

which mimics the near constancy of dark energy density through time as the universe undergoes expansion (devoid of the model's NMDs), such that:

$$\frac{\Delta E_{\text{dark}}}{\Delta V_u} \approx \text{constant}$$

These two constant (energy density) ratios, describe very different and unrelated physical scenarios. The very similar proportionality displayed by both (dimensionless) Δa_{scale} and ('dimensional') ΔR , as well as ΔV_u and ΔV_R , makes *these* two (temporal change based) ratios approximately equal. We reiterate that V_R is a (photon-point) motion based volume, whereas V_u is an expanding scale (line segment) based volume.

If we accept the model's alternative energy 'density' relationship, then the 'plain vanilla' dark energy, that Max Tegmark refers to, may be regarded as symptomatic of a (i.e. our) completely different mechanism; a mechanism that is largely independent of (i.e. only marginally influenced by) the universe's expansion. This mechanism (similarly) requires the existence of an unforeseen energy (E_R^*), or more appropriately E_r^* (as defined in section 7.6). Subsequently, dark energy (E_{dark}) may no longer be in need of 'dispensation'.

7.11 Summary and closing remarks

In this Section we conjectured that the model's various non-local mass distributions (NMDs), i.e. the various m_r^* distributions, and the overall/total energy distribution (E_r^*) — associated with the superposition of (multiple) rotating space-warps — can influence the energy (and frequency) of electromagnetic radiation. We hypothesised — in relation to the observation (in our solar system) of photons arriving from (standard candle) type 1a supernovae — that: Earth bound EM radiation (during its journey) has been *blueshifted*, by way of encountering the (non-local) energy distribution/dispersion of the (multiple) GQ-RSWs and NMDs scalar fields — such that: $\Delta E_r^* = h^*[\Delta(\delta\nu_r)]$ where h^* is a (new) proportionality constant with dimensions $[ML^2/T]$. Further, section 7.7 established that the quantification of this 'supplementary' blueshift effect/*offset* ($\delta\nu$), as measured by its *change* between (distant) emission/absorption and reception/measurement at Earth locations in the present epoch $[\Delta(\delta\nu)]$, is only slightly dependent upon

the expansion of the universe; as compared to the (cosmological) cosmic scale factor redshift, which is solely determined by the expansion of the universe. In other words, this (lesser and) hitherto unrecognised cosmological blueshift is almost totally dependent upon an Earth *approaching* photon (and/or EM radiation) gaining energy as it climbs into the (distance, or rather volume dependent) scalar energy field (E_r^*) of the solar system 'centered/based' RSWs and NMDs.

Support for this conjecture/hypothesis lies in the remarkable 'coincidence' that the dark energy transition redshift ($z_t \sim 0.44$) — i.e. from decelerating universal expansion to an accelerated expansion — matches the establishment 'time' of lunar spin-orbit resonance in the solar system's very early history (i.e. ≈ 4.55 billion 'lookback' years ago), see section 7.3. This synchronisation feature is arguably *the* major prerequisite required for 'generating' the model's RSW and NMD scalar fields. Other features supportive of dark energy were critically evaluated in section 7.5.

An important finding of this Section was that the (through time) manner of the EM radiation's (frequency based) energy change is very similar to, and effectively mimics, the presence of an apparent constant 'dark' energy *density* and accelerated expansion of the universe. This second (and currently accepted) interpretation is implied by the redshift-magnitude relation of type 1a supernovae (and observations of baryon acoustic oscillations) — in the assumed *absence* of RSWs and their 'conjointly existing' NMDs. Further, by way of the overall/total energy distribution (exhibiting dispersion) that surrounds (multiple) RSWs and NMDs in our solar system, we need to adjust/correct the measured (and assumed) value of the universe's expansion-based cosmological redshift in order to compensate for this new cosmological blueshift effect. Upon implementing this adjustment (it turns out that) the universe is quite conceivably (completely) free of dark energy and undergoing a gravitation-based *deceleration* in its expansion.

Concerns raised by this (suggested) return to what was once conventional gravitational cosmology have been briefly addressed, but not with any rigour; e.g. "the age problem", and the status of the late-time integrated Sachs-Wolfe effect. The objective of this Section has been to broadly outline, with the assistance of a not insubstantial degree of idealisation, a conceivable *alternative* to dark energy. This was achieved by way of drawing upon content within the model and explanation of a 'real' Pioneer anomaly presented throughout this paper — with this model being molded and governed by the awkward observational evidence associated with the Pioneer anomaly (and other peripheral issues such as the Earth flyby anomaly).

8 Summary, Discussion and Conclusions

Due to the length and broad/multidisciplinary scope of this paper, rather than merely present a conclusion, this (final) Section contains both: a summary of the major features and findings of the model/mechanism,

as well as a fairly protracted discussion of its ramifications. As such, this concluding Section is interspersed with a diverse range of constitutive conclusions. Sections 8.1 to 8.3 are largely model-specific; they outline the model's: primary conceptual features, major quantitative results and equations, as well as a number of predictions and applications. A somewhat different approach is evident in sections 8.4 and 8.5, in that they are largely devoted to discussing and encapsulating the wider and widest ramifications (respectively) of the new model/mechanism. In section 8.6 a brief final retrospective commentary is delivered. In the interests of being productive and progressive, some new information has been introduced throughout this Section in order to achieve this treatise's ultimate aims. An underlying theme of this Section (particularly prominent in section 8.2) is the philosophy of science and philosophy of physics based apologetics⁴²⁷ given to appease the sceptical reader's hostility towards both a real and non-systematic based Pioneer anomaly and the unorthodox explanation (of it) 'reasonably' pursued herein.

8.1 The model's major results, predictions, applications & features

This paper has developed a model for what many scientists believe is the impossible or highly unlikely case of a 'real' (non-systematic based) Pioneer anomaly, i.e. one where thermal-radiation/heat based effects and other (external, on-board, and computational) systematics are present but play only a minor role. The new model explains the four principal observational features/constraints of the Pioneer anomaly; these being: (1) an mean anomalous (inward) deceleration (a_P) of $8.74 \pm 1.33 \times 10^{-10} \text{ m s}^{-2}$ (in the outer solar system and beyond); (2) the (quasi-stochastic) temporal variation of the anomaly around this long-term mean/average value; (3) an apparent violation of general relativity's weak equivalence principle (WEP) — in that the anomaly affects the motion of spacecraft and 'low' mass celestial bodies but not the motion of 'high' (or large) mass celestial bodies such as planets and moons; and (4) the lower value of the (Pioneer 11 based) anomaly — on approach to Saturn encounter.

A major qualitative feature of the model is that the anomalous 'deceleration' — which is generally referred to (simply) as an *acceleration magnitude* — acts in the (opposite) direction of the translational *path* of a (low mass) body, as compared to being Sun-directed or Earth-directed. Furthermore, the anomalous acceleration itself — although not its line-of-sight measurement — is independent of a body's speed⁴²⁸ and direction of motion, i.e. it is velocity independent. The energy source 'driving' the anomalous (Pioneer) acceleration is solar system based, involving (amongst many other things) three-body Sun-planet-moon celestial systems (see Introduction) — with each moon

needing be in a 1:1 or synchronous spin-orbit resonance 'around' its host planet. For an Earth-based observer the mean *measured* anomaly, although essentially constant in the far outer solar system — by way of the line-of-sight observations approaching a maximum constant value (asymptote) at large distances from the barycentre — and always present to some degree, is *not* position independent; especially when a spacecraft lies between Jupiter and Saturn, as was the case for Pioneer 11 on its approach to Saturn encounter. For each Sun-planet-moon system the model establishes a "cut-off mass", i.e. the maximum (passive gravitational⁴²⁹) mass of a body that can be influenced. These (numerous and different) cut-off masses (all) reduce with increasing distance away from their 'source' region, by way of an inverse r cubed relationship⁴³⁰ — i.e. an inverse spherical volume relationship. A rough ("rule of thumb") guide is that solar system bodies (of average density) with a diameter of approximately 1 to 2 kilometres lie around these cut-off masses; Table 7 in section 6.6 gives the actual (moon specific and) distance dependent cut-off *mass* values.

The model, whose basis and major features were encapsulated in the Introduction/Section 1, proposes two new physical field mechanisms associated with each (participating) "gravito-quantum" Sun-planet-moon system — of which (quantitatively) there are five dominant contributors involving Jupiter's four Galilean moons and Saturn's Titan. Apart from a very minor contribution from Neptune's Triton, effects attributable to all other Sun-planet-moon systems are either non-existent (e.g. Earth's moon) or negligible (e.g. Uranus' largest moon Titania⁴³¹). These five major superpositioned ('circumferentially' sinusoidal-like) three dimensional rotating space-deformations or rotating space-warps (RSWs) — existing (in a 'planar' disk slice) as opposing perturbations 'above and below' (and 'upon') the (equilibrium) spacetime curvature produced by general relativistic gravitation⁴³² — account for the path-based Pioneer anomaly. Quantitatively, a *root sum squared* (RSS) value is required, with this (RSS) proper deceleration (superposition) value (a_P) — cf. the (Pioneer S/C) observation based value (a_P) — 'formalised' in section 8.3. This RSS (anomalous 'acceleration magnitude') value is related to (both) the (sinusoidally varying) constant acceleration/curvature *field* amplitudes $[(\Delta g)_i]$ and the ensuing (effectively equal) proper acceleration amplitude (components) of a *moving body* $[(\Delta a)_i]$ — by way of these field effects/sinusoids producing (component) sinusoidal/oscillatory *speed* variations around the moving body's equilibrium (speed) value. Note that the '*i*' subscript (as used) here indicates both:

⁴²⁹Where the word "gravitational" is used here in a wider sense than normal, referring to both: standard (general relativistic) gravitational fields, *and* the supplementary acceleration/gravitational field type proposed herein.

⁴³⁰Where ' r ' is the separation distance of a body from a given (lunar-based) source region.

⁴³¹The eighth largest moon in the solar system.

⁴³²There is no change in the overall/net spacetime curvature arising from the inclusion of rotating space-warps (recall section 3.1). For a diagrammatic representation see Figures 4 to 7 in Section 3 and Figure 8 in Section 6.

⁴²⁷Reasoned arguments or writings in justification (or the defense) of something.

⁴²⁸Noting that no celestial body in the solar system exists at rest (i.e. in the absence of some kind of motion) relative to the solar system's barycentre.

the index of summation of the RSS components, and that multiple instantiations of the particular variable are being referred to. Importantly, each unsteady/oscillatory motion/speed component also produces a component (rate of) *speed shortfall* $[(\delta a)_i]$ — and a (component) translational kinetic energy loss (rate) — *relative* to (speed) predictions/expectations that *omit* the presence of the supplementary gravitational/accelerational mechanism, i.e. where the *predicted* kinetic energy (K.E.) is assumed (effectively) steady/translational⁴³³. Additionally, the model (necessarily) proposes/introduces a separate (i.e. second) field mechanism feature — involving the magnitude and distribution of a new scalar physical quantity⁴³⁴ called “*non-local mass*” (m^*) — that pertains to both the appeasement of the apparent violation of the weak equivalence principle, and the dispersion of energy away from each supplementary field’s source region⁴³⁵.

The model’s **quantitative results** are divided into three categories. Firstly, the (outer solar system and beyond) long-term average constant (idealised maximum/asymptotic) ‘Pioneer’ anomaly value (a_p) is: $8.65 \pm 0.66 \times 10^{-10} \text{ m s}^{-2}$. This is a path-based result, and (subsequently) the model’s proposed average line-of-sight-measurement value of the anomaly associated with the Pioneer 10 spacecraft (01 Jan 1987 to 22 July 1998) is $8.52 \pm 0.66 \times 10^{-10} \text{ m s}^{-2}$. An average line-of-sight value of $7.61 \pm 0.66 \times 10^{-10} \text{ m s}^{-2}$ is associated with the Pioneer 11 spacecraft (01 Jan 1987 to 01 Oct 1990). Note that for a number of reasons including: data arc duration/length, general spacecraft performance (e.g. gas leaks), magnitude of solar radiation pressure, and level of solar activity (Anderson et al. 2002, p.34), *the Pioneer 10 data is/has been considered (significantly) superior to the Pioneer 11 data.*

Secondly, in Section 3 the model is able to account for aspects of the temporal variation in the Pioneer 10 data. Noting that $10 \text{ mHz} \equiv 0.652 \text{ mm/s}$:

- a) The model-based variation in speed, around an equilibrium speed, of 16 to 19 mHz due to the RSWs, combined with the (root mean square) raw measurement noise/residual of “a few mHz”, sum (i.e. add up) to match the overall observation-based (post-fit) residual/noise of 20 to 25 mHz.
- b) The Earth-based *diurnal* residual confirms the Pioneers’ exceptional navigational precision and accuracy. Section 3.4 argued that although a “true-annual” residual exists, its amplitude is not significant, and the (more appropriately labelled) \sim annual residual (C. Markwardt’s nomenclature) has been incorrectly interpreted as Earth-based.
- c) The Callisto-Titan beat frequency amplitude of 0.733 mm/s (i.e. 11.24 mHz) and temporal duration of 357.9 days — corrected to 356.1 days

⁴³³The kinetic energy associated with a spacecraft’s spin/rotational motion may be safely ignored.

⁴³⁴A *physical quantity* is expressed as the product of a numerical value (i.e. a number) and a physical unit.

⁴³⁵Note that this source ‘region’ is centered upon a point on the moon’s *spin axis* in the case of the (cylindrical-like or parallel planar-like) rotating space-warps, whereas it is centered upon a moon’s core (or *central point*) in the case of the (spherical-like) non-local mass distributions.

from the perspective of the Pioneer 10 spacecraft — matches the amplitude ($\approx 0.7 \text{ mm/s}$) and duration of the \sim annual oscillatory term (i.e. 355 ± 2 days, or $0.0177 \pm 0.0001 \text{ rad/day}$ angular velocity).

- d) Noting that Io-Europa-Ganymede obey a 4:2:1 (spin and) orbital resonance, the (unusual) 200-day correlation time sensitivity of the \sim annual residual can be attributed to this duration being almost exactly four times the 50.1 day (lesser known) 7-to-3 Ganymede-Callisto orbital resonance duration.

Thirdly, the initial/establishment values of non-local mass $[m^*(r_1)]$ or more simply/succinctly m_1^* , specific to each Sun-planet-moon (or barycentre-planet-moon) system, and their spatial distribution $[m^*(r)]$ or rather m_r^* around their (lunar) source region are discussed in section 6.6 and quantitatively displayed in Table 7. Note that this subscript based nomenclature is tailored to equalities involving the continuity of non-local mass.

Primary **predictions** specific to the model’s conceptualisation and quantification are:

- a) That the accuracy of celestial body and spacecraft “orbit determination”, e.g. future interplanetary missions and near-Earth objects (NEOs) — as well as the proposed Laser Interferometer Space Antenna (LISA) mission — will benefit from adjusting for the (model’s) Pioneer anomaly-like influence.
- b) For Pioneer 11 ‘between’ Jupiter and Saturn the amplitude of temporal variation in the anomalous acceleration should be muted, in proportion to the lessening of the anomalous acceleration magnitude.
- c) Asteroid 1862 Apollo (1.7 km in diameter with a mass $\approx 5.1 \times 10^{12} \text{ kg}$) is an ideal asteroid for examining the model’s distance dependent non-local mass cut-off values — i.e. the maximum (passive gravitational) mass that can be influenced (subsection 6.6.10). This is because its orbital path spans the transition zone of four of the five dominant Sun-planet-moon contributors — with Saturn-Titan being out of ‘range’ (for an asteroid of this mass).

It should be noted that due to the very small magnitude of the (Pioneer) anomaly, verification or falsification of these predictions is not easily (nor inexpensively) achieved; it will take time, effort, and resources.

Major **applications** of the multiple (and superpositioned) instantiations of the two proposed (supplementary) field mechanisms include:

- a) Section 7’s proposal that a foreground effect related to the non-local mass distributions (NMDs) leads to a supplementary cosmological blueshift of incoming electromagnetic radiation (and *massless* photons); for example, the ‘light’ from “standard candle” type Ia supernovae, and “standard ruler” baryon acoustic oscillations (re: galaxy clustering). This effect arguably dispenses with the need to enact the existence of (mysterious) *dark energy*.
- b) Subsection 6.2.4 gives a qualitative account of how the *Earth flyby anomaly* can be attributed to the existence of rotating space-warps (RSWs), if the RSWs are ‘locally’ refracted by the Earth’s (~ 10 billion times stronger) gravitational field so that

their planes of rotation are (all) parallel with the Earth's equatorial plane. This qualitative account complements (and validates) the quantitative modelling and empirical prediction formula of the Earth flyby anomaly proposed by (Anderson et al. 2008).

- c) Due to the large moons of the solar system achieving spin-orbit resonance very early in the solar system's history, explanatory accounts of *solar system formation* — at least as regards asteroids and comets of less than approximately 1 or 2 km diameter (respectively)⁴³⁶ — will be similarly affected by the overlooked/supplementary (anomalous) speed-retardation effect, as the Pioneer spacecraft have been in the present era. Over one million years the extremely small speed retardation rate can produce a significant ≤ 27.3 km/s reduction in speed⁴³⁷.

Other possible applications include: (d) that an explanation of the anomalous *alignment of the cosmic microwave background radiation (anisotropy) with the ecliptic plane* may have its basis in the new model; noting that the planes of space-warp rotation, associated with Jupiter's equatorial plane and its four Galilean moons, are closely aligned with ($< 2^\circ$ inclination to) the solar system's (Earth based) ecliptic plane (see subsection 3.5.3). Further, by way of having (indirectly) argued that other (non-general relativistic) gravitational 'sources' can exist, it may be the case that: (e) *galaxies*, i.e. systems of billions of roughly equally massive stars — (at least) when compared to circumstances in our solar system — that are fairly evenly (and non-centrally) distributed throughout, exhibit more than simply Newtonian gravitation; i.e. galaxies might exhibit an additional non-Euclidean geometry contribution. A myriad of field interaction effects (for example) could possibly be involved.

8.2 Physical and conceptual supplements inherent to the model

In this paper the Pioneer anomaly is treated as a real (i.e. non-systematic based) "gateway observation" that suggests a need for new physics, with the Earth flyby anomaly being supportive of this stance. The

⁴³⁶ Although larger/higher mass bodies can be affected 'on occasion' — for example Halley's comet at $\approx 2.2 \times 10^{14}$ kg (as discussed in subsection 6.4.6) — this range of size values applies to a more 'continual' anomalous influence. For a comet of (say) 1 km diameter with a (typical comet) density of 0.5×10^3 kg m⁻³ and (thus a) mass of 0.26×10^{12} kg, the rotating space-warp based anomalous speed/motion retardation effects are largely 'continuous' — applying at distances of less than approximately: 18 AU from Jupiter, 7.5 AU from Saturn, and 5 AU from Neptune (via Table 7). For a 0.5 km diameter comet (cf. 1 km), or a (typical density) 0.3 km diameter asteroid — noting that asteroids have an average density of about 2.5×10^3 kg m⁻³ or 2.5 g/cm³ — these distances increase eight-fold, and thus the speed retardation/loss effect is made effectively (or at least increasingly) distance-independent.

⁴³⁷ This speed reduction arguably cleanses the solar system of a great deal of (relatively) low mass material, by way of this matter losing kinetic energy and spiralling into the Sun. By way of comparison the mean orbital velocities of Venus, Jupiter, and Neptune are: 35.0, 13.1, and 5.4 km/s.

'boutique', unique and very specific new physics model proposed herein, although completely independent of general relativistic gravitational sources, needs to (and does) coexist (harmoniously) with general relativity. General relativity (GR) is neither considered wrong (in any way) nor in need of modification; and on the whole (i.e. "all in all"), general relativity 'rules' (i.e. dominates) Gravitation. An "exception to the rule" solitary new source of gravitation is proposed — i.e. a new source of non-Euclidean space-time geometry is proposed. This supplementary source type is based upon (a many atomed/moleculed) internally inexpressible fractional quantum mechanical (spin-based) *energy* being expressed externally, so as to appease systemic (i.e. universal) energy conservation — see the Introduction for a more elaborate account/overview.

Basically, this additional source-type of non-Euclidean (spatial) geometry, in addition to GR's (mass, momentum, and energy based) non-Euclidean geometry/spacetime curvature, affects a (low-mass) body's motion in a manner that 'parallels' both: solar radiation pressure, and the Poynting-Robertson and Yarkovsky effects. This (remark) is true in the sense that the Pioneer anomaly is a supplementary/additional effect upon celestial motion, not necessarily equally applicable to all sizes/scales of matter, and it acts so as to merely alter or perturb a body's motion in a comparatively minor manner — at least over (celestial/cosmologically) brief time scales. The boutique/unique nature of the model relates (in part) to the unique fact that (with regard to quantum mechanics) intrinsic angular momentum (i.e. *spin*) has neither a classical limit nor a classical analogue. We also note that the development of GR preceded/predated quantum mechanics (QMs) and that (to date) a unification of GR and QMs has not been convincingly validated; as such, the 'new' physical model hypothesised is *not* inconsistent with the physics of *today*.

The primary conceptual move supporting the new model/mechanism is the implementation of an energy ('transfer'/re-expression) based mechanism that spans (microscopic) systems described by quantum mechanics and (macroscopic) systems 'described' by space-time curvature, so as to resolve a unique/solitary conflict scenario pertaining to *discrete* quantum mechanical atomic/molecular systems moving (along a geodesic) in *analog* curved spacetime. The implementation of a (model-specific) energy conservation principle is dependent upon (and consequential to) a proposed supplementation of our understanding of time — specifically a background/hidden stance (or 'take') on the sequencing of events on a universal scale — that takes issue with the conventional (ubiquitous non-simultaneity) *interpretation* of light signal-based measurements; notwithstanding the fact that 'different' clocks run at different rates. This (temporal) interpretive supplementation is implied by quantum entanglement and quantum non-locality. Support for this (energy and time based) conceptual supplementation is buttressed by an appreciation of the following *four* (largely philosophy of science and philosophy of physics based) *issues/discussions*.

Firstly, one's attitude to the Pioneer anomaly is

crucial. Are we in a period of (what Thomas Kuhn calls) “normal science”, or not (by way of the existence of a number of disconcerting anomalies)? The former (of these two stances) will tend to: dwell on the strengths of a ‘research programme’ cf. concentrate upon and investigate its weaknesses; dismiss or seek to explain away the (creditably established) Pioneer and Earth flyby anomalies; and bypass or downplay the need to question/re-access core concepts/notions such as mass, time, and energy — much less enter into an interconnected re-conceptualization of aspects of these fundamental physical ‘quantities’⁴³⁸. In broad terms, ‘normal science’ undergoes incremental advances, and a big risk factor is an associated sense of confidence that (on the whole) “all is well”, such that ‘opportunities’ may not be fully examined/explored. Recently, Hawking & Mlodinow (2010, p.13) confidently proclaimed/concluded that: “... *philosophy is dead* [italics added].” Sympathising with the latter of the two schools of thought given at the beginning of this paragraph, one hears a faint echo of the confident proclamation attributed to Lord Kelvin (William Thomson) and his contemporaries — at the turn of the (19th to 20th) century⁴³⁹. Indeed, philosophical thinking can (easily) appreciate two *hidden assumptions* in this ‘morbid’ stance/quote: (1) that we *are* in a period of “normal science”, and (2) the primacy given to the role of mathematics (over conceptual ‘wrestling’/evaluation); as well as appreciating a type of subtle elitism peculiar to physicists — who (by their behaviour) display no pressing need to draw upon philosophical expertise. The downside risk here is one of physicists tending to become an island unto themselves, albeit with ‘selected’ visitors welcome, confident that the *mathematical* nature of (current) physical models/theories (largely) contains the seeds of its future development⁴⁴⁰. Philosophers can clearly highlight the “straw-man” nature of the argument given to arrive at Stephen Hawking and Leonard Mlodinow’s (moribund) proclamation/conclusion (see http://www.philosophynow.org/issue82/Hawking_contra_Philosophy⁴⁴¹); and as a ‘society’ they (and philosophers of science and physics) give a decidedly greater weighting/emphasis to seeking a deeper understanding of what (the ‘weirdness’ of) quantum entanglement and non-locality might entail — as compared to (arguably) a latent *bias* within physics to downplay this awkward/non-conformist aspect of scientifically verified reality.

Secondly, minor imperfections regarding special and (particularly) general relativity (SR and GR) have been presented, involving: the intractability of (total) systemic energy (and mass); the global/systemic (cf. relative) nature of rotation and acceleration; the physical implications of GR’s (mathematical technique of)

general covariance; the reliance of GR’s formulation upon SR, and the scaffolding role played by the general principle of relativity in establishing the Einstein field equations; and (lastly) lingering doubts regarding SR’s clock problem/twin paradox. These issues allow us to (at least) question the totality of GR’s physical (and conceptual) scope. To this we might add two questions: (1) Can everything be relative? and in response to the question: “Why is there something rather than nothing?” (2) Can something and nothing exist independently of the concept of (a physical) everything? These two questions, and relativity’s imperfections, strongly suggest the need (on some level, in some manner) for a ‘universal’ (i.e. global or systemic) perspective. By way of quantum entanglement our new model uncovers and then exploits/utilises (such) a universal/global physical perspective.

Thirdly, armed with an open mind to physical reconceptualisation and an appreciation of SR’s and GR’s imperfections⁴⁴², the model’s (necessary) next step was to question an agenda of reductionism to a unified theory of everything, such that a fairly clear divide between microscopic and macroscopic realms/domains is maintained cf. overcome — which additionally casts into doubt the existence of the graviton particle. Assisting this ‘divided’ stance was: QMs and GR having so little in common, both physically (re: scale and ‘nature’) and in terms of their laws/mathematics; and (also) the fact that the (*linear and rotational* based) classical vortex theory of propellers and wind turbines (see subsection 4.2.7) requires a power (or at least energy) based governing expression (recall Equation 15: $E_p = m\Gamma_e f$) — cf. microscopic physics where a *force* and/or energy basis is sufficient. Subsequently, a “*pairism*” of ‘dual’ concepts is embraced⁴⁴³, and this structural tool is then used to delineate different facets of physical-reality/ontology. ‘Pairs’ particularly relevant to the model are:

- macroscopic vs. microscopic (domains and laws);
- analog vs. digital (i.e. continuous vs. discrete);
- Euclidean vs. non-Euclidean (re: geometry);
- local vs. non-local (re: physical ‘interactions’);
- internal vs. external (re: a physical system);
- relative vs. systemic/global (physical perspective);
- local vs. global (re: physical representation);
- dynamical phase cf. geometric phase (re: QMs);
- force (at a given time) cf. a process based energy;
- real cf. virtual (re: physical energy);
- spacetime cf. (distinct) space and time⁴⁴⁴;
- mathematical cf. conceptual (re: theory advance);
- symmetry cf. asymmetry (re: theoretical physics);
- linear vs. rotational (re: physical circumstances);

⁴⁴²This in no way should detract from the reverence deservedly bestowed upon the scientist/physicist (and philosopher) Albert Einstein.

⁴⁴³With “pairism” obeying “dualism” only in the sense of: the division of something conceptually into two opposed or contrasted aspects, or the state of being so divided. The new term ‘pairism’ is preferred because ‘dualism’ has a lot of additional conceptual baggage associated with it — more so in philosophy than in physics. Furthermore, the expression “duality” has quite different meanings in these two distinct academic fields, and markedly different applications.

⁴⁴⁴Written as “space-time” herein.

⁴³⁸Even though physicists admit that there are deficiencies concerning/around the understanding of these concepts.

⁴³⁹“There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.”

⁴⁴⁰For example, 11 dimensional M-theory with its dearth of clear-cut experimentally falsifiable predictions.

⁴⁴¹The article’s author Christopher Norris (at the time of writing) was Professor of Philosophy at Cardiff University.

reductionist cf. non-reductionist (physics agenda);
 unification cf. complementarity (re: guiding aim);
 the phenomenal/observational vs. the ‘noumenal’;
 with this last ‘pair’ having been discussed in some detail in the Introduction. Excluding the first three pairs, the second/latter of these paired concepts are given at least equal priority in the model/mechanism presented herein. In standard physics, with regard to its: overall perspective, self reflectivity, and ultimate objectives — i.e. “the big picture” — these (second) notions are generally considered of equal or *lesser* importance. Herein, our (non-reductive) dualist/‘pairist’ leanings (necessarily) *equally* recognises both sides of such ‘divides’.

Fourthly (and finally), rather than downplay the ‘weirdness’ of quantum entanglement and non-locality, we recognise that relativity’s light speed limit (conceivably) places physics at risk of being in something like a (scientific) “double bind” or a “Catch-22” (in its common idiomatic “no-win situation” usage)⁴⁴⁵. Combining this dilemma with the different (and ‘occasionally’ incompatible) treatment of time in quantum mechanics cf. special and general relativity (Albert & Galchen 2009), Section 4 argued for the introduction of a supplementary/additional ontological scenario involving a (hidden) universal/systemic ‘noumenal’ background ‘scanning process’ that occurs ‘between’ incrementally different staccato phenomenal (or able to be observed) temporal moments⁴⁴⁶ (see section 4.4). From a scientific observer’s (restricted) local and relative phenomenal/measurement-based perspective, time (only) appears continuous/analog, whereas at a deeper level of understanding it involves a digital ‘phenomenal’ aspect. Importantly, by proposing a (non-relative) relationship between: a body’s motion/speed, light/EM radiation as an observing ‘agent’, and the background universe itself — with the latter two not being independent of each other (cf. the case implicit in SR and GR⁴⁴⁷) — a (noumenal/universal perspective) supplementary interpretation of the Lorentz transformations and SR’s need for spacetime interval invariance was proposed. Six ramifications ensuing from: the aforementioned considerations (albeit a brief discussion⁴⁴⁸), and the noumenal–phenomenal (physical) *complementarity* arising from these (temporal and interpretive) supplementations, are now presented.

- a) A universe-encompassing (background/hidden) event/‘time’ simultaneity, albeit with *different*

⁴⁴⁵“Gregory Bateson and Lawrence S. Bale describe double binds that have arisen in science that have caused decades-long delays of progress in science because science had defined something as outside of its scope (or ‘not science’) . . .” — extracted from Wikipedia: *Double Bind*, Dec-Jan 2011-12.

⁴⁴⁶That is, separate (digital) instantiations of a universal-wide reality, with each ‘phenomenal’ discrete (incrementally different) moment separated by a (noumenal process-based) ‘pause’ — that is beyond direct observational/phenomenal appreciation.

⁴⁴⁷Particularly SR’s Principle of the (*observed*) invariance of the speed of light (in a vacuum), regardless of an inertial observer’s or light source’s state of (relative) motion/speed.

⁴⁴⁸Encapsulating Section 4 for the purposes of this brief summary unfortunately results in some degree of ambiguity.

rates of *measured* time duration between events — implies a (deformable/curvable) *space* substratum, that in the (idealised) absence of all: mass, motion, energy, etc. would be Euclidean (i.e. a ‘flat’ space).

- b) The ‘latent’ all-encompassing (‘beyond’ direct measurement) reality/event simultaneity allows *energy conservation* to be applicable in certain ‘universal’ situations, and it supports/strengthens the case for a well coordinated background inertial frame, particularly when non-local QM spin entanglement is involved — as is the case in our model/mechanism.
- c) From a noumenal/global perspective special relativistic time dilation, and subsequently length contraction and mass dilation, are alternatively/complementarily attributed to a *loss*⁴⁴⁹ in available-to-measurement specific energy — by way of the specific energy associated with relative motion ($\frac{1}{2}v^2$). Residual unease pertaining to the clock/twin paradox is vanquished, and maximum relative specific K.E. is bounded by a c^2 limit.
- d) Perceiving a standard/phenomenal specific energy ($e = E/m$) increase (from zero *up*) as alternatively a “drawing-down” from a maximum ‘potential reservoir’ value of $e = c^2$, that (in idealized conditions) would exist homogeneously throughout the universe if it were ‘completely’ empty. This *reversed*/‘opposed’ perspective (further) builds upon the use of ‘paired’ or complementary perspectives.
- e) GR’s need for a tensor based generally covariant mathematical formulation is seen to be a consequence of (further) adding gravitational effects to the (already existing) effects of (special relativistic) high-speed relative motion, and as such (from a noumenal and reversed perspective) a double “draw-down” (from the spatially homogeneous $e = c^2$ “maximum potential reservoir”) is involved.
- f) Finally, noumenal–phenomenal ontological complementarity will have an impact upon: (1) how quantum mechanics can/should be interpreted; and (2) the belief that randomness, and hence indeterminism/uncertainty, are deeply inherent to Nature⁴⁵⁰.

Note that the model does not establish these ramifications for the reason of discrediting special and general relativity. As previously mentioned, the theoretical structures of special and general relativity are neither: wrong, nor in need of modification; rather, by way of the phenomenon of quantum entanglement, some of their ontological assumptions (and foundations) have been questioned⁴⁵¹ and GR’s scope/domain of application is (merely) considered to be in need of a

⁴⁴⁹This loss is achieved by way of an accumulation of numerous incremental time losses (relative to maximum *availability*) — see subsection 4.5.6. Further, this situation is dependent upon ‘time passing’ as systemic/universal (extremely small/short finite duration) discrete ‘moments’.

⁴⁵⁰To what extent (“behind the scenes”) noumenal based *non-local hidden variables and processes* will impact upon these issues is (itself) an open and debatable issue.

⁴⁵¹With this questioning, and the ensuing ontological supplementation, having (minimally consequential) “time of an event” based *numerical* implications/ramifications.

(solitary) additional/supplementary physical circumstance (that is itself beyond GR's ken). Finally, we also note that although GR's non-Euclidean geometry is expressed in terms of a metric tensor of (phenomenal) spacetime, the new model's supplementary acceleration/gravitational field is necessarily expressed in terms of (distinct) curved space and (a systemic/universal) 'time' — by way of (i.e. in response to) utilising/adopting a noumenal perspective.

8.3 Major mathematical equalities and relationships of the model

The model/mechanism (developed throughout this paper) benefits from circumstantial **simplifications** such as: its field *perturbation* basis; the geodesic (i.e. unforced) motion of (the lunar based) atoms/molecules involved in the mechanism; the prevalence of geometry (in a number of guises⁴⁵²); and the use of a simple scalar (total/aggregate) energy — that is proportional to the total *number* of atoms/molecules (N_m) within a lunar non-rigid (predominantly solid) body (i.e. within a moon). Indeed, (quantifiable) energy is the physical linking (or bridging) means whereby the aggregate of (each and every) atom's or molecule's virtual (and 'localised') quantum mechanical based (spin) energy is re-expressed collectively/singularly and *externally* as a real (distributed) macroscopic (non-Euclidean geometry based) *rotating* acceleration/gravitational field. At a 'fixed' point in space this field induces a *sinusoidally* varying field amplitude/strength. Each of these "rotating space-warps" exists conjointly with a non-local mass field. In tandem with this energy conservation 'bridging' effect, and recalling the model's five distinct sizes of particles and systems (outlined in the Introduction), we note that the model's 'physicality' stretches from the (common and virtual) spin phase offsets of (lunar atomic/molecular residing) *elementary fermion particles* through to space-time curvature(s) (and non-local mass distributions) that span/encompass *the universe* as a whole (i.e. in its entirety).

This (aforementioned) macroscopic re-expression of total fictitious/non-inertial QM spin-based (virtual) *energy*: (1) addresses a unique discrepancy that can arise when a *digital system* is moving in *analog* curved spacetime; (2) maintains universal/systemic energy conservation; and (3) is time-irreversible; so as to ensure (ongoing) universal/systemic *stability* (without fail). The *irreversibility* of this energy re-expression allows the model/mechanism to be (effectively) regarded as a second type of non-reversible/irreversible time evolution of a QM 'system' — in addition to the irreversible process of a (quantum mechanical) measurement. Note that the large scale nature of the (superpositioned) rotating space-warp (RSW) based 'gravitational' field undulations (of amplitude Δg) — whose ultimate basis is spatial curvature (cf. space-time curvature in GR) — ensures that the sinusoidal field strength (amplitudes) at , and proper accelera-

tions acting upon, a *moving* spacecraft (Δa) effectively equal the Δg_{field} values (see subsection 3.2.3).

The **compatibility** of each (RSW based) Δa with **special and general relativity** is ensured by way of: (1) each supplementary acceleration/gravitational field's rotating sinusoidal perturbation having a (universally) *position invariant* constant 'amplitude' — i.e. (at any given 'universal moment') the maximum possible amplitude (Δa) is the same everywhere — although changes to Δa will propagate 'outwards' at the speed of light; and (2) by way of the secondary/supplementary Δa field amplitude being envisaged as a different (and independent) type of energy 'draw-down'⁴⁵³ — such that a second class/type of non-Euclidean geometry (involving space-time cf. spacetime) can coexist with GR's gravitation ('in its own right')⁴⁵⁴.

The three major equalities of this paper are firstly, Equation 12 (noting that $\Delta g_{\text{field}} \approx \Delta a_{\text{field}}$ at a body):

$$a_p = \overline{a_p(t)} = \sqrt{\sum (\delta a_{\text{proper}})_i^2} = \sqrt{\sum (\Delta a_{\text{field}})_i^2}$$

representing the average superposition-based overall/resultant effect of all the fixed and constant amplitude (i.e. time invariant and position invariant amplitude) gravitational/accelerational space-warp fields *upon* spacecraft (and low mass body) acceleration; i.e. the (model's) path-based constant *anomalous Pioneer 'deceleration'* value — *relative* to predictions that do not include the supplementary (RSW) field type. Each individual (field-based) Δa 'equals' and corresponds to its respective (and consequent) spacecraft based δa (speed *shortfall* rate) value (see section 3.2). Further, we note that each δa is equal to (and 'derived' from) a (single cycle) anomalous speed loss (δv) 'divided by' its (respective) sinusoidal period/duration (Δt).

Secondly, we have Equation 16 — rewritten here with some minor nomenclature modifications — which is applicable to a *single* (barycentre/Sun-planet-moon) 'gravito-quantum' rotating space-warp system:

$$\Delta E_{\text{qm}}^{\text{Vi}} = \left[\frac{1}{2} \hbar (\Delta t)^{-1} \eta \right] N_m = \Delta E_{\text{gr}}^{\text{Re}} = \frac{1}{2} \Delta a^2 \Delta t^2 m_1^*$$

It describes the (irreversible) external *re-expression* of: a *non-inertial* additive-based ($\times N_m$ atoms/molecules) *virtual* fractional QM spin (exact) energy (i.e. $\Delta E_{\text{qm}}^{\text{Vi}}$), (conjointly) as the product of: (1) a *real/actual* acceleration/gravitational field perturbation-based specific energy — with this specific energy proportional to perturbation/'wave' *amplitude* ($\Delta a = \Delta a_w = \Delta a_o \eta$) *squared* — and (2) an initial (spherical surface-based) distributed non-local mass value [m_1^* , i.e. $m^*(r)$ at $r = r_1$]. We note three things: (a) the (quantum-based) efficiency factor: $0 \leq \eta \leq 1$; (b) that with a *process* based Δt term on both sides of the equality, some form of simultaneity is required — implicating the presence of quantum entanglement and non-locality; and finally, (c) that with constant \hbar , and Δt , N_m and η effectively fixed (over human time scales i.e. decades), the Δa values are also 'effectively' fixed.

⁴⁵²These include: non-Euclidean geometry; (quantum mechanical) geometric phase; and classical geometry — (the latter) in connection with celestial orbits and kinematics.

⁴⁵³As compared to GR's 'draw-down' situation — mentioned near the end of section 8.2 [bullet points d) and e)].

⁴⁵⁴This ensures that the aforementioned virtual energy to real energy 're-expression' does not create new energy.

Thirdly, Equation 20: $m_r^* V_r = \text{constant} = m_1^* V_1$ describes m^* “continuity”, via the product of (QM entanglement based) non-local mass and (enclosed) spherical volume, relative to establishment/inception conditions (at the “reference radius” r_1). This relationship⁴⁵⁵ ensures that the (non-local) field energy $[\Delta E_{\text{gr}}^{\text{Re}}(r)]$, at any given point/radius in the ‘far’-field (where $r > r_1$), diminishes (i.e. undergoes dispersion) away from the (lunar) source region. Note that the initial non-local mass value (m_1^*) exhibits the idiosyncrasy of (concurrently) being *both*: a contributor to systemic *total* ‘gravitational’ energy (in $\Delta E_{\text{gr}}^{\text{Re}}$), and a local value (i.e. a point-like instantiation value) that is *homogeneously distributed* over/upon a (lunar-centered) spherical surface (of radius r_1).

The model has at least four similarities or parallels to magnetism (see subsection 6.2.10). In particular, we note that a (further) *simplifying* aspect of the model is the *common* (spin-based) geometric phase offset and (common) spin axis orientation applicable to *every* constituent fermion ‘particle’ within (the atoms and molecules constituting) a particular moon. This situation is dependent upon *externally* imposed (barycentre/Sun-planet-moon) orbital and lunar spin characteristics, and is ‘bookended’ by the appeasement of any ensuing *virtual* rate of intrinsic angular momentum (i.e. spin energy) imbalance as a single (real) *external* rotating space-warp (RSW) and conjoint (external) non-local mass distribution. *Importantly*, this re-expression process is demanded/facilitated by (the ‘rigidity’ or inflexibility of) *quantised* electromagnetic spin-orbit coupling within atomic and molecular systems — when confronted by a (relative) geometric phase offset that is $< 2\pi$ radians (i.e. less than half a fermion wavelength).

Furthermore, with *geometry* — both pure/classical geometry (via barycentre/Sun-planet-moon celestial orbits) and a (quantum mechanical) geometric phase offset angle — playing a vital role in the model, the introduction of a fixed “universal constant” (reference) *angle*: $\phi = \tan^{-1}(8\pi)^{-1} \approx 2.28^\circ$ was necessary — in order to ‘best-fit’ the (Pioneer spacecraft based) experimental/observational data to a model. With regard to each (and every) particular Sun-planet-moon (gravito-quantum) system, this angle is essential in determining both: the efficiency factor (subsection 6.3.5) and the reference radius $r_1 = 8\pi r_o$ (subsection 6.5.3) mentioned in relation to Equation 20 (above)⁴⁵⁶, as well as the optimum and weighted acceleration amplitudes, i.e. Δa_o and Δa_w respectively (see subsections 6.5.2 and 6.5.3). This (fixed/universal-constant) reference angle is also used to determine the different Sun-planet-moon (atomic/molecular, and fermion based) (relative) geometric (spin) phase offsets $[(\beta)_i]$ — common to all atoms/molecules, and their constituent elementary fermion particles, within a specific (and ‘geometrically’ appropriate) moon. Each (lunar based) spin phase offset (β) determines (and corresponds to) its particular efficiency factor (η). This (‘final’ or post-orbit) virtual/fractional spin-phase off-

set — *relative* to initial spin phase (i.e. at lunar orbital loop commencement) — is indicative of an (‘over-spin’ or) ‘*precession*’ of the background inertial (spin) frame — *relative* to the actual unchanged (electromagnetic force constrained/dominated) atomic/molecular spin-orbit configuration. Ultimately, this relative (and virtual/latent) spin frame precession arises from (lunar/third-body) geodesic motion in curved space-time (over closed loop/orbital duration Δt). For a pi radian (i.e. one quarter fermion wavelength) phase offset $\eta = 1$, whereas $\eta = 0$ for both a zero/null radian ($\beta = 0$) and $\beta \geq 2\pi$ radian (relative) geometric spin phase offsets. A triangle function (interpolation) is seen to apply between these three (phase offset vs. efficiency factor ‘coordinate’) values (recall Figure 9).

A central physical relationship of the model — applying to each individual/single Sun-planet-moon system — is the inter-relationship existing between (virtual/fractional) quantum mechanical intrinsic angular momentum (i.e. spin): $\frac{1}{2}\hbar\eta = \frac{1}{2}\hbar_w$ and (ensuing, real) RSW-based perturbation amplitude $\Delta a_o\eta = \Delta a_w$. Subsection 6.5.6 argued that this angular momentum to (gravitational) acceleration relationship is a new and unique physical relationship. Facilitating our understanding of the physical ‘connection’ existing between these two physical ‘quantities’, we employ the *conceptual* notion of (a relative) *twist*, in the sense of both a (spin) *turning* and a (space) *warping* respectively, as a conceptual bridging device — noting that at a *physical* level the first of these (two) conceptualisations is particularly inappropriate. Thus, a *virtual* (QM spin phase based) *turning* (or precession) relative to an actual electromagnetic force dictated/constrained QM spin (and orbital) configuration — and achieved over a (lunar celestial spin and) orbit-based process time Δt — is physically linked/related to a space *warping* (acceleration/gravitational) perturbation that rotates (also) with period Δt , and in the same sense/direction as lunar spin. Note that the ‘gravitational’ equilibrium (i.e. unperturbed) condition or state, and a moving body’s unperturbed *kinematic* condition/state, are determined by standard gravitational theorisation and the (conventional) gravitational sources it encompasses. This physical ‘twist’ relationship may be (additionally) considered as a second type/face of spin entanglement — albeit involving a (spin) *energy transmutation* into (a new type of) ‘gravitational’ field energy — with both ‘sides’ of this relationship dependent upon their own (distinct) macroscopic ‘rotational’ *process* (of equivalent duration/period Δt).

Non-local mass (m^*) is considered to be a second type/class of mass, primarily because its associated acceleration/gravitational field (i.e. a rotating space-warp) does *not* affect *all masses* in an equivalent manner (cf. GR). Non-local mass varies with distance (r) from its source region, diminishing in an inverse-cubed manner (i.e. inverse of the volume enclosed); and subsequently its effect upon a passive (gravitational) mass (m_p) is not distance independent. More precisely, this mass ‘interaction’ is an “all or nothing effect”; “all” if $m^*(r) \geq m_p$ and “nothing” if $m^*(r) < m_p$ (at the body, i.e. at the m_p), as is similarly (and generally)

⁴⁵⁵ Note the contrast with density (ρ) where $\rho = m/V$.

⁴⁵⁶ Where r_o is lunar (elliptical) semi-major axis (length), usually denoted (in standard nomenclature) as a or r_a .

the case with the *energy* based (and EM radiation interacting with matter based) photoelectric effect. This weak equivalence principle defying feature of the new model/mechanism also utilises a point-mass idealisation as regards the ‘interaction’ of m^* (at r) with a compact/condensed m_p (see subsection 6.6.7), albeit for quite different reasons to those applicable in traditional gravitation theory. By its very nature, non-local mass is devoid of both (tangible) matter/‘material’ and hence *local* active gravitational mass⁴⁵⁷.

8.4 A crucial distinction and the model’s broader implications

In this section, further implications and ramifications peripheral to the model (itself) are presented. The discussion takes the form of a series of brief remarks.

Assuming a real and non-systematic based Pioneer anomaly has given us no choice other than: (1) to considerably re-examine the (physically) foundational concepts of time, mass, and energy; and (2) establish a model that is independent of general relativity (GR) but nevertheless (is) physically compatible (and coexistent) with GR. We addressed the misconception that GR’s success, and/or its principle-based approach to a theory of gravitation — i.e. principles/principle of: equivalence, general relativity, and general covariance — denies *all* other sources (and forms) of non-Euclidean geometry (see section 6.7 in particular).

Crucial to this new understanding has been the fundamental distinction drawn by Sir Arthur S. Eddington (and reprised by Matthew Stanley, see subsection 4.7.2) — that quantum mechanics (QMs) and GR provide insight into *how* we see the world (i.e. its epistemological characterisation), rather than *what* the world [entirely] is (i.e. its ontological characterisation). Thus, physical observations and their mathematical/theoretical modelling and understanding do not necessarily provide an unambiguous picture of ‘what’ the world is, free of subtlety and/or oversights.

Eddington’s fundamental conceptual (cf. empirical) distinction facilitated the introduction of the (non-reductive) paired/complementary phenomenal *and* noumenal perspectives proposed herein; particularly its implementation in achieving a supplementary/complementary interpretation of SR’s Lorentz transformations by way of appreciating that: the ‘world’/universe, an observer’s motion/speed, EM radiation/light itself, and observations utilising EM radiation are interwoven — when considered/‘viewed’ from a noumenal perspective. As such, SR’s and GR’s use of “spacetime” and general covariance need not necessarily be understood as simply “how *and* what the world is.” Although there is a reduction in explanatory simplicity, this new approach is not in defiance of Occam’s razor because there is a (more than) compensatory increase in explanatory capability.

⁴⁵⁷ Whether or not *non-local* mass (itself) is an *active* gravitational mass has not been pursued. Being a secondary effect arising from Sun-planet-moon motion it is (most) likely that any (active gravitational) contribution will be very minor, if not entirely negligible; the latter either by way of its cosmological extent or simply via its very different ‘nature’.

Further implications and ramifications of these related (philosophical) epistemological–ontological and (pragmatical/physical) phenomenal–noumenal dichotomies fills out the remainder of this section and parts of section 8.5.

The *unavoidability* of “observer dependence” — regarding measurements made at extremely low energies or momentums, or over extremely short distances or durations — exhibited in microscopic QMs is (herein considered) similarly present in macroscopic physics, but it is exhibited in an altogether different way; i.e. involving an observer’s (relativistic) high speed motion (and substantial specific kinetic energy). This macroscopic observer dependence is downplayed and masked by SR’s phenomenal (spacetime) perspective, whereas it has been unmasked by our new/complementary noumenal (distinct space and time, i.e. space-time) and global/systemic perspective (see section 4.5).

Within a many-body barycentric solar system (for example), the tactile/sensory basis and *historically* problematic hypothesis of a gravitational *force* — acting upon a moving object — was superseded by general relativity’s curved spacetime based gravitation. Note that in a *many-bodied barycentric system*, the use of ‘gravitational accelerations’ (at different points in space) — that apply to all bodies irrespective of their (passive gravitational) mass — has retained its pragmatic validity, e.g. regarding an interplanetary spacecraft’s “orbit determination”. The model’s (unique/specific) implementation of: (1) non-local mass (m^*), which effectively involves a *dematerialisation* of the concept of ‘mass’ (cf. matter); and (2) a (general) scepticism regarding (both) the graviton particle’s existence and the physical reduction of gravitation to a (boson) particle “exchange force” basis; is *arguably* the second-stage of a two-stage process of “gravitational dematerialisation”, if we consider/designate GR’s (pro-curved) “de-forcing” innovation/advancement as the first-stage of this dematerialisation of “gravitation” — in the broadest imaginable sense of the word.

In response to: (1) the question “Can everything be relative?” which distinguishes a (systemic) whole from (relationships involving) its constituent parts; and (2) the observation that spacetime on a cosmological scale is uncurved (i.e. ‘flat’ such that in GR $k = 0$); as well as (3) Section 7’s querying of dark energy; it was proposed (subsection 4.4.3) that whole universe/global curvature ($k = \pm 1$ in GR) may not (actually) be physically realisable. As such, the assumption that (*relativistic*) gravitation has a ‘whole universe’ (or cosmological) *physical* application — as compared to a theoretical implication — is quite conceivably unjustified. Pursuant to this stance, the model proposed (subsection 4.4.10) that in the *idealised* scenario of a complete absence of any mass, momentum and physical field energy in the universe, a homogeneous space ‘substratum’/continuum exhibiting Euclidean geometry would ‘exist’; i.e. an uncurved empty Euclidean space ‘container’. In (actual) reality, this idealised background uniformity/homogeneity is *locally* (but not globally) curved/deformed by its contents (up to the scale of its largest substructures). The (root) moti-

vation and basis for this change in stance has been an embracing of quantum entanglement and quantum non-locality, together with an openness to (and ‘demarginalisation’ of) their ontological implications.

8.5 Big picture ‘meditations’ arising from the model proposed herein

In this subsection, discussion of the implications of the new model is extended to ‘big picture’ topics⁴⁵⁸.

The Nobel laureate Daniel Kahneman, an Israeli-American psychologist noted for his work on judgment and decision-making, states (in an interview⁴⁵⁹) — regarding forming an impression on the basis of the information you have — that:

...we can’t live in a state of perpetual doubt — so we make up the best story possible and live as if the story were true.
...we like the stories to be good stories.

When dealing with a complicated problem, *correctness* differs depending upon the task and its perceived solution. Scientific instrumentation and machinery either work (as planned) or they don’t; similarly, an elaborate computer program either ‘runs’ or it doesn’t, and a mathematical proof or a logical deduction is either valid or invalid. The investigation of a complex scientific circumstance or problem is (predominantly) not as straightforward as in these cases, e.g. star formation. Scientific ‘correctness’ (or truth) in the presence of *conceptual* complexity or intricacy, especially if it is combined with limited/restricted knowledge and/or ambiguous information, is often provisional with its maturity requiring: time, effort, insight, and new or (at least) refined information. It appears that the lack of a ‘timely’ (and generally favoured) explanation for the well credentialed Pioneer anomaly has led to a dispelling of lingering doubt by virtue of embracing a thermal-radiation/heat based explanation, which *is* a viable (and ‘good’) story; although this scenario leaves the well credentialed Earth flyby anomaly (isolated and) in need of a similarly ‘good’ explanation. Herein, we have looked beyond the (psychological) weight of ongoing doubt to both: query the correctness of this favoured explanation, and construct/formulate an alternative model that is unavoidably/necessarily conceptually (cf. mathematically) intricate⁴⁶⁰.

This emphasis upon conceptualisation has necessitated the copious use of both bracketed words and the (forward) slash (or virgule) punctuation character throughout this document, primarily to add detail and/or clarify the meaning of a word, phrase, or sentence — cf. the slash’s (other) “choice between two words” application. Additionally, various punctuation marks, especially the: comma, semi colon, and em

dash (i.e. —), as well as scare quotes (i.e. ‘...’), are liberally employed in order to minimise ambiguity. Note that scare quotes (herein) are used to alert the reader to the non-standard or special use of a word or phrase.

Philosophers of science warn about being overly demanding or prematurely dismissive of a recently formulated theory/model that matches the evidence, especially one that is conceptually ‘different’. In other words, the exactitude demanded of well established theoretical models in physics, as exhibited by mathematical/computational reasoning, should not be equally (or blindly) applied to a recently formulated theory/model⁴⁶¹. Deficiencies will surely exist, but these do not logically entail that the young model is entirely ‘wrong’ or unworthy of ongoing consideration.

To simply proclaim that general relativity, because of its success, denies the Pioneer anomaly is unjustified and hence unscientific (although understandable); whereas a conceptual enriching and supplementation of physical concepts, driven by a creditably established and striking observation, (although possibly flawed) *is* scientific — especially if the model/mechanism proposed can be falsified, and encompasses all the Pioneer anomaly’s awkward observational constraints. Herein, the latter were not easily ‘fitted’, e.g. by way of (mere) theory modification; and a valid (and progressive) model’s fit to the evidence is necessarily complicated/difficult, in so much that physical reality/ontology and fundamental physical concepts have required (innovative) modification. The physical model/mechanism established herein supports the saying/proverb that: “truth is stranger than fiction⁴⁶²”, in the sense that (regarding the explanation for the Pioneer and Earth flyby anomalies proposed herein):

‘*a posteriori* scientific knowledge’ driven by a striking new scientific result⁴⁶³ is stranger than and beyond any ‘*a priori*’ reality or theory conceivable by means of human reason and imagination alone⁴⁶⁴.

This point of view is supported by the model’s use of virtual (relative) geometric phase and the model’s *introduction* of: non-local mass; (real) constant amplitude sinusoidal perturbations upon the pre-existing relativistic gravitational field; and a latent ontological ‘phenomenal–noumenal’ complementarity; with the latter somewhat debasing the centrality of the ‘observer’ (and physicists) as regards achieving progress towards a further/deeper understanding of physical reality — e.g. quantum entanglement (and causation).

The reversal of (an observer’s) *locational* perspective arising from Nicolaus Copernicus’ seminal (16th century) celestial motion proposal has recently been revisited with the embrace of the ‘multiverse’. A number of cosmologists (and theoretical physicists) now be-

⁴⁵⁸In the sense of crucial features, facts and issues that contribute to mankind’s understanding and explanation of the physical world/universe (that we ‘find’ ourselves in).

⁴⁵⁹Liz Else, New Scientist, Vol. 212, No. 2839, pp. 34–35, “Nobel psychologist reveals the error of our ways”.

⁴⁶⁰This approach appears to be anathema to some theoretical physicists, particularly when the complex conceptual discussion is in a language other than their first language.

⁴⁶¹The (“intricate defensive”) strategy of continually “raising the bar” of acceptance for a new theory/approach, is (ultimately) symptomatic of preferring the *status quo*.

⁴⁶²The genesis of this saying is (commonly) attributed to: Lord Byron’s poem *Don Juan*, Canto XIV (1823).

⁴⁶³That is, a new physical explanation deduced/derived from a new and anomalous scientific experimental finding.

⁴⁶⁴Inclusive of mathematical deduction and hypotheses, and drawing upon all (pre-discovery) scientific knowledge.

lieve that we only observe (figuratively speaking) “the tip of the (cosmological) iceberg”, with the rest (i.e. other universes) hidden — in the sense of *never* being directly amenable/available to observations. It has been proposed herein that something similar is occurring with respect to ‘time’ (recall subsection 4.4.8)⁴⁶⁵. Interrelated to this *temporal* aspect — but occurring in the classical Copernican “*reversal* of perspective” sense — is an energetics/energy aspect, that pertains to a body’s specific energy (see subsection 4.5.3). Recalling section 8.2, this reversed (specific energy) perspective is dependent upon a description of quantum entanglement based situations/phenomena requiring the prioritisation of a (complementary) noumenal perspective over the standard (amenable to scientific measurements/observations) ‘phenomenal’ perspective.

The uniqueness of the model/mechanism hypothesised herein (implies and) is illuminated by a (heuristic) two-by-two categorisation/bifurcation of the laws of nature into four main categories/classes. The first bifurcation involves a distinction pertaining to laws describing (quantum mechanical) microscopic ‘reality’ as compared to macroscopic reality, with the largest molecules demarcating the extent of the microscopic realm/domain (recall subsections 4.2.6 and 4.2.7). Note that electromagnetism has distinct laws in each domain, and that a total physical micro-macro separation is not being espoused. The second bifurcation involves a distinction between local (i.e. speed of light limited) and non-local based processes/laws. Clearly, all (known) physical laws, and almost all (known) physical phenomena — except for quantum entanglement and quantum non-locality — have a local basis cf. a non-local basis; with often neglected (engineering based) fluid mechanics and the mechanics of solids lying in the local macroscopic class of physical laws. The (unique and boutique) model/mechanism proposed herein involves a (repeating) cyclic *process* that spans the *non-local* microscopic-macroscopic ‘divide’ or bifurcation (previously articulated). By way of the exemplar of aircraft propellers and horizontal-axis wind turbines, subsection 4.2.7 sought to show that a force (and/or energy) basis to local macroscopic physics, as is the case with local microscopic physics, is not a fail-safe (core and/or working) assumption. When a vortex/circulation based theoretical (potential flow) approach is used to describe this linear *and* rotational mechanism (in its three-dimensional entirety), the ‘physical quantity’ of power (in addition to momentum, pressure, force and energy) is crucial/necessary. Consequently, and notwithstanding the unquestionable validity of the (three force) unification of: the strong force, the weak force, and electromagnetism, a four *force* unification (i.e. theory of *everything*) unification agenda is quite conceivably misguided (recall sections 4.2 and 4.3). This claim was borne out in the model by the virtual offset of (geometric phase and) intrinsic angular momentum (i.e. QM spin), applicable to each and every elementary fermion particle ‘comprising’ an atom or molecule, being *equivalent* to the

spin offset of the atom/molecule in its (systemic) entirety, whereas the total QM intrinsic angular momentum (offset) associated with an entire moon is determined by way of an additive *summation* of the (equal) spin (offset) values of its multitudinous ‘constituent’ atoms and molecules.

A second ‘story’ that physicists currently seek to (and do freely) tell — in addition to a preferred thermal-radiation/heat (or thermal reaction force) based explanatory story/account of the Pioneer anomaly — relates to the future direction of physics and its ultimate goals. This (‘big picture’) challenge involves dealing with what is colloquially referred to as “a final theory”. A projected solution, and generally considered ‘good story’, is the often mentioned/discussed: string theory, and the more (recent and) elaborate M-theory — requiring 11 cf. 10 spatial dimensions and incorporating supersymmetry — espoused by a majority of theoretical physicists. If this paper’s explanatory story and findings (both ‘physical’ and conceptual) are correct, it follows that this (almost unassailable) unification (and reductive) agenda will (continue to) suffer stagnation as regards its experimental confirmation and (to a lesser extent) its predictive ability. In hindsight, the pursuit of a reductive four-force unification goal, although understandable by way of historical unification successes, is seen (in its current form) as somewhat blinkered and too prescriptive. Additionally, dark matter and dark energy are not integrally (corralled and) woven into this agenda, and the “make up” of both these entities has (to date) not been assuredly determined. In light of this current “dark daze” and dubious “theory of everything” progress, to suggest a more generalist approach to physical reality (in its widest sense) — particularly involving/incorporating the stronger elements/aspects of philosophical practice — is not unreasonable.

Bearing in mind that “certainty is [sometimes] the enemy of progress”, the phenomenal–noumenal distinction (and its complementarity) proposed herein is at risk of encountering resistance and delay (within physics); arguably/possibly similar to that which befell an appreciation of the usefulness of the number “zero” (and the subtle concept of nothing) in some cultures in the past (Kaplan 2000; Seife 2000). Zero, in the sense of “the absence of a number”, somewhat parallels the conceivable existence of a ‘noumenal physicality’ that is not susceptible to *direct* physical observation. By way of quantum probability and entanglement, and (indirectly via) Bell’s inequality, the existence of non-local (hidden) variables (and processes) has been entertained for many years. Furthermore, nothingness⁴⁶⁶, abstract objects, (Kant’s version and other ‘takes’ on) the phenomenal–noumenal distinction, and continental philosophy in general (cf. analytic philosophy) are examples of ongoing active areas of philosophical investigation; and it is not unreasonable to presume that the elaborate explanation of the Pioneer anomaly proposed herein has partially tilled the ground of this *potentially* rich field of indirect (analytical) ‘scientific’ investigation.

⁴⁶⁵ As was the case in Section 8.2, “time” — for want of a better word — is (occasionally) being used in an extended and ‘beyond’ (direct) measurement manner.

⁴⁶⁶ See (for example) the Stanford Encyclopedia of Philosophy, <http://plato.stanford.edu/entries/nothingness/>

8.6 A closing conditional comment

Lastly, in this “summary, discussion and conclusions” Section (i.e. Section 8), some retrospective remarks are made and then a conditional scenario is put forward.

It was totally unexpected at the outset of this research project that a new scientific model designed specifically to account for the Pioneer anomaly’s (solar system based) awkward observational constraints — by way of introducing two separate cosmological scale/size mechanisms⁴⁶⁷, i.e. (gravito-quantum) rotating space-warps (RSWs) and non-local mass distributions (NMDs) — could conceivably also resolve or at least enlighten (via two distinct cosmological foreground effects) both the cosmic microwave background (CMB) anomalies and the dark energy issue (by way of the RSWs and NMDs respectively). The model, within which energy and geometry feature prominently (cf. force and position coordinates) also produces a viable explanation of the (unresolved) Earth flyby anomaly, whilst imposing (only) very minor changes to the solar system’s historical behaviour (i.e. its early history and subsequent evolution) — because only ‘low’ mass bodies are anomalously/additionally affected. Furthermore, the model’s conceptual structure casts doubt upon (both) a theory of everything unification agenda and the veracity of the dark matter hypothesis; the latter (hypothesis) in the sense that it assumes gravitational theorisation is ‘complete’, and thus any observed transgression cannot be gravitationally-real — e.g. “flat galactic rotation curves”.

The broad scope of the model proposed in this treatise — which involves reconsideration (to some extent) of: time, mass, quantum entanglement, QM (relative) geometric phase, and physical ‘reality’ (in general) — means that the model’s very existence raises a (big picture) concern. The nature of this concern is that: *if* the new model is viable/correct (i.e. “on the right track”) *then* to ignore its progressive implications and ramifications — e.g. by way of antipathy towards deep (philosophically inclined) reasoning, or by way of preferring a systematic based explanation of the Pioneer anomaly — does more than simply maintain the *status quo*. The downside risk of overlooking the model’s ability to be a catalyst for change/action, is that an (ultimate) form of (scientific) conceptual stagnation might be unnecessarily perpetuated, in so much that ongoing and future deliberations (specifically) aimed at establishing the conceptual and physical *cruce* of (some or all of) these (six) aforementioned issues/anomalies — particularly the (seemingly unrelated) CMB anomalies and the (what is) dark energy issue — would effectively be misappropriating time, effort, and resources; notwithstanding: the new information to be (surely) gleaned by these efforts, the erudition and skill of the participating scientists, and the complexity involved in performing the research.

Putting to one side this possible counterfactual and meta-scientific discussion, this research has been primarily a comprehensive investigation of direct and peripheral evidence pertaining to the Pioneer anomaly.

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⁴⁶⁷With the two (three dimensional) scalar fields associated with these mechanisms both extending to ‘infinity’.

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